The Official Guide for GMAT Review, 1lth edition
Supplementary Explanations for the Data Sufficiency Sample Questions
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Please read this first

I make no guarantees as to your results from using the methods in these explanations.

Most of the methods described in these explanations will work well for most test-takers in most situations, but few methods are perfectly applicable to all situations or all test-takers. Use your judgment.

On the other hand, if you find a method in these explanations challenging to apply, don't give up on it too quickly—give it a fair shot. It is natural for *any* new technique to initially feel unfamiliar and time-consuming. If you keep using the same techniques on the GMAT that you're *already* comfortable with, you can expect to keep getting the same score! If you would like to *change* your GMAT score, the only way to do so is to *change* your GMAT techniques—even though at first any new technique is likely to feel uncomfortable and to slow you down.

This document provides supplementary explanations to some of the Data Sufficiency Sample Questions in *The Official Guide for GMAT Review*, 11th ed. The *Official Guide* can be purchased from amazon.com at this link: http://www.amazon.com/gp/product/0976570904/sr=8-1/qid=1154718341/ref=pd_bbs_1/002-6026248-8506436?ie=UTF8
The *Official Guide* provides explanations to all the problems in the *Guide*. However, many of those explanations are insufficient (no pun intended) to give you insight into how to attack GMAT math and Data Sufficiency **systematically**. The explanations in this document should be used as a supplement to rather than a substitute for the explanations in the *Official Guide*. For problems or statements for which the explanation provided by the *Official Guide* is adequate, no further explanation is provided here.

These explanations illustrate the techniques described in the "GMAT Data Sufficiency Manual", and therefore these explanations should be used in conjunction with that manual. The manual is available at http://www.geocities.com/freelanceteacher. Terms in this font are described in more detail in the manual—use the index.

You may have found some of the problems explained in this document to be easy. Even so, I encourage you to study these supplementary explanations carefully. The explanations will give you systematic methods and notation that will serve you well on more challenging problems.

Abbreviations used in these explanations:

? = Question	(S1) = statement 1 alone	CN = choosing numbers
G = given	(S2) = statement 2 alone	√ = consistent with the givens
S = sufficient	(tog) = both statements together	✕ = inconsistent with the givens
I = insufficient		FA = free answer

1.

?: how much is 20 percent of a certain number?	Begin by identifying the Question. (Notice that this problem has no initial givens.)
?: $.2x$	Rephrase the question as a mathematical expression. 20% = .2 "of" = multiply We can use x to stand for the "certain number." Write down this mathematical expression, and label it with a *?*, as shown.

(S1) G: 10% of the number is 5	Start with the easier statement, statement one. Write down *S1* to remind yourself that you're working on statement one. Remember that the statement represents a given. There is no initial given, so the only relevant given is statement one.
(S1) G: $.1x = .5$ ⇒ x = one number ⇒ $.2x$ = one number ⇒ (?: one answer)	Rephrase the statement as an equation; write the equation down, and label it with a *G*. 10% = .1; "of" = multiply; "the number" is x "is" = equals We **could** solve this equation for x; the equation will have exactly *one* solution. Therefore, there will be exactly *one* value for $.2x$. Therefore, there will be exactly *one* answer to the Question which is consistent with the relevant given. Therefore, statement one is sufficient.
S (S1) G: $.1x = 5$ ⇒ x = one number ⇒ $.2x$ = one number ⇒ (?: one answer)	Write down an *S* to remind yourself that statement one is sufficient. The complete systematic scratchwork for statement one is shown at left.

Notice that it would be a waste of time to actually **solve** the equation. All we have to do is determine that it will have only **one** solution. To save time on data sufficiency, avoid unnecessary work. You don't need to *determine* the answer to the Question; you just need to determine whether there's *more than one* answer to the Question.

?: .2*x*

(S2) G: 40 percent of twice the number is 40	Now evaluate the other statement, statement two. The only relevant given now is statement two. **Ignore** statement one. Although statement one is sufficient, we did not actually figure out the one answer which is consistent with statement one; therefore, we do *not* have a free answer to use for statement two.
(S2) G: .4 (2*x*) = 40	Write down *(S2)* to remind yourself that now you are working on statement two. Rephrase the statement as an equation; write the equation down, and label it with a *G*. "twice the number" = 2*x* 40% = .4; "of" = multiplied by; "is" = equals
S (S2) G: .4 (2*x*) = 40 *x* = one number ⇒ .2*x* = one number ⇒ ?: one answer	We *could* solve this equation for *x*; the equation will have exactly *one* solution. Therefore, there will be exactly *one* value for .2*x*. Therefore, there will be exactly one answer to the Question that is consistent with the relevant given. Therefore, statement two is sufficient. Write down an "S" to remind yourself that statement two is sufficient. The complete systematic scratchwork for statement two is shown at left.

Use 12TEN. Since each of the statements are sufficient alone, we can now pick choice D; there is no need to evaluate the statements together.

As with statement one, it would be a waste of time to actually **solve** the equation in statement two. It's enough to see that it will have exactly **one** solution. On data sufficiency, avoid needless work.

Here is the complete systematic scratchwork for #1:

?: .2*x*

S (S1) G: .1*x* = 5 ⇒ *x* = one number ⇒ .2*x* = one number ⇒ ?: one answer	S (S2) G: .4 (2*x*) = 40 *x* = one number ⇒ .2*x* = one number ⇒ ?: one answer

You might have found that this question was easy enough that you did not need to be so systematic in your scratchwork. Study the systematic scratchwork carefully anyway, as you will find it useful on more challenging problems.

MORALS

(1) Try to rephrase the Question and givens into mathematical expressions and equations. (2) Avoid needless work on data sufficiency.

ADDITIONAL NOTE

For completeness, here is how to actually solve the equations in statements one and two.

(S1) G: $.1x = 5$	
$x = 5 \div .1$	Solve for x by dividing both sides by .1
$.1\overline{)5}$	
$.1\overline{)5.0}$	We cannot divide by a decimal, so get rid of the decimal by moving the decimal point for .1 one place to the right. To make this adjustment legal, we have to move the decimal point for 5 one place to the right as well.
$1\overline{)50}$ = 50 $x = 50$ $.2x = .2(50) = 10$?: 10	

(S2) G: $.4\ (2x) = 40$ $.8x = 40$	
$x = 40 \div .8$	Solve for x by dividing both sides by .1
$.8\overline{)40}$	
$.8\overline{)40.0}$	We cannot divide by a decimal, so get rid of the decimal by moving the decimal point for .8 one place to the right. To make this adjustment legal, we have to move the decimal point for 40 one place to the right as well.
$8\overline{)400}$ = 50 $\Rightarrow x = 50$ $.2x = .2(50) = 10$?: 10	

Notice that the one answer that is consistent with statement one ("10") is also the one answer that is consistent with statement two. This is how GMAT data sufficiency will **always** work: *If each statement is sufficient alone, then the one answer that is consistent with statement one will be the same as the one answer that is consistent with statement two.* This is simply a fact about how the GMAT is written.

2.

Part: flour Part: baking powder Whole: mix	This is a "parts and whole" problem. Identify the two parts and the whole
?: $\frac{\text{baking powder}}{\text{flour}}$	Identify the Question. Rephrase the Question as a mathematical expression. Write down the expression, and label it with a *?*. The question is asking for a part:part ratio, not a part:whole ratio.

(S1) G: When mix = 10, flour = 9.9	Write down *S1* to remind yourself you are working on statement one. Statement one gives us the whole and a part.
S (S1) G: When mix = 10, flour = 9.9 ⇒ baking powder = one number ⇒ $\frac{\text{baking powder}}{\text{flour}}$ = one number ⇒ (?: one answer)	Since we are given the whole and one of the parts, we **could** determine the other part (baking powder), which will have exactly *one* possible value. So there will be exactly *one* possible value for the part:part ratio. So there will be exactly one answer to the Question which is consistent with all the relevant givens. So statement one is sufficient.

(S2) G: When mix = 30, baking powder = .3	Now we move on to statement two. Ignore statement one. Although statement one is sufficient, we did not actually figure out the one answer that is consistent with statement one, so we don't have a free answer. Statement two gives us the whole and a part.
S (S2) G: When mix = 30, baking powder = .3 ⇒ flour = one number ⇒ $\frac{\text{baking powder}}{\text{flour}}$ = one number ⇒ (?: one answer)	Since we're given the whole and one of the parts, we *could* determine the other part (flour), which will have exactly *one* possible value. So there will be exactly *one* possible value for the part:part ratio. So there will be exactly one answer to the Question which is consistent with all the relevant givens. So statement two is sufficient.

12TEN: Each of the statements is sufficient alone, so we can now pick choice D without evaluating the statements together.

Here is the complete systematic scratchwork for #2.

Part: flour
Part: baking powder
Whole: mix
?: $\frac{\text{baking powder}}{\text{flour}}$

S (S1) G: When mix = 10, flour = 9.9	S (S2) G: When mix = 30, baking powder = .3
⇒ baking powder = one number	⇒ flour = one number
⇒ $\frac{\text{baking powder}}{\text{flour}}$ = one number	⇒ $\frac{\text{baking powder}}{\text{flour}}$ = one number
⇒ (?: one answer)	⇒ (?: one answer)

This question was easy enough that you might not have needed to be so systematic in your scratchwork. Study the systematic scratchwork carefully anyway, as you will find it useful on more challenging problems.

Notice that for both statements, we did not figure out the **answer** to the Question. We just figured out that there was **only one** answer to the Question. Actually figuring out the answer to the Question would be a waste of time. Avoid unnecessary calculations on data sufficiency.

Explanation to #2 continued on next page …

For completeness, here is how to actually figure out the answer to the Question.

(S1) G: When mix=10, flour = 9.9 baking powder = mix - flour $= 10 - 9.9$ $= .1$ $\Rightarrow \frac{\text{baking powder}}{\text{flour}} = \frac{.1}{9.9}$ $= .1 \div 9.9$ $= 1 \div 99$	You cannot divide by a decimal, so shift the decimal point in 9.9 one place to the right (converting "9.9" into "99"); to make this adjustment legal, you have to shift the decimal point in .1 one place to the right as well (converting ".1" into "1").
$\Rightarrow \frac{\text{baking powder}}{\text{flour}} = \frac{1}{99}$ $\Rightarrow$ (? : $\frac{1}{99}$)	

(S2) G: When mix=30, baking powder = .3 flour = mix - baking powder $= 30 - .3$ $= 29.7$ $\Rightarrow \frac{\text{baking powder}}{\text{flour}} = \frac{.3}{29.7}$ $= .3 \div 29.7$ $= 3 \div 297$	You cannot divide by a decimal, so shift the decimal point in 29.7 one place to the right (converting "29.7" into "297"); to make this adjustment legal, you have to shift the decimal point in .3 one place to the right as well (converting ".3" into "3").
$\Rightarrow \frac{\text{baking powder}}{\text{flour}} = \frac{3}{297}$ $= \frac{1}{99}$ $\Rightarrow$ (? : $\frac{1}{99}$)	Reduce $\frac{3}{297}$ by cancelling a three from the numerator and denominator. $3 \div 3 = 1$ and $297 \div 3 = 99$.

Notice that the one answer which is consistent with statement one ("$\frac{1}{99}$") is also the one answer which is consistent with statement two. This is how GMAT data sufficiency will **always** work: *If each statement is sufficient alone, then the one answer that is consistent with statement one will be the same as the one answer which is consistent with statement two.* This is simply a fact about how the GMAT is written.

3.

?: $\|x\|$	
(S2) G: [a] $x^2 = 4$	Start with the easier statement, statement two. The only relevant given is statement two.
[b] $x = 2$ or -2	Solve [a] for x.
CN $x = 2$	Choose numbers. 1. Choose numbers. Write down your number. Label your number by writing down the variable you are setting equal to the number.
CN $x = 2$ √	2. Check whether your number is consistent or inconsistent with all the relevant givens. We already know from [b] that x=2 **is** consistent with the given, so there's no need to plug it back into [a]. Make a √ to indicate that our number *is* consistent with all the relevant givens.
CN $x = 2$ √ $\|x\|=\|2\|=2$	Our number is consistent with the relevant given, so move on to step 3. 3. Use your number to answer the Question. Label your work. Don't write "|2|=2"; instead, write "$\|x\|=\|2\|=2$".
CN $x = 2$ √ (?: 2) $\|x\|=\|2\|=2$	We have found that, when x=2, the answer to the Question is "2". Write down the answer to the Question. Label the answer to the Question with a *?*.
CN $x = -2$	4. Choose another number, trying to get a different answer than "2"; back to step 1. 1. Again, write down the number and label it with a variable.
CN $x = -2$ √	2. Check whether your number is consistent or inconsistent with all the relevant givens. We already know from [b] that $x = -2$ is consistent with the given, so there is no need to plug it back into [a]. Make a √ to indicate our number *is* consistent with all the relevant givens.
CN $x = -2$ √ $\|x\|=\|-2\|=2$	Our number is consistent with the relevant given, so move on to step 3. 3. Use your number to answer the Question. Label your work. Don't write "| -2 |= 2 "; instead, write "$\|x\|=\|-2\|=2$".
CN $x = -2$ √ (?: 2) $\|x\|=\|-2\|=2$	We have discovered that, when $x = -2$, the answer to the Question is "2". Write down the answer to the Question; label the answer to the Question with a *?* and circle it.
S (S2) G: $x^2 = 4$ $x = 2$ or -2 CN $x = 2$ √ (?: 2) $\|x\|=\|2\|=2$ CN $x = -2$ √ (?: 2) $\|x\|=\|-2\|=2$	From [b] we know that there are no other numbers that are consistent with statement two. We have gotten *exactly one* answer to the question ("2") which is consistent with all the relevant givens, so statement two is sufficient. Shown to the left is the complete systematic scratch work I would recommend for choosing numbers to evaluate statement two.

?: |*x*|

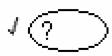
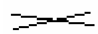
(S1) FA (?: 2)	Now move on to statement one. Write down *S1* to remind yourself that you're working on statement one now. Now the only relevant given is statement one; *ignore statement two*. Since statement two was sufficient, the one answer which was consistent with statement two ("2") is a free answer which we automatically know must also be consistent with statement one. Write down this free answer; label it with a *?* and circle it.
G: $x = -\|x\|$	Write down statement one; label it with a *G*.
CN $x = 0$	Choose numbers. 1. Choose numbers. We already have one answer to the Question ("2", our free answer). Now we are trying to get a *different* answer, so there is no point choosing *x*=2 or -2. Instead, let's choose 0, which is usually the first number you should choose on Data Sufficiency. Write down your number. Label the number with the variable you are setting equal to the number.
CN $x = 0$ $-\|x\| = -\|0\| = -0 = 0$	2. Check whether your number is consistent or inconsistent with the all the relevant givens. Continue to label your work. Don't write down "-|0| = -0 = 0"; instead, write "-|*x*| = -|0| = -0 = 0".
CN $x = 0$ √ $-\|x\| = -\|0\| = -0 = 0$	Make a √ to indicate that our number *is* consistent with all the relevant givens.
$x = 0$ √ $-\|x\| = -\|0\| = -0 = 0$ $\|x\|=\|0\|=0$	Since our number is consistent with all the relevant givens, we move on to step three. 3. Use your number to answer the Question. Continue to label your numbers. Don't write down "|0|=0"; instead, write "|*x*|=|0|=0".
CN $x = 0$ √ (?: 0) $-\|x\| = -\|0\| = -0 = 0$ $\|x\|=\|0\|=0$	We have discovered that, when *x*=0, the answer to the Question is "0". Write down the answer to the Question; label the answer to the Question with a question mark.
I (S1) FA (?: 2) G: $x = -\|x\|$ CN $x = 0$ √ (?: 0) $-\|x\| = -\|0\| = -0 = 0$ $\|x\|=\|0\|=0$	Normally, we would have to choose at least two numbers. However, because of our free answer, we've already gotten two different answers ("2" and "0") so we can see that statement one is insufficient without choosing any more numbers. Write down an *I*. Shown to the left is the complete scratch work I would recommend for evaluating statement two.

12TEN: Only statement 1 is sufficient, so the correct choice is A; there is no need to consider the statements together.

Here is the complete recommended systematic scratchwork for #3 (remember that we evaluated statement two first):

?: $|x|$

I (S1) FA (?: 2)	S (S2) G: $x^2 = 4$
G: $x = -|x|$	$x = 2$ or -2
CN $x = 0$ ✓ (?: 0)	CN $x = 2$ ✓ (?: 2)
$-|x| = -|0| = -0 = 0$	$|x|=|2|=2$
$|x|=|0|=0$	CN $x = -2$ ✓ (?: 2)
	$|x|=|-2|=2$

Check your notation. The only correct notation for choosing numbers for data sufficiency is:  or .

All our notation was correct. If any of our notation did *not* match one of these two patterns, we would know that we had made a mistake. For example, suppose we had tried choosing x=0 for statement two, and suppose that our notation for statement two had looked like this:

$$x=0 \quad ?:\ 0$$

Hopefully we would see that we'd forgotten to check whether our number was consistent or inconsistent with all the relevant givens.

We have provided a detailed description of how to solve #3 by choosing numbers, carefully illustrating all the steps and notation in the systematic choosing numbers method.

You may feel that statement two is so easy that you don't need to systematically choose numbers to evaluate it. Even so, please study the explanation carefully. It will give you a clearer idea of the systematic steps and notation for choosing numbers, which will serve you well on more challenging problems.

By the way, here is a different way to solve the equation in statement two:

$x^2 = 4$

$\Rightarrow\ x^2 - 4 = 0$

$\Rightarrow\ (x + 2)(x - 2) = 0$

$x+2=0$ or $x-2=0$

$x = -2$ or $x=2$

Here is an algebraic approach to statement one:

(S1) G: $x = -\|x\|$ $\Rightarrow$ $-x = \|x\|$	multiplying both sides by -1
$\Rightarrow$ [a] $\|x\| = -x$	reversing the equation
$\Rightarrow$ $x \leq 0$	For any positive x, $\|x\|=x$. For any nonpositive x, $\|x\| = -x$. So [a] tells us that x is nonpositive.

Statement one says that $|x|$ is *opposite* in sign to x. But positive numbers always have the *same* sign as their absolute value. So statement one is just a fancy way of telling us that x is nonpositive.

So x can be any nonpositive number. So there are many different possible values for $|x|$. So there are many different answers to the Question which are consistent with all the relevant givens. So statement one is sufficient.

4.

?: Is $r > .27$?	Rephrase the Question using the symbol >. Write down the Question; label it with a *?*. Include the word "is"; do *not* just write: ?: r>.27?

(S2) G: $r = .3$	Start with the easier statement. Since the question is about a decimal, it's helpful to rephrase the statement as a decimal.
(?: Yes)	.3 > .27, so the only possible answer to the Question is "yes".
S (S2) $r = .3$ (?: Yes)	Since there's exactly one answer to the Question which is consistent with the relevant givens, statement two is sufficient; label it with an *S*. The complete suggested scratchwork for statement two is shown to the left.

(S1) FA (?: Yes)	Write down *S1* to indicate that we're working on statement one now. Since statement two is sufficient, the one answer to the Question which is consistent with statement two ("yes") is a free answer which we automatically know must be consistent with statement one as well. Write down your free answer, label it with a *?*, and circle it.
G: $r > .25$	Rephrase the statement as decimal. The only relevant given now is statement two; statement one is *not* relevant.
CN $r = .26$	Choose numbers. 1. Choose numbers There are two good reasons to choose r=.26. First, when there's a boundary, we should usually start by picking a number that is close to the boundary. Second, since we have already obtained an answer of "yes", we should be trying to choose numbers that will give us an answer of "no"; therefore it would be a waste of time to pick, say, r=.28, or r=.3, or r=1 million.
CN $r = .26$ ✓	2. Check whether your number is consistent or inconsistent with the relevant givens. .26>.25, so our number *is* consistent. Make a check mark.
CN $r = .26$ ✓ (?: No)	3. Use your numbers to answer the Question. .26<.27, so the answer to the Question is "no". Write down the answer, label it with a *?*, and circle it.
I (S1) FA (?: Yes) G: $r > .25$ CN $r = .26$ ✓ (?: No)	Normally we would have to choose at least two numbers to evaluate statement one. However, in this case we had a free answer ("yes"). Therefore, we've already gotten two different answers ("yes" and "no") so we know statement one is insufficient without choosing any more numbers; label it with an *I*. Suggested scratchwork for statement one is shown at left.

Remember that we evaluated statement two first.

?: Is $r > .27$?	
I (S1) FA (?: Yes) G: $r > .25$ CN $r = .26$ √ (?: No)	S (S2) $r = .3$ (?: Yes)

Here is a mistake to avoid when choosing numbers for statement one:

THE MISTAKE: Some people will avoid choosing a number like r=.26 because it seems "inconsistent with the Question." This will make it *seem* like the only answer is "yes", which will make it *seem* like statement one is sufficient.

WHY IT'S A MISTAKE: This is the mistake of treating the Question like a given. We **should** avoid picking numbers that are inconsistent with the givens. We should **not** avoid picking numbers that are "inconsistent with the Question." In fact, *"inconsistent with the Question" is a meaningless phrase which you should avoid using.* Since our first answer to the Question was "yes", we should be **trying** to pick a number like .26 that will give us an answer of "no".

HOW TO AVOID THIS MISTAKE: (1) Write down the Question. Label it with a *?* and include the verb "is": you are more likely to treat "r>.27" as a given than "?: Is r>.27?" (2) Remember that, in step four, you should be trying to get a *different* answer to the Question. So, if your first answer was "yes", you should be *trying* to get an answer of "no".

Here is another mistake to avoid when choosing numbers for statement one:

THE MISTAKE: Some people will ask themselves, "Is r=.26 consistent with the relevant given?" Then they will say, "**Yes**, r=.26 *is* consistent with the given." This will make them think that the *answer to the Question* is "**yes"**. This will make it seem like there is only one answer to the Question ("yes") which will make it seem like statement one is sufficient.

WHY IT'S A MISTAKE: This is the mistake of treating the given like a Question. r=.26 is *consistent with the given*, but that does not mean that the *answer to the Question* is "yes".

HOW TO AVOID THIS MISTAKE: (1) Write down the given. Label it with a *G* to remind yourself that it's a given, not a Question. (2) Do *not* ask yourself "Is r=.26 consistent with statement one?" This is bad wording because it prompts you to respond "yes" or "no". *Instead*, ask yourself "Is r=.26 consistent or inconsistent with statement one?" This is better wording because it prompts you to respond "consistent" or "inconsistent", rather than "yes" or "no". (3) **Never** write down the word *yes* to indicate that a number is consistent with the givens; instead, use a √. Notice that, if you made the mistake of treating the given like a Question, your notation would look like this:

r=.26 yes

Hopefully, you would see immediately that this notation is incorrect because it contains neither a check mark nor a cross-out, and this would alert you that you must have made a mistake.

5.

G: a list of numbers contains *n* odd integers ?: sum of the list of numbers	The Question had a hidden given.

(S1) G: *n*=8	Start with the easier statement.
CN list of numbers: 1, 1, 1, 1, 1, 1, 1, 1	Choose numbers. 1. Choose numbers. Let's choose eight 1's. (It's good to choose 1, and it's good to choose the same number for multiple variables) We write down our numbers. There is no specific "variable" which we're setting the numbers equal to, so we just label the numbers as the "list".
CN list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 √	2. Check whether our numbers are consistent or inconsistent with the relevant givens. We have chosen eight numbers, so our list is consistent with statement one; the eight numbers are all odd, so our list is consistent with the initial given. Make a check mark.
CN list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 √ (?: 8) 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8	3. Use your numbers to answer the Question.
CN list of numbers: 3, 3, 3, 3, 3, 3, 3, 3	4. Choose more numbers, trying to get a different answer than "8"; back to step 1. 1. Choose numbers.
CN list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 √	2. Check whether your numbers are consistent or inconsistent with the givens. Make a check mark to show that our numbers are consistent with the givens.
CN list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 √ (?: a different answer)	3. Use your numbers to answer the Question. Usually we should use our numbers to determine an answer to the question. In this case, however, it is obvious the sum of the numbers in our new list *will* be different from the sum of the numbers in the old list. So it's not necessary to actually add up the numbers in the list.
I (S1) G: *n*=8 CN list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 √ (?: 8) 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8 CN list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 √ (?: a different answer)	We see that our two different lists give us two different answers to the Question. Therefore statement one is insufficient.

G: a list of numbers contains *n* odd integers
?: sum of the list of numbers

(S2) G: $n^2 = 64$	The relevant givens now are the initial given and statement two. Statement one was insufficient so there's no free answer.
[2a] $n = 8$ or -8	Solve the equation for *n*.
CN $n = -8$	1. Choose numbers: First, we have to choose a number for *n*.
CN ~~n = -8~~	2. Check whether your number is consistent or inconsistent with all the relevant givens. *n* = -8 is consistent with statement two, However, *n*=-8 is *in*consistent with the initial given, because it is impossible for a list to contain -8 numbers. Since *n*=-8 is *not* consistent with *all* the relevant givens, we need to throw this number out. Cross it out and pick a new number for *n*. Of course, if it was obvious to you that *n* = -8 would be inconsistent with the initial given, then there was no need to choose it in the first place.
CN *n* = 8; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1	1. Choose a new *n*. Choose your list of numbers. Reuse your numbers from the statement one.
CN *n*=8; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 ✓	2. Check consistency with the givens. From [2a] we know that *n*=8 is consistent with statement two. Since we picked *n*=8 and our list contains 8 odd integers, our list is consistent with the initial given.
CN *n* = 8; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 ✓ (?: 8)	3. Use your numbers to answer the Question.
CN *n*=8; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3	4. Choose more numbers, trying to get a different answer than "8". 1. Choose numbers. We know that we have to choose *n*=8; we should continue trying to reuse our old numbers from statement one.
CN *n*=8; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 ✓	2. Check consistency with the givens. *n*=8 is consistent with statement two. Since we picked *n*=8 and our list contains 8 odd integers, our list is consistent with the initial given. Make a check mark.
CN *n*=8; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 ✓ (?: a different answer)	3. Answer the Question.
I (S2) G: $n^2 = 64$ *n* = 8 or -8 CN ~~n = -8~~ CN *n* = 8; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 ✓ (?: 8) CN *n*=8; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 ✓ (?: a different answer)	We see that we can get two different answers to the Question that are consistent with all the relevant givens. Therefore, statement two is insufficient.

Since both statements are insufficient alone, we need to evaluate the statements together.

G: a list of numbers contains n odd integers ?: sum of the list of numbers	
(tog)	Write down *(tog)* to remind yourself that you're now evaluating the statements together. The relevant givens are now the initial given, statement one, and statement two.
CN $n = 8$; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1	Choose numbers. 1. Choose numbers Don't forget to choose a number for n. Continue reusing your old numbers.
CN $n=8$; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 √	2. Check whether your numbers are consistent or inconsistent with all the relevant givens. $n=8$ is clearly consistent with statement one. (In fact, $n=8$ is the only number for n which is consistent with statement one.) From our previous work, we already know that our numbers *are* consistent with all the other givens in the problem as well.
CN $n = 8$; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 √ (?: 8)	3. Use your numbers to answer the Question.
CN $n=8$; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3	4. Choose more numbers, trying to get a different answer to the Question. 1. Choose numbers. Continue reusing old numbers.
CN $n=8$; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 √	2. Check whether your numbers are consistent or inconsistent with all the relevant givens.
CN $n=8$; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 √ (?: a different answer)	3. Use your numbers to answer the Question.
I (tog) CN $n = 8$; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 √ (?: 8) CN $n=8$; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 √ (?: a different answer)	We see that we can get two different answers to the Question which are consistent with all the relevant givens. Therefore, the statements together are still insufficient. Write down an *I*.

12TEN: Nothing is sufficient; pick choice E.

If the problem had given that the list has "*n* distinct odd integers", then our numbers would be inconsistent with that givens. Since the problem did *not* specify that the integers are distinct, our numbers *were* consistent with the givens. If picking the same number multiple times makes you uncomfortable, you can still get #5 right by choosing a set of eight distinct numbers. However, choosing the same number for multiple variables is a good habit to get into.

initial G: a list of numbers contains *n* odd integers
?: sum of the list of numbers

I (S1) G: $n=8$

CN list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 ✓ (?: 8)

$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8$

CN list of numbers: 3,3,3,3,3,3,3,3 ✓ (?: a different answer)

I (S2) G: $n^2 = 64$

$n = 8$ or -8

CN ~~n = -8~~

CN $n = 8$; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 ✓ (?: 8)

CN $n=8$; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 ✓ (?: a different answer)

I (tog)

CN $n = 8$; list of numbers: 1, 1, 1, 1, 1, 1, 1, 1 ✓ (?: 8)

CN $n=8$; list of numbers: 3, 3, 3, 3, 3, 3, 3, 3 ✓ (?: a different answer)

12TEN: Nothing is sufficient; pick choice E.

This illustrates the systematic approach for evaluating the statements *together*. Notice that choosing numbers can simplify the process of evaluating the statements together. And notice that the strategy of reusing our numbers simplified the process of choosing numbers, especially when we had to evaluate the statements together.

If this problem was easy for you, then it would not be necessary to use all the systematic steps and notation described above. On the other hand, if you found yourself getting confused, you definitely should use this systematic approach. In any case, study this explanation carefully so you will understand how to use the systematic approach on more difficult problems.

Here is a MISTAKE that a student might make while choosing numbers for either statement.

Step one: The student chooses the list of eight 1's.

Step two: The student confirms that the list is consistent with the givens.

Step three: The student says to themself, "Now I need to check whether my numbers answer the Question." Then, the student responds, "Yes—the numbers **do** answer the Question. (The answer is '8'.) So the statement must be sufficient."

This is the mistake of choosing only one set of numbers. The student forgot to execute step four ("choose *more* numbers, trying to get a *different* answer to the Question"). Just because you can find an answer to the Question does not mean that a statement is sufficient. There is always at least one answer to the Question. The only issue is whether there is **more than one** answer.

To avoid this mistake, use good vocabulary when thinking about step three. *Don't* say, "I'm checking whether my numbers answer the Question." *Instead*, say, "I'm using my numbers to answer the Question." And don't forget step four—on data sufficiency, aside from some exceptional cases, you always have to choose at least two sets of numbers.

7.

There are two sets: women and redheads. People can be in one set only, in the other set only, in both sets, or in neither set. So this is an “overlapping sets” problem.

	women	**men**	**total**
red			
not red			
total			100

Use an “overlapping sets table”.

Each cell of this table represents a part:whole ratio, expressed as a percentage. The Whole is all the people in the group.

	women	**men**	**total**
red	?		
not red			
total			100

Identify the Question. Label the appropriate cell with a “?”.

The Question is asking for a part:whole ratio. The Part is women with red hair; the Whole is all the people in the group.

(S1)

	women	**men**	**total**
red	?=.05W		
not red			
total	W		100

Make a new copy of the table for statement one. Enter the given’s into the table.

5% is a part:whole ratio. The Part is women with red hair. The Whole is not *all* the people in the group but only all the *women* in the group.

I (S1)

	women	**men**	**total**
red	?=.05*W*		
not red			
total	*W*		100

W could be anything. So .05*W* could be anything. So there are many different answers to the Question which are consistent with all the relevant givens. So statement one is insufficient.

MISTAKE:

	women	**men**	**total**
red	?=.05		
not red			
total			100

This table is **WRONG**!

It is tempting to simply enter .05 in the top-left cell. This make it *seem* like there’s only one answer to the Question (“5%”)—which makes it *seem* like statement one is sufficient.

This would have been the correct table if the statement had said, “Of *all the people* in the group, 5 percent have red hair.”

MISTAKE:

	women	**red**	**total**
men			
not red			
total			

This is the ***wrong*** way to set up the overlapping sets table.

(S2)

	women	**men**	**total**
red	?	$.1M$	
not red			
total		M	100

The relevant given is now statement two; ignore statement one.

Make a new copy of the table for the new relevant given. Enter the givens into the table.

10% is a part:whole ratio. The Part is men with red hair. The Whole is not *all* the people in the group but only all the *men* in the group.

I (S2)

	women	**men**	**total**
red	?	$.1M$	
not red			
total		M	100

M could be anything. Even if we knew what M is, we still would not know the answer to the Question. So there are many different answers to the Question which are consistent with all the relevant givens. So statement two is insufficient.

(tog)

	women	**men**	**total**
red	$?=.05W$	$.1M$	
not red			
total	W	M	100

Since both statements are insufficient alone, we need to evaluate the statements together. Write down *tog*.

Now the relevant givens are statement one and statement two; make a new table for the new relevant givens.

I (tog)

	women	**men**	**total**
red	$?=.05W$	$.1M$	
not red			
total	W	M	100

W could be anything. So $.05W$ could be anything. So there are many different answers to the Question which are consistent with all the relevant givens. So the statements together are still insufficient.

Use 12TEN: Nothing is sufficient, even when we evaluate both statements together, so the correct choice is E.

Complete scratchwork for #15:

I (S1)

	women	**men**	**total**
red	$?=.05W$		
not red			
total	W		100

I (S2)

	women	**men**	**total**
red	?	$.1M$	
not red			
total		M	100

I (tog)

	women	**men**	**total**
red	$?=.05W$	$.1M$	
not red			
total	W	M	100

If you were nervous about your evaluation of the statements together, you could boost your confidence by choosing numbers:

(tog)
CN

	women	men	total
red	?=.05W	.1M	
not red			
total	W=0	M	100

We need numbers for all the cells. We will *choose* a number for W, and then *determine* numbers for the other cells.

Choose W=0. We should usually start by choosing 0, especially where it's a boundary value for the variable.

Make a new copy of the table for W=0.

CN √

	women	men	total
red	?=0	10	10
not red	0	90	90
total	W=0	100	100

Use W=0 to determine the other cells in the table.

5% of 0 is 0, so these numbers are consistent with statement one. 10% of 100 is 10, so these numbers are also consistent with statement two. Therefore, the numbers are consistent with all the relevant givens.

When we choose these numbers W=0, the answer to the Question is "0".

CN

	women	men	total
red	?=.05W	.1M	
not red			
total	W=100	M	100

Now choose another number, trying to get a different answer than "0". Choose W=100. We should usually try to pick boundary numbers.

Make a new copy of the table for W=100.

I (tog)
CN √

	women	men	total
red	?=0	10	10
not red	0	90	90
total	W=0	100	100

CN √

	women	men	total
red	?=5	0	5
not red	95	0	95
total	W=100	0	100

Use W=100 to work out the other cells in the table.

5% of 100 is 5, so these numbers are consistent with statement one. 10% of 0 is 0, so these numbers are also consistent with statement two. Therefore, the numbers are consistent with all the relevant givens.

We have found two different answers to the Question ("0%" and "5%") which are consistent with all the relevant givens. So the statements together are insufficient.

If choosing W=0 and W=100 made you nervous, you could have chosen different numbers. But choosing boundary values is a good habit and certainly makes the calculations easier for this problem. Don't assume that the variables are positive unless that is required by the givens.

9.

The best way to attack #9 is by using algebra, as described in the *Official Guide*'s explanation. However, you could also get it correct by choosing numbers.

?: Is $\underline{a}>b$?

(S1) G: $2a>2b$	
CN $a=1$, $b=0$	1. Choose numbers. It's good to choose 0 and choose 1.
CN $a=1$, $b=0$ ✓ $2a=2(1)=2$, $2b=2(0)=0$	2. Check whether your numbers are consistent or inconsistent with all the relevant givens. Label your work.
CN $a=1$, $b=0$ ✓ (?: Yes) $2a=2(1)=2$, $2b=2(0)=0$	3. Use your numbers to answer the Question.
CN $a=0$, $b=1$	4. Choose more numbers, trying to get an answer of "no".
CN ~~a=0, b=1~~ $2a=2(0)=0$, $2b=2(1)=2$	2. Check whether your numbers are consistent or inconsistent with all the relevant givens. Label your work.
S (S1) G: $2a>2b$ CN $a=1$, $b=0$ ✓ (?: Yes) $2a=2(1)=2$, $2b=2(0)=0$ CN ~~a=0, b=1~~ $2a=2(0)=0$, $2b=2(1)=2$	We have gotten **one** answer ("yes"). If you continue choosing numbers, you will not be able to find any numbers that give an answer of "no" which are also consistent with all the relevant givens. So eventually, you should stop choosing new numbers and conclude that the statement is sufficient.

(S2) FA (?: Yes) G: $a+c>b+c$	Since statement one was sufficient, it gives us a free answer: we know that "yes" must be consistent with statement two as well.
CN $a=0$, $b=1$, $c=0$	1. Choose numbers, trying to get an answer of "No." We need numbers for all the variables, including c, even though c is not mentioned in the Question. It's good to choose 0.
CN ~~a=0, b=1, c=0~~ $a+c=0+0=1$, $b+c=1+0=1$	2. Check whether your numbers are consistent or inconsistent with all the relevant givens. Label your work.
S (S2) FA (?: Yes) G: $a+c>b+c$ CN ~~a=0, b=1, c=0~~ $a+c=0+0=1$, $b+c=1+0=1$	So far we've got only **one** answer to the Question ("yes", our free answer) which is consistent with the relevant givens. If you continue choosing numbers, you will not be able to find any numbers which give an answer of "no" which are also consistent with all the relevant givens. So eventually, you should stop choosing new numbers and conclude that the statement is sufficient.

12TEN: Each of the statements is sufficient alone; pick choice D.

?: Is $\underline{a}>b$?

S (S1) G: $2a>2b$	S (S2) FA (?: Yes)
CN $a=1, b=0$ ✓ (?: Yes) $2a=2(1)=2$, $2b=2(0)=0$ CN ~~$a=0, b=1$~~ $2a=2(0)=0$, $2b=2(1)=2$	G: $a+c>b+c$ CN ~~$a=0, b=1, c=0$~~ $a+c=0+0=1$, $b+c=1+0=1$

12TEN: Each statement is sufficient alone; pick choice D.

Here is a mistake to avoid when choosing numbers for #9:

THE MISTAKE: While executing step two of choosing numbers for statement one, some people will ask themselves, "Is a=0, b=1 consistent with the relevant given?" Then they will say, "**No**, a=0, b=1 is *not consistent with the given*." This will make them think that the *answer to the Question* is "**no**". This will make it seem like there are two answers to the Question ("yes" and "no") which will make it seem like the statement is insufficient.

WHY IT'S A MISTAKE: This is the mistake of treating the given like a Question. When we notice that our numbers are inconsistent with the *given*, that doesn't mean we have obtained an answer to the *Question*. Remember that, when a number is inconsistent with givens, you should *not* proceed to step three—i.e., when a number is inconsistent with the givens, you should *not* use the number to answer the Question. Instead, you should just throw the inconsistent numbers out.

ASPECTS OF THE SYSTEMATIC METHODS WHICH WILL HELP YOU TO AVOID TREATING THE GIVEN LIKE A QUESTION: (1) Write down the given. Label it with a *G* to remind yourself that it's a given, not a Question. (2) Use good phraseology. Do **not** ask yourself, "Is a=0, b=1 consistent with the relevant given?" This is bad phrasing because it prompts you to respond with the words "yes" or "no". **Instead**, ask yourself "Is a=0, b=1 consistent or inconsistent with the relevant given?" This is better phrasing because it prompts you to respond with the words "consistent" or "inconsistent", rather than "yes" or "no". (3) **Never** write down the word *no* to indicate that a number is inconsistent with the givens; instead, just cross the numbers out. Notice that, if you made the mistake of treating the given like a Question, your notation would look like this:

a=0, b=1 no
$2a=2(0)=0$, $2b=2(1)=2$

Hopefully, you would see immediately that this notation is incorrect because it contains neither a check mark nor a cross-out, and this would alert you that you must have made a mistake.

12.

The best way to attack #12 is by using algebra, as described in the *Official Guide*'s explanation. However, if you didn't know how to solve #12 with algebra, you could still have gotten it correct by choosing numbers, as described below:

?: Does *jkmn*=1?	We rephrased the Question as a mathematical equation.

(S1) G: $\frac{jk}{mn}=1$	
CN *j*=1, *k*=1, *m*=1, *n*=1	1. Choose numbers. It's good to choose 1. It's good to choose the same number for multiple variables.
CN *j*=1, *k*=1, *m*=1, *n*=1 √ $\frac{jk}{mn}=\frac{1\cdot1}{1\cdot1}=1$	2. Check whether your numbers are consistent or inconsistent with all the relevant givens. Label your calculations. Don't write "$\frac{1\cdot1}{1\cdot1}=1$". Instead, write "$\frac{jk}{mn}=\frac{1\cdot1}{1\cdot1}=1$".
CN *j*=1, *k*=1, *m*=1, *n*=1 √ (?: Yes) $\frac{jk}{mn}=\frac{1\cdot1}{1\cdot1}=1$ *jkmn*=1·1·1·1=1	3. Use your numbers to answer the Question. Label your work. Don't write "1·1·1·1=1". Instead, write "*jkmn*=1·1·1·1=1". #12 has a yes/no Question, not a number Question, so the answer is "yes", not "1".
CN *j*=2, *k*=2, *m*=2, *n*=2	4. Choose more numbers, trying to get an answer of "no".
CN *j*=2, *k*=2, *m*=2, *n*=2 √ $\frac{jk}{mn}=\frac{2\cdot2}{2\cdot2}=1$	2. Check whether your numbers are consistent or inconsistent with all the relevant givens. Label your calculations.
CN *j*=2, *k*=2, *m*=2, *n*=2 √ (?: No) $\frac{jk}{mn}=\frac{2\cdot2}{2\cdot2}=1$ *jkmn*=2·2·2·2=8	3. Use your numbers to answer the Question. Label your work. #12 has a yes/no Question, not a number Question, so the answer is "no", not "8".
I (S1) G: $\frac{jk}{mn}=1$ CN *j*=1, *k*=1, *m*=1, *n*=1 √ (?: Yes) $\frac{jk}{mn}=\frac{1\cdot1}{1\cdot1}=1$ *jkmn*=1·1·1·1=1 CN *j*=2, *k*=2, *m*=2, *n*=2 √ (?: No) $\frac{jk}{mn}=\frac{2\cdot2}{2\cdot2}=1$ *jkmn*=2·2·2·2=8	We have obtained **two** answers to the Question ("yes" and "no") which are consistent with all the relevant givens. So statement one is insufficient.

?: Does *jkmn*=1?

(S2) G: $j=\frac{1}{k}, \quad m=\frac{1}{n}$	Statement one was insufficient, so there's no free answer.
CN *k*=1, *n*=1 $j=\frac{1}{k}=\frac{1}{1}=1, \quad m=\frac{1}{n}=\frac{1}{1}=1$	1. *Choose* numbers for *k* and *n*; then use those numbers and statement two to *determine* numbers for *j* and *m*.
CN *k*=1, *n*=1 ✓ $j=\frac{1}{k}=\frac{1}{1}=1, \quad m=\frac{1}{n}=\frac{1}{1}=1$	2. Since we used statement two to determine *j* and *m*, our numbers are automatically consistent with statement two.
CN *k*=1, *n*=1 ✓ (?: Yes) $j=\frac{1}{k}=\frac{1}{1}=1, \quad m=\frac{1}{n}=\frac{1}{1}=1$	3. Use your numbers to answer the Question.
CN *k*=2, *n*=2 $j=\frac{1}{k}=\frac{1}{2}, \quad m=\frac{1}{n}=\frac{1}{2}$	4. Choose new numbers, trying to get an answer of "no". Again, **choose** *k* and *n*, then **determine** *j* and *m*.
CN *k*=2, *n*=2 ✓ $j=\frac{1}{k}=\frac{1}{2}, \quad m=\frac{1}{n}=\frac{1}{2}$	2. Since we used statement two to determine *j* and *m*, our numbers are automatically consistent with statement two.
CN *k*=2, *n*=2 ✓ (?: Yes) $j=\frac{1}{k}=\frac{1}{2}, \quad m=\frac{1}{n}=\frac{1}{2}$ $jkmn=\frac{1}{2}\cdot 2\cdot\frac{1}{2}\cdot 2=1$	3. Use your numbers to answer the Question. Label your calculations. Don't write "$\frac{1}{2}\cdot 2\cdot\frac{1}{2}\cdot 2=1$". Instead, write "$jkmn=\frac{1}{2}\cdot 2\cdot\frac{1}{2}\cdot 2=1$". #12 has a yes/no Question, not a number Question, so the answer is "yes", not "1".
S (S2) G: $j=\frac{1}{k}$ CN *k*=1, *n*=1 ✓ (?: Yes) $j=\frac{1}{k}=\frac{1}{1}=1, \quad m=\frac{1}{n}=\frac{1}{1}=1$ CN *k*=2, *n*=2 ✓ (?: Yes) $j=\frac{1}{k}=\frac{1}{2}, \quad m=\frac{1}{n}=\frac{1}{2}$ $jkmn=\frac{1}{2}\cdot 2\cdot\frac{1}{2}\cdot 2=1$	So far we have found only **one** answer to the Question ("yes") which is consistent with all the relevant givens. If you continue choosing numbers, you will not be able to find any numbers which give an answer of "no" which are also consistent with all the relevant givens. So eventually you should stop choosing new numbers and conclude that the statement is sufficient.

12TEN: Only statement 2 is sufficient, so choice B is correct.

Explanation continued on next page …

?: Does $jkmn=1$?

I (S1) G: $\frac{jk}{mn}=1$ CN $j=1, k=1, m=1, n=1$ ✓ (?: Yes) $\frac{jk}{mn}=\frac{1\cdot1}{1\cdot1}=1$ $jkmn=1\cdot1\cdot1\cdot1=1$ CN $j=2, k=2, m=2, n=2$ ✓ (?: No) $\frac{jk}{mn}=\frac{2\cdot2}{2\cdot2}=1$ $jkmn=2\cdot2\cdot2\cdot2=8$	S (S2) G: $j=\frac{1}{k}$, $m=\frac{1}{n}$ CN $k=1, n=1$ ✓ (?: Yes) $j=\frac{1}{k}=\frac{1}{1}=1$, $m=\frac{1}{n}=\frac{1}{1}=1$ CN $k=2, n=2$ ✓ (?: Yes) $j=\frac{1}{k}=\frac{1}{2}$, $m=\frac{1}{n}=\frac{1}{2}$ $jkmn=\frac{1}{2}\cdot2\cdot\frac{1}{2}\cdot2=1$

12TEN: Only statement 2 was sufficient, so choice B is correct.

Here is a MISTAKE that a student might make while choosing numbers.

Step one: The student chooses $j=k=m=n=1$ for statement one.

Step two: The student confirms that the list is consistent with the givens.

Step three: The student correctly notices that, with these numbers, the answer to the Question is "**yes**". The student then says to themself, "**Yes**, statement one is sufficient".

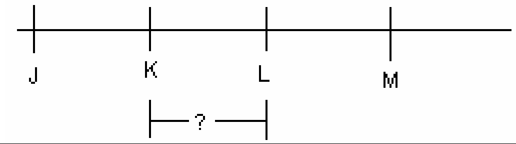
This is the mistake of choosing only one set of numbers. The student forgot to execute step four ("choose *more* numbers, trying to get a *different* answer to the Question"). It is especially easy to make this mistake on yes/no Questions. "Yes" and "sufficient" are both positive-sounding words, so it's easy to think that one answer of "yes" means "sufficient".

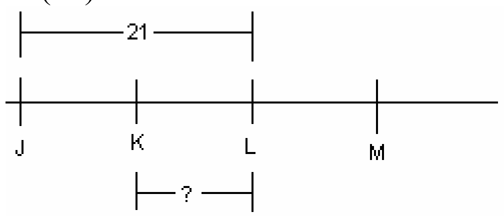
Here's another MISTAKE you might make on statement one.

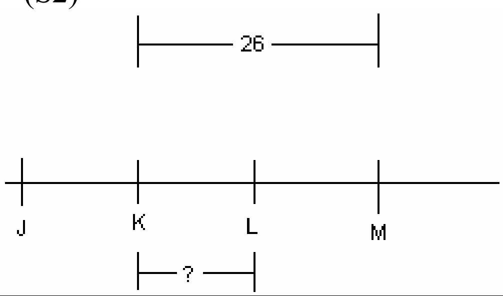
CN $j=2$, $k=2$, $m=2$, $n=2$ yes $\frac{jk}{mn}=\frac{2\cdot2}{2\cdot2}=1$	This student has said to themself, "**yes**, these numbers are consistent with statement one," and this has confused them into thinking that "the answer to the Question is **yes**". This makes it *seem* as if statement one allows only one answer ("yes"), which makes it *seem* as if statement one is sufficient.

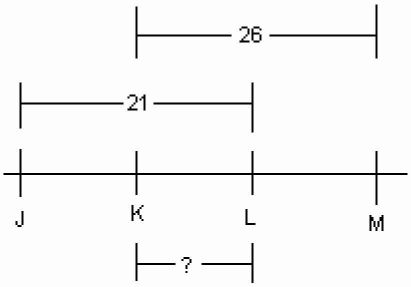
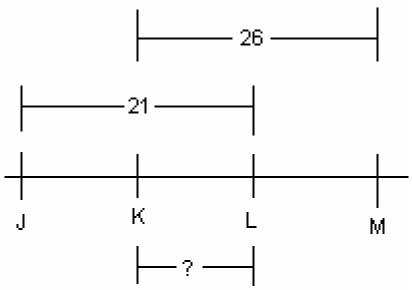
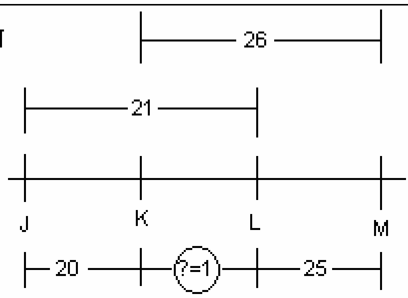
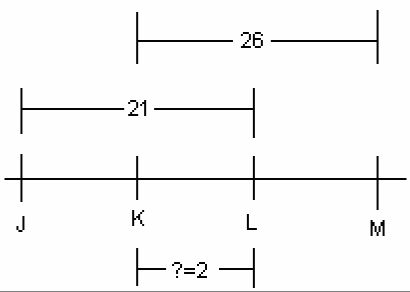
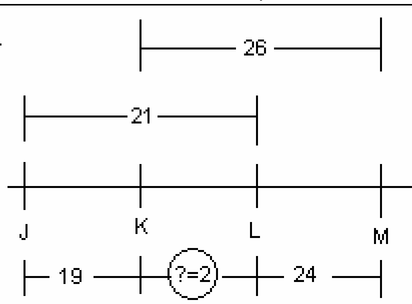
This is the mistake of treating the given like a Question. The student should notice that their notation includes neither a checkmark nor a cross-out, and that should alert them that they've made a mistake.

13.

	When a geometry problem doesn't give you a diagram, draw your own diagram. Use the diagram to identify the Question.

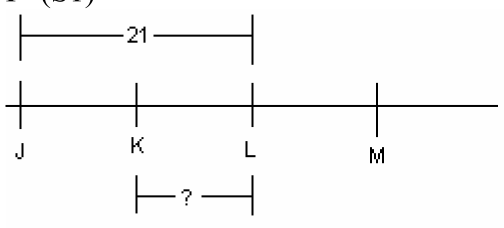
I (S1) 	Draw a new diagram for statement one. Build the given into the diagram. KL can be anything less than 21. So there many answers to the Question that are consistent with all the relevant givens. So statement one is insufficient.

I (S2) 	Draw a new diagram for statement two. Build the new given into the diagram. KL can be anything less than 26. So there many answers to the Question that are consistent with all the relevant givens. So statement two is insufficient.

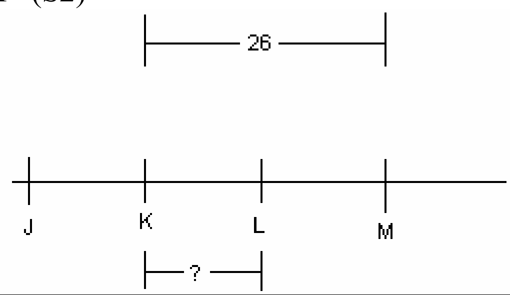
(tog) 	Draw a new diagram for the statements together. Build all the relevant givens into the diagram. Now choose numbers.
CN 	1. Choose numbers. We need numbers for *all* the segment lengths. We will *choose* a number for KL, and then use that number, together with the givens, to *determine* our numbers for the other lengths. Let's choose KL=1.
CN ✓ 	Use the number we chose for KL to determine numbers for the other lengths. 2. Check whether your numbers are consistent or inconsistent with all the relevant givens. 3. Use your number to answer the Question. The answer to the Question is "1".
CN 	4. Choose new numbers, trying to get a different answer to the Question. Draw a new diagram. Let's choose KL=2.
CN ✓ 	Use the number we chose for KL to determine numbers for the other lengths. 2. Check whether your numbers are consistent or inconsistent with all the relevant givens. 3. Use your number to answer the Question. The answer to the Question is now "2".
I (tog)	We have obtained **two** different answers to the Question ("1" and "2") which are consistent with all the relevant givens. So the statements together are insufficient.

Complete suggested scratchwork for #13:

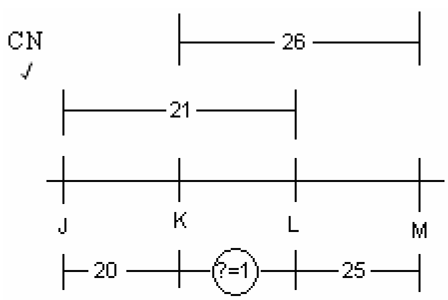
I (S1)

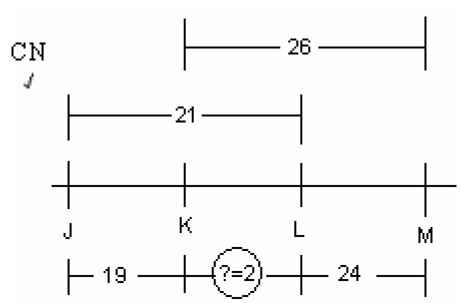


I (S2)



I (tog)





MORAL: You can use choosing numbers for geometry problems, even when a problem does not have clearly identified "variables". Build your numbers into the diagram.

You need numbers for *all* the variables in both the Question and the relevant givens. So we needed numbers for all the segment lengths. However, you will not necessarily want to *choose* numbers for all the variables. Instead, you may want to choose numbers for some of the variables, and then use those numbers, together with the givens, to *determine* numbers for the other variables. For #13, we *chose* numbers for KL, and then *determined* values for the other lengths.

The Question is, "What is the distance from K to L?" Therefore, when we choose KL=1, the answer to the Question is "1". When we choose KL=2, the answer to the Question is "2". Don't feel uncomfortable about choosing a number for a variable when the Question is asking for the value of that same variable.

A MISTAKE you might make when evaluating the statements together is to choose KL=1, get an answer of "1", and assume that, since you've "gotten an answer", the statements together are sufficient. This is the mistake of choosing only one set of numbers. Remember that "getting an answer" doesn't mean "sufficient". You can *always* get an answer. The issue is whether you can get *more than one* answer—and the only way to tell whether you can get more than one answer is to choose more than one set of numbers.

14.

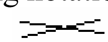
?: Is *k* prime? G: *k* is an integer	The initial information has a hidden given.

Consult the *Official Guide* for an explanation of statement one.

I (S2) FA (?: Yes) G: 1<*k*<6 CN *k*=4 √ (?: No)	Since statement one is sufficient, it gives us a free answer for statement two.The one answer that is consistent with statement one and the initial given is "yes", so our free answer for statement two is also "yes". Now it would be a waste of time to choose *k*=3 or *k*=5, since we've already gotten one answer of "yes". Instead, we try to pick a number that will give us an answer of "no".

1̲2TEN: Only statement 1̲ is sufficient—pick choice A.

MISTAKE

S (S2) FA (?: Yes) G: 1<*k*<6 CN *k*=4 Yes	This illustrates the **mistake** of treating the given like a Question. The student has said to themselves, "Y̲e̲s̲, *k*=4 is consistent with the statement," which confused them into thinking that they've gotten an *answer to the Question* of "y̲e̲s̲". This makes it appear that the only answer to the question is "yes", make makes statement two seem sufficient. The student can catch their mistake by noticing that they have wrong notation: the student's notes for *k*=4 match neither  nor , which are the only two correct patterns

MISTAKE

Some students may avoid picking *k*=4 for statement two "because it's inconsistent with the Question." This will make them think that the only answer is "yes", which will make them think that statement two is sufficient. This is the mistake of treating the Question like a given. "Inconsistent with the Question" is a meaningless phrase that you should never use. Since we have already gotten an answer of "yes", we should be *trying* to choose a number (like "4") which will give us an answer of "no".

To defend against this mistake, use good notation when you write down the Question. You are more likely to treat

k prime

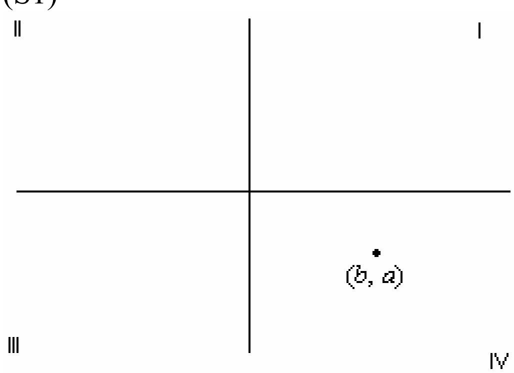
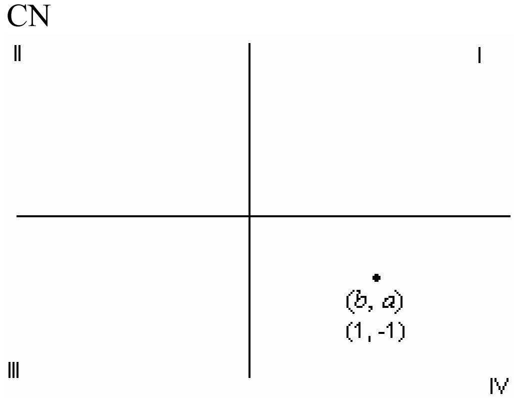
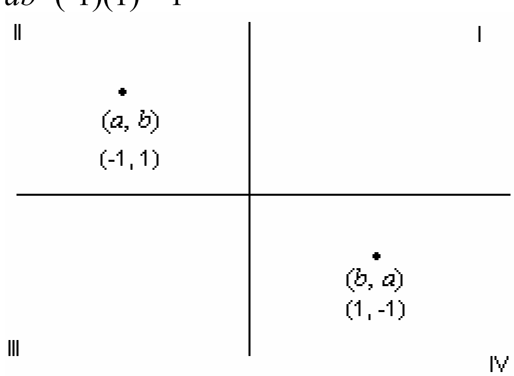
as a given than

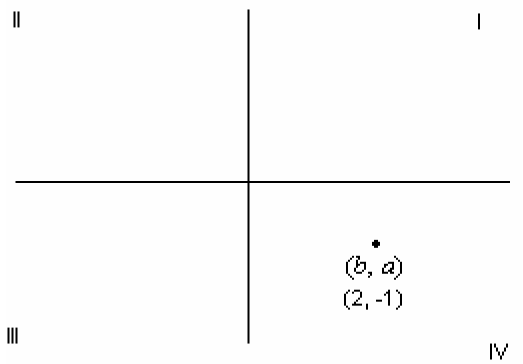
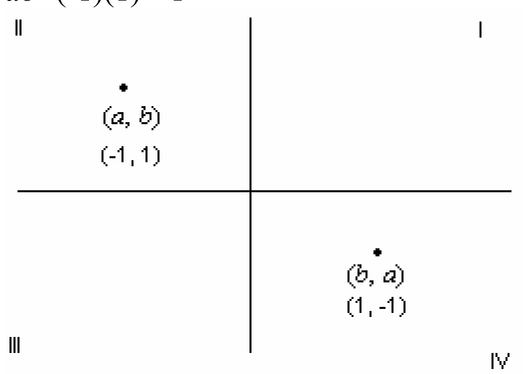
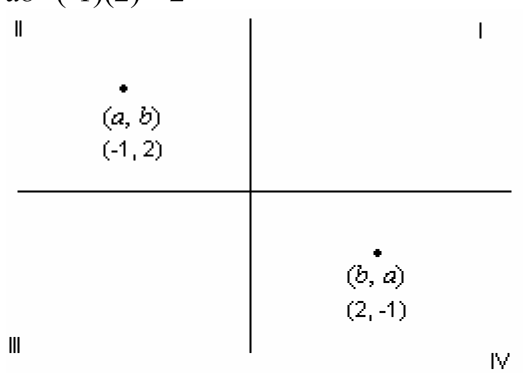
?: Is *k* prime?

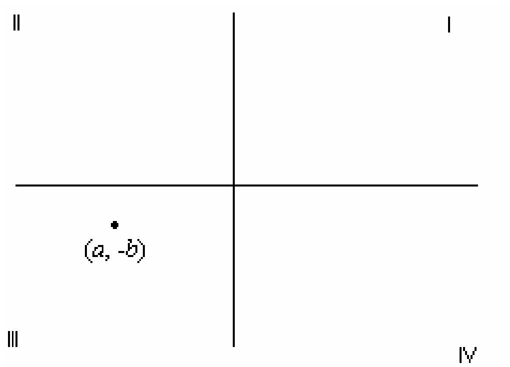
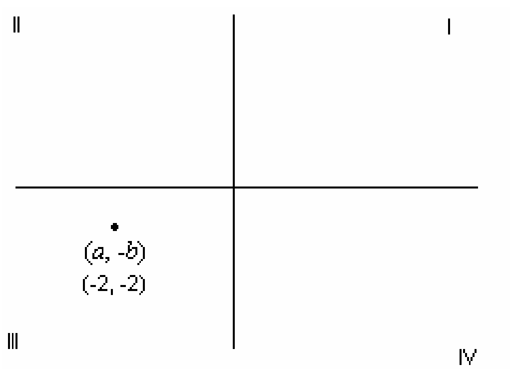
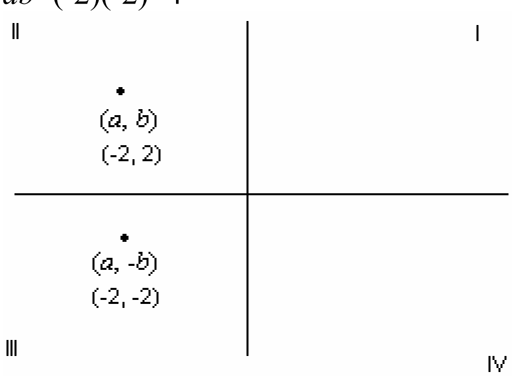
15.

The explanation in the *Official Guide* describes a good way to solve #15. If you could not come up with that approach on your own, however, you could still get #15 right by choosing numbers.

?: In what quadrant does point (a, b) lie? G: $ab \neq 0$	The word “if” indicates givens.

(S1) 	When the GMAT provides you with a diagram, *copy it*. Build the statement into the diagram.
CN 	1. Choose numbers. We are choosing a = -1 and b=1. (It’s good to choose 1 and it’s good to choose –1.) Label your numbers by building them into the diagram.CN
CN ✓ ?: II ab=(-1)(1)=-1 	2. Check whether your numbers are consistent or inconsistent with the relevant givens. (-1)(1)≠0, and (1, -1) *is* in quadrant IV, so our numbers *are* consistent with all the relevant givens. 3. Use your numbers to answer the Question. (-1, 1) is in quadrant II, so the answer to the Question is “II”.

CN  	Let's continue our evaluation of statement one. 4. Choose new numbers, trying to get a different answer than "II". Let's choose *a*=2 and *b*=-1. Draw a new diagram. Build your new numbers into the new diagram.
S (S1) CN ✓ (?: II) *ab*=(-1)(1)=-1  CN ✓ (?: II) *ab*=(-1)(2)=-2  	2. Check whether your numbers are consistent or inconsistent with the relevant givens. (-1)(2)≠0, and (2, -1) *is* in quadrant IV, so our numbers *are* consistent with all the relevant givens. 3. Use your numbers to answer the Question. (-1, 2) is in quadrant II, so the answer to the Question is still "II". So far, we have found only **one** answer to the Question ("II") which is consistent with all the relevant givens. If you continue choosing numbers, you will not be able to find any numbers that give a different answer which are also consistent with all the relevant givens. (For example, if you pick *a*=1, *b*=-1, you will have to throw those numbers out because they are not consistent with statement one. For another example, if you decide to pick *a*= -1, *b*=0, you will have to throw those numbers out too, because they are not consistent with the initial given.) So eventually, you should stop choosing new numbers and conclude that statement one is sufficient.

(S2) FA (?: II)  	Since statement one was sufficient, the one answer which is consistent with statement one and the initial givens ("II") is a free answer which we automatically know must also be consistent with statement two. Draw a new diagram. Build statement two into the diagram.FA I
CN $-b=-2 \Rightarrow b=2$  	1. Choose numbers, trying to get a different answer than "II". When a statement is about a mathematical expression, it can be convenient to choose a number for the mathematical expression. Since statement two is about "-*b*" rather than *b*, it is convenient to pick a number for "-*b*" instead of for *b*. Let's choose $a=-2, -b=-2$. Then, we use our value for $-b$ to *determine* a value for *b*.
S (S2) FA (?: II) CN ✓ (?: II) $-b=-2 \Rightarrow b=2$ $ab=(-2)(-2)=4$  	2. Check whether your numbers are consistent or inconsistent with the relevant givens. 3. Use your numbers to answer the Question: (-2, 2) is in quadrant II, so the answer to the Question is still "II". So far, we have found only **one** answer to the Question "II" which is consistent with all the relevant givens. If you continue choosing numbers, you will not be able to find any numbers that give a different answer which are also consistent with all the relevant givens. (For example, $a=2$, $-b=-2$ are inconsistent with statement two.) So eventually, you should stop choosing new numbers and conclude that statement two is sufficient.

12TEN: Each of the statements is sufficient, so we pick choice D.

By the way, it is helpful to rephrase the initial given as: "$a\neq0$ and $b\neq0$"

16.

If #16 isn't obvious to you, choose numbers.

?: Is $x>1.8$?

I (S1) G: $x>1.7$	S (S2) G: $x>1.9$
CN $x = 1.8$ ✓ (?: No)	CN ~~$x=1.8$~~
CN $x = 1$ million ✓ (?: Yes)	CN $x = 1$ million ✓ (?: Yes)

12TEN: Only statement 2 is sufficient, so pick choice B.

We rephrased the Question as a mathematical inequality.

For statement one, we chosen a number close to the boundary ($x>1.7$), and a number far from the boundary.

Statement one was *insufficient*, so we do *not* have a free answer for statement two. It is a good idea to reuse your numbers, so we reused $x=1.8$ and $x=1$ million for statement two. $x=1.8$ turned out to be inconsistent with statement two, so we had to throw that number out.

If it wasn't obvious to you already, choosing a few numbers for statement two should make it clear that there is no way to get an answer to the Question of "no" that will be consistent with statement two.

MISTAKE

Some students may avoid picking a number like $k=1.8$ for statement one "because it's inconsistent with the Question." This will make them think that the only answer is "yes", which will make them think that statement one is sufficient. This is the mistake of treating the Question like a given. "Inconsistent with the Question" is a meaningless phrase that you should never use. There is nothing "bad" about getting an answer to the Question of "no".

To defend against this mistake, use good notation when you write down the Question. You are more likely to treat

$$x>1.8$$

as a given than

?: Is $x>1.8$?

MISTAKE

Step one: The student chooses $x=1.8$ for statement one.

Step two: The student confirms that the list is consistent with the givens.

Step three: The student correctly notices that, with these numbers, the answer to the Question is "no". The student then says to themself, "No, statement one is not sufficient".

This is the mistake of choosing only one set of numbers. The student forgot to execute step four ("choose *more* numbers, trying to get a *different* answer to the Question"). It is especially easy to make this mistake on yes/no Questions. "No" and "insufficient" are both negative-sounding words, so it is easy to think that one answer of "no" means "insufficient".

17.

The *Official Guide* describes a good way to solve #17. You could also choose numbers:

?: Is n+1 odd? G: n is an integer	
S (S1) G: n+2 is even CN n+2=2 ⇒ n=0 ✓ (?: Yes) n+1=0+1=1 CN n+2=4 ⇒ n=2 ✓ (?: Yes) n+1=2+1=3	S (S2) FA (?: Yes) G: n-1 is odd CN n-1=3 ⇒ n=4 ✓ (?: Yes) n+1=4+1=5

12TEN: Each of the statements is sufficient alone; pick choice D.

For each statement, if you continue choosing numbers, you will not be able to find any numbers that give a different answer which are also consistent with all the relevant givens. So eventually, you should stop choosing new numbers and conclude that each statement is sufficient alone.

#17 has a yes/no Question, not a number Question, so our answers are"yes", "yes", and "yes"—not "1", "3", and "5".

The word "if" indicates givens.

When a statement is about a mathematical expression based on a variable, it can be convenient to choose a number for the mathematical expression, rather than for the variable itself. Since statement one was about n+2 rather than n, it was convenient to *choose* numbers for n+2, rather than for n, and then use those numbers to *determine* values for n. Since statement two was about n-1 rather than n, it was convenient to *choose* a number for n-1, rather than for n, and then use that number to *determine* a value for n.

Since statement one was sufficient, it gave us a free answer ("yes") for statement two.

The easiest trap to fall into for #17 is to confuse n, n+1, n+2, and n-1. Therefore, an important strategy for #17 is labeling each number with the variable or expression which that number represents. It's also useful to write down the Question and the statements.

18.

?: Is $1<x<2$?

I (S1) G: $0<x$
CN x=1 ✓ (?: No)
CN x=1.5 ✓ (?: Yes)

I (S2) G: $x<3$
CN x=1 ✓ (?: No)
CN x=1.5 ✓ (?: Yes)

I (tog)
CN x=1 ✓ (?: No)
CN x=1.5 ✓ (?: Yes)

12TEN: Nothing is sufficient; pick choice E.

x=1 is a good number to start with for statement one because it's *close* to the boundary set by the statement (0). Usually, we would also want to choose a number (like *x*=1 million) that is *far* from the boundary. In this case, however, it doesn't make sense to proceed to choose a number that's far from the boundary, since such a number would just give us the same answer to the Question as *x*=1 ("no").

Statement one was *insufficient*, so we do *not* have a free answer for statement two.

Notice how reusing our numbers from statement one for statement two and for the statements together simplified our work and saved us time.

MISTAKE: If you don't think to choose a number (like *x*=1.5) that is between 1 and 2, then statements one and two will seem sufficient. Remember, after you get an answer of "no", your next step should be to choose a new number, **trying** to get an answer of "yes". The only way to get an answer of "yes" for #18 is to choose a number between 1 and 2.

Some people would avoid picking a number like 1.5 because they unconsciously assume that *x* has to be an integer. Remember that variables don't have to be integers unless the givens *say* they have to be integers. Be willing to choose nonintegers.

20.

The best way to solve #20 is with algebra. However, you could also use choosing numbers. First, let's see how to solve #20 by choosing numbers:

?: Is *x* negative?

S (S1) G: $9x>10x$ CN ~~x=0~~ $9x=9(0)=0$, $10x=10(0)=0$ CN ~~x=1~~ $9x=9(1)=9$, $10x=10(1)=10$ CN $x=-1$ √ (?: Yes) $9x=9(-1)=-9$, $10x=10(-1)=10$	I (S2) FA (?: Yes) G: $x+3$ is positive CN $x=0$ √ (?: No) $x+3=0+3=3$

12TEN: Only statement 1 is sufficient; pick choice A.

Statement one: It's good to choose 0 and choose 1, but they turn out not to be consistent with statement one. If you try choosing more positive numbers for *x*, you will find that none of them are consistent with statement one. Eventually you should stop choosing numbers and conclude that statement one is sufficient.

Since statement one is sufficient, it gives us a free answer ("yes") for statement two.

Notice how, for both statements, we labeled the work we did for step two (checking whether our numbers were consistent or inconsistent with the relevant givens).

MISTAKE

I (S1) G: $9x>10x$ CN $x=0$ no $9x=9(0)=0$, $10x=10(0)=0$ CN $x=-1$ √ (?: Yes) $9x=9(-1)=-9$, $10x=10(-1)=10$	This illustrates the **mistake** of treating the given like a Question. The student has said to themselves, "No, $x=0$ is not consistent with the statement," which confused them into thinking that they have gotten an *answer to the Question* of "no". The student can catch their mistake by noticing that they have wrong notation: the student's notes for $x=0$ match neither √ (?) nor ~~—~~, which are the only two correct patterns

MISTAKE: Some students may avoid choosing a number like $x=0$ for statement two "because it's inconsistent with the Question." This will make them think that the only answer is "yes", which will makes statement two seem sufficient. This is the mistake of treating the Question like a given. "Inconsistent with the Question" is a meaningless phrase that you should never use. Since we already have an answer of "yes" for statement two (our free answer) we should be *trying* to choose a number (like $x=0$) that will give us an answer of "no".

Now, let's see the better way to solve #20, using algebra:

?: Is $x<0$?	Rephrase the Question as a mathematical inequality.
S (S1) G: $9x > 10x$ $\Rightarrow 0x > 1x$ $\Rightarrow 0 > x$ $\Rightarrow$ [a] $x < 0$ (?: Yes)	To solve the inequality for *x*, we need to get all the *x*'s on one side of the inequality. We can get all the *x*'s onto the right-hand side by subtracting 9*x* from both sides. Inequality [a] makes it clear that the answer to the Question is "yes".
I (S2) (?: Yes) G: $x+3 > 0$ $\Rightarrow$ [b] $x > -3$ CN $x = 1$ million √(?: no)	Statement one is sufficient, so it gives us a free answer ("yes"). *Rephrase* the statement as a mathematical inequality. Solve the inequality for *x* by subtracting 3 from both sides. It may be obvious to you from [b] that statement two is insufficient. If it is not obvious, choose numbers. Because of [b], it is clear that "*x* = 1 million" is consistent with statement two—there is no need to plug "*x* = 1 million" back into the statement for step two.

<u>1</u>2TEN: Only statement <u>1</u> is sufficient; pick choice A.

Some people will not know how to solve statement one for *x*. The key to solving for *x* is to get all the *x*'s on one side of the equation. Notice that it would not help much to divide both sides of the inequality by 9, since dividing by 9 does not help us to get both *x*'s on the same side of the inequality.

The reason that some people won't think of subtracting 9*x* from both sides is that they are used to subtracting to "undo" addition, and there doesn't seem to be any addition going on in statement one. If statement one had said "$4 + 9x > 10x$", then it might be more obvious that, since 9*x* is being **added** to the left side, we have to **subtract** 9*x* in order to move it to the right side.

It might help to think of statement one as "$0 + 9x > 10x$". This shows that in statement one, the 9*x* really is being added to the left side, so that it makes sense to subtract 9*x* to move it to the right side.

MISTAKE

For statement one, you might be tempted to "cancel" the *x* from both sides of the inequality. This is not legal. "Cancelling" the *x* really means, "divide both sides of the inequality by *x*." However, we don't know yet whether *x* is positive or negative (that's what we're trying to figure out), so if we divide the inequality by *x* we won't know whether or not we should flip the ">" sign into a "<". So we shouldn't divide by *x* in the first place.

For the same reason, you cannot get all the *x*'s on one side by dividing both sides by 9*x*. We cannot divide the inequality by 9*x* because we don't know yet whether 9*x* is positive or negative.

MORAL: You cannot divide an inequality by a variable unless you *already* know whether the variable is positive or negative.

Note

From this point on, to avoid infringing GMAC's copyright, I will avoid copying the Question and givens into this document. Please remember, however, that in your own scratchwork for data sufficiency, I recommend that you should almost always write down the Question, and that you should usually write down the givens as well.

21.

(S2) CN n=0	1. We need numbers for both n and m. We will choose a number for n, and determine a number for m. You should usually start by choosing 0.
CN n=0, m=0 $6m=9(0) \Rightarrow 6m=0 \Rightarrow m=0$	Use the number we **chose** for n, together with statement two, to **determine** m.
CN n=0, m=0 ✓ $6m=9(0) \Rightarrow 6m=0 \Rightarrow m=0$	2. Check consistency with the givens. Since we used statement two to determine m, we **already** know that our n and m **are** consistent with statement two—there is no need to plug them back into statement two.
CN n=0, m=0 ✓ (?: Yes) $6m=9(0) \Rightarrow 6m=0 \Rightarrow m=0$ $2m - 3n = 2(0) - 3(0) = 0$	3. Answer the Question. Label your work. #21 has a yes/no Question, not a number Question, so the answer is "yes", not "0".
CN n=2	4. Choose a new n, trying to get an answer of "no."
CN n=2, m=3 $6m=9(2) \Rightarrow 6m=18 \Rightarrow m=3$	Use the number we **chose** for n, together with statement two, to **determine** our new m.
CN n=2, m=3 ✓ $6m=9(2) \Rightarrow 6m=18 \Rightarrow m=3$	2. Check consistency with the givens. Since we used statement two to determine m, we **already** know that our n and m **are** consistent with statement two, so there is no need to plug our numbers back into statement two.
S (S2) CN n=0, m=0 ✓ (?: Yes) $6m=9(0) \Rightarrow 6m=0 \Rightarrow m=0$ $2m - 3n = 2(0) - 3(0) = 0$ CN n=2, m=3 ✓ (?: Yes) $6m=9(2) \Rightarrow 6m=18 \Rightarrow m=3$ $2m - 3n = 2(3) - 3(2) = 0$	3. Answer the Question. Label your work. #21 has a yes/no Question, not a number Question, so the answer is "yes", not "0". If you keep trying new numbers, you will find that any numbers that are consistent with statement two will give you an answer of "yes". So you should eventually stop choosing new numbers and conclude that statement two is sufficient.

MORAL: We need numbers for all the variables. But, rather than **choosing** numbers for *all* of the variables, it is often easier to **choose** numbers for *some* of the variables, and then use the givens to **determine** values for the other variables.

It's easier to choose appropriate numbers if you rephrase statement two by solving it for m, which gives you: "$m = \frac{3}{2}n$". It is often a good idea to solve an equation for one of the variables!

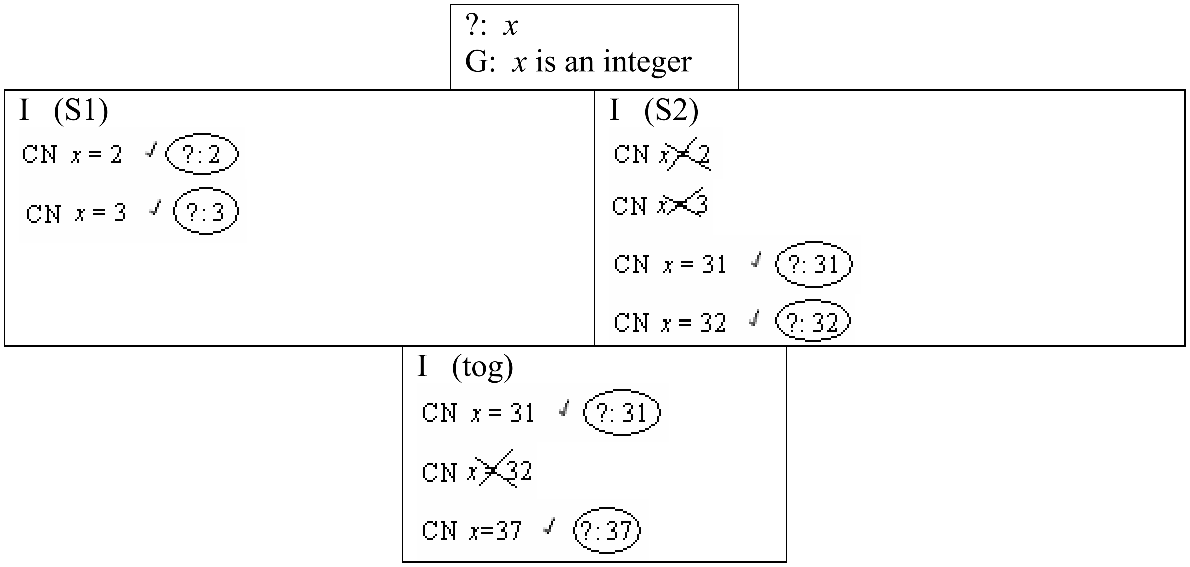
It is better to attack #21 with algebra than by choosing numbers. Here is one way to attack statement two using algebra:

?: Does 2*m*=3*n*?	We will rephrase the Question by solving the equation for *m*. Shown at left is the first step in solving the equation in the Question for *m*.
?: Does $m = \frac{3}{2}n$?	This is the second and final step in solving the equation for *m*. It is often a good idea to solve equations for one of the variables.

We have already mentioned (on the previous page) that statement two can be solved for *m* to give "$m = \frac{3}{2}n$". This should make it plain that there is only one possible answer to the Question that is consistent with statement two: "yes". So, statement two is sufficient.

MORAL: It is often a good idea to solve equations for one of the variables. **This applies not only to equations in the givens, but also to equations in the *Question*.**

22.



12TEN: Nothing is sufficient; the correct choice is E.

The initial information contains a hidden given.

Statement one is probably obvious enough to you that you didn't have to formally choose numbers to evaluate it.

Since statement one is insufficient, it does **not** give us a free answer for statement two.

We tried to reuse our numbers from statement one (x=2 and x=3) on statement two; but the numbers were not consistent with statement two, so we threw them out. We tried to reuse our numbers from statement two (x=31 and x=32) for the statements together; but x=32 was inconsistent with statement one, so we had to throw it out.

The Question is "What is the value of x?" Therefore, when we choose x=31, the answer to the Question is "31"; when we choose x=37, the answer to the Question is "37"; etc.

Don't feel uncomfortable about choosing a number for a variable when the Question is asking for the value of that same variable. We can use the same choosing numbers method for such situations as we would normally use in any other situation.

A MISTAKE you might make when statement one is to choose x=2, get an answer to the Question of "2", and assume that, since you've "gotten an answer", statement one is sufficient. This is the mistake of choosing only one set of numbers. Remember that "getting an answer" doesn't mean "sufficient". You can *always* get an answer. The issue is whether you can get *more than one* answer—and the only way to tell whether you can get more than one answer is to choose more than one set of numbers.

You might have committed this mistake while evaluating statement two, or while evaluating the statements together, as well.