GMAT Data Sufficiency Manual
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Read this first

I make no guarantees as to your results from using the methods in this manual.

Most of the methods described in this manual will work well for most test-takers in most situations, but few methods are perfectly applicable to all situations or all test-takers. Use your judgment.

On the other hand, if you find a method in this manual challenging to apply, don't give up on it too quickly—give it a fair shot. It is natural for *any* new technique to initially feel unfamiliar and time-consuming. If you keep using the same techniques on the GMAT that you are *already* comfortable with, you can expect to keep getting the same score! If you would like to *change* your GMAT score, the only way to do so is to *change* your GMAT techniques—even though at first any new technique is likely to feel uncomfortable and to slow you down.

The techniques described in this manual are illustrated in the Supplementary Explanations to the *GMAT Official's Guide*'s Data Sufficiency Diagnostic Questions and Sample Questions, available at http://www.freelance-teacher.com. Therefore, this manual should be used in conjunction with those supplementary explanations.

Problems in the *Official Guide*'s Data Sufficiency Sample Questions (section 6.3) will be referred to with the "#" symbol. For example, "#6" refers to Data Sufficiency Sample Question #6, on page 278 of the *Official Guide*. For another example, #91 refers to Data Sufficiency Sample Question #91, on page 285 of the *Official Guide*. Problems in the data sufficiency portion of the *Official Guide*'s Quantitative Diagnostic Test (section 3.1, pp. 24-25) will be referred to with a "D"; e.g., "D26" means problem number 26 of the math Diagnostic, on page 24 of the *Official Guide*.

Terms in this font are discussed in more detail elsewhere in the manual. Consult the index to find information on each topic.

There are two types of GMAT math problems: "data sufficiency" and "problem solving". "Problem solving" questions are "normal" math problems such as you might encounter in math classes or on other tests. "Data sufficiency" is an unusual type of math problem that appears only on the GMAT. This manual focuses on data sufficiency, not problem solving. Some of the data sufficiency techniques and concepts covered in this manual are useful for problem solving as well, but some are not.

Abbreviations used in this manual:

? = Question	(S1) = statement 1 alone	CN = choosing numbers
G = given	(S2) = statement 2 alone	✓ = consistent with the givens
S = sufficient	(tog) = both statements together	✗ = inconsistent with the givens
I = insufficient		FA = free answer

Basics

The components of a data sufficiency problem

Each data sufficiency problem consists of the "initial information" and two "statements". For example, on #11 in the *Official Guide* Data Sufficiency Sample Questions, the initial information is "If $5x + 3y = 17$, what is the value of x?" Statement one is "x is a positive integer". Statement two is "$y = 4x$".

There are two types of initial information: the "Question", and the "initial givens". On #11, the Question is "what is the value of x?" The initial given is "$5x + 3y = 17$".

This manual will capitalize the word "Question" to remind you that it refers to one **part** of a data sufficiency problem, rather than to the entire problem.

The initial information will always contain exactly one Question. The initial information may or may not contain givens. For example, for #4, there is no initial given, only a Question ("Is r greater than 0.27?").

The word "if" indicates givens. Examples: D25, D27, D30, #15, #17

There are two main types of Question: "number Questions" and "yes/no Questions". If the answer to the Question is a number, then it's a number Question. If the answer to the Question is "yes" or "no", then it's a yes/no Question. For example, the Question for #1 is a number Question. The Question for #4 is a yes/no Question.

What are the statements? Are they Questions, or givens?
The statements are givens.

To summarize, Data Sufficiency problems have three components:
(1) the Question,
(2) the initial givens (if any), and
(3) the statements (which are additional givens).

Hidden givens

The initial information will occasionally include a "hidden given".

For example, in #22 the Question is "What is the value of x?" We know that this is the Question because they are *asking* us the value of x, as opposed to *telling* us the value of x.

The initial given for #22 is "x is an integer." This is a given because they are *telling* us that x is an integer, as opposed to *asking* us *whether* x is an integer. This is a hidden given because at first it *appears* to be part of the *Question*.

Examples: D26, D28, #5, #14, #22

The data sufficiency choices

Here is a table that summarizes the circumstances for which each of the data sufficiency choices is correct. Remember that *S* stands for "sufficient", and *I* stands for "insufficient".

choice	statement one alone	statement two alone	both statements together
A	S	I	
B	I	S	
C	I	I	S
D	S	S	
E	I	I	I

Notice from this table that *we only need to evaluate the statements* ***together*** *when* ***both*** *statements are* ***insufficient alone***; in this case, the correct choice is C or E. If either statement is sufficient alone, then there is no need to evaluate the statements together; in this case, the correct choice is A, B, or D.

We can use the mnemonic "12TEN" to remember when to pick each choice:

1	(A)	only statement 1 is sufficient
2	(B)	only statement 2 is sufficient
T	(C)	the statements are **only** sufficient together
E	(D)	each statement is sufficient alone
N	(E)	nothing is sufficient, even when the statements are considered together

T stands for "together"; pick choice (C) when the statements are only sufficient together. *E* stands for "each"; pick choice (D) when each of the statements is sufficient alone. *N* stands for "nothing"; pick choice (E) when the statements are insufficient together, so that nothing is sufficient.

Keep in mind that we pick choice C when the statements are sufficient together *but both statements are insufficient alone*. I.e., we pick choice C when the statements are **only** sufficient together. If either statement is sufficient alone, then the correct choice is A, B, or D, and there is no need to evaluate the statements together.

The meaning of "sufficient" and "insufficient"

*In data sufficiency, the words "sufficient" and "insufficient" have precise, **technical** definitions that are **different** from their definitions in ordinary speech.* Therefore, when doing data sufficiency, you have to ignore the ordinary definition of "sufficient" and focus only on the technical definition.

Here are the technical definitions:

sufficient	**exactly one** answer to the Question which is consistent with all the relevant givens
insufficient	**two or more** answers to the Question which are consistent with all the relevant givens

Don't worry if you don't fully understand these definitions yet; you will see many illustrations of how to use the definitions in the supplementary explanations.

What does it mean if you can't find *any* answers to the Question which are consistent with all the relevant givens?

It means you screwed up. The GMAT is designed so that there will always be at least one answer to the Question that is consistent with all the relevant givens.

A common *mistake* is to think that, if you can't find any answers that are consistent with the givens, then the statement is insufficient. But remember that insufficient means "**two or more** answers", not "**zero** answers." There will *never* be *zero* answers.

Students commonly say that a statement is sufficient if you "can get an answer to the Question". This is *poor* wording because, as we have just seen, you can *always* get an answer to the Question. The issue is not *whether* you can get an answer, but whether you can get *more than one* answer.

Of course, in order to use these technical definitions, we have to know what the "relevant" givens are.

When evaluating…	**…the relevant givens are:**
statement one alone	the initial givens and statement one
statement two alone	the initial givens and statement two
the statements together	the initial givens, statement one, and statement two

Students sometimes find the concept of evaluating statements "together" to be confusing or mysterious. Hopefully, this table makes it clear that there is nothing mysterious about evaluating the statements together. "Evaluating the statements together" just means treating both statements as relevant givens. There is no need to somehow "combine" the statements. Again, to "evaluate the statements together", you just have to treat both statements as relevant givens.

Examples of evaluating the statements together: D25, D26, D27, #5, #7, #13, #18, #22

Free answers

When a statement is sufficient, we know that there is exactly one answer to the Question that is consistent with that statement and with the initial givens.

It turns out that the GMAT is constructed so that, when a statement is sufficient, its one answer will *also* be consistent with the *other* statement. This is simply a fact about the way that the GMAT happens to be written.

Therefore, when a statement is *sufficient*, it gives you a "free answer" which you automatically know is consistent with the other statement as well.

Sometimes you will be able to determine that the first statement you evaluate is sufficient—i.e., that it allows only one answer to the Question—without actually calculating what the one answer *is*. In such a case, even though the first statement was sufficient, you will have no free answer to use for the second statement.
Examples: #1, #2

Please notice that when a statement is *insufficient*, there is *no* free answer.

Notice that the free answer strategy never applies when you are evaluating the statements *together*.

Don't worry if you don't fully understand free answers yet. Consult the supplementary explanations for examples of how to apply the free answer strategy.
Examples: D31, D33, #3, #4, #9, #14, #17, #20

We will use the abbreviation "FA" to indicate "free answer".

Write it down

It is helpful to have the most important components of the problem written down on your scratch paper. That way, you can focus your attention on your scratch paper without having to continually glance back at the screen. For this reason, on data sufficiency you should almost always write down the Question, and you should usually write down the givens as well.

It is also helpful to write down reasoning steps and stuff that you have figured out while solving the problem. Writing this stuff down has at least two advantages: (1) You will only have to figure stuff out once. If you ever forget what you figured out, you can just consult your written notes to remind yourself, rather than having to figure it out all over again. (2) It's easier to check your work for mistakes if you can review each step on paper, rather than trying to remember the steps in your head.

Labeling

When you write down numbers or calculations, you should "label" them so you know what they represent. For example, don't write down "14". Instead, write down "x=14". For another example, don't write down "14+56=70". Instead, write down "x+y=14+56=70".

Rephrasing

It is often useful to "rephrase" the Question. For example, you might want to rephrase the Question as a mathematical equation or expression. Or, if the Question is an equation, you might want to *solve* the equation for one of the variables.

Example: D29, D33, #1, #2, #4, #12, #16, #20, #21

It is often useful to rephrase the givens. For example, you might want to rephrase the given as a mathematical equation. Or, if the given is an equation, you might want to *solve* the equation for one of the variables.

Example: #1, #3, #4

Avoid needless calculations

On data sufficiency, you do not actually need to *determine* the answer to the Question. You just have to determine whether there is *more than one* answer to the Question.

In many cases, it is helpful to actually calculate answers to the Question in order to determine whether the Question has exactly one answer or more than one answer. In many other cases, however, you can determine whether the Question has exactly one answer or more than one answer **without** calculating any specific answers. A *common mistake* on data sufficiency is to waste time calculating answers in situations where it's unnecessary.

Examples: D25, D26, D29, #1, #2

Yes/no Questions

Consider a data sufficiency problem with a yes/no Question.

Suppose that, when you evaluate a statement, you find that the *only* answer which is consistent with the relevant givens is "yes". Is the statement sufficient or insufficient?

It's sufficient. We have gotten exactly one answer to the Question ("yes"), and "exactly one answer" means sufficient.

Now suppose that, when you evaluate a statement, you find that the *only* answer which is consistent with the relevant givens is "no". Is the statement sufficient or insufficient?

It's **sufficient**. We have gotten exactly one answer to the Question ("no"), and "exactly one answer" means sufficient.

A *common mistake* is to think that an answer of "no" indicates **insufficient**. This is an easy mistake to make because the word "no" is negative-sounding, and the word "insufficient" is also negative-sounding, so it's tempting to think that they mean the same thing.

We have seen that, if the only answer that is consistent with the relevant givens is "yes", then the statement is sufficient. And, if the only answer that is consistent with the relevant givens is "no", then the statement is also sufficient. So when can a statement be *insufficient*?

A statement is insufficient when the answers "yes" and "no" are **both** consistent with the relevant givens. In that case, there would **two** answers which are consistent with the relevant givens, and "two answers" means "insufficient".

sufficient	**insufficient**
"Yes" is the *only* answer consistent with all the relevant givens.	"Yes" and "no" are *both* consistent with all the relevant givens.
"No" is the *only* answer consistent with all the relevant givens.	

Systematic method for data sufficiency

step	**comments and notation**
1. Identify the Question.	Write down the Question. Label it with a question mark.
2. Identify the initial givens, if any.	You should usually write down the initial givens. Label them with a *G*.
3. Evaluate the easier statement first.	Write down *(S1)* or *(S2)* to indicate which statement you're evaluating. You should usually write down the statement; remember that the statement is a given, so label it with a *G*. When you are done evaluating the statement, write down an *S* or an *I* to indicate whether the statement is sufficient or insufficient.
4. Then evaluate the other statement.	Again, write down either *(S1)* or *(S2)* to indicate which statement you're evaluating. If the first statement you evaluated was *sufficient*, and if you figured out the one answer to the Question that is consistent with that statement and with the initial givens, then write that answer down as a free answer that is automatically consistent with the second statement as well. Label the free answer with a question mark, and circle it. If the first statement you evaluated was *insufficient*, then there is no free answer. Again, you should usually write down the statement, labeling it with a *G*. When you are done evaluating the statement, again write down an *S* or an *I* to indicate whether the statement is sufficient or insufficient.
5. If either statement was sufficient alone, you can now pick choice A, B, or D *without* having to evaluate the statements together. If both statements were insufficient alone, you *do* need to evaluate the statements together; in this case the correct choice will be C or E.	If you need to evaluate the statements together, write down *(tog)*; when you are done evaluating the statements together, you should again write down an *S* or an *I* to indicate whether the statements together are sufficient or insufficient.

Additional comments on the systematic method for Data Sufficiency:

1. Identify the Question. If the Question is a yes/no Question, write down the verb. For example, if the Question is "Is $x > 4$", do *not* write: ?: x>4? Instead, write: ?: Is x>4? It is often useful to rephrase the Question.
2. Identify the initial givens, if any. It is often useful to rephrase the initial givens.
3. Evaluate the easier statement first. Don't spend more than a second deciding which statement is easier; if you have no guess as to which statement is going to be easier, just evaluate statement one first. Oftentimes, the shorter statement will be easier. If you start by evaluating statement two, it is especially important to label it as *(S2)* so that you don't start thinking of it as statement one. It is often useful to rephrase the statement.
4. Then evaluate the other statement. If evaluating the statement seems like it will be time-consuming, consider just taking a guess and moving on to the next problem. Again, it is often useful to rephrase the statement.
5. If necessary, evaluate the statements together. If evaluating the statements together seems like it will be time-consuming, consider just taking a guess and moving on to the next problem.

Don't worry if you don't completely understand some of these steps or comments yet; they will become clearer when you study the supplementary explanations.

Choosing numbers for data sufficiency

Systematic method for choosing numbers for data sufficiency

steps	**notation**
1. Choose numbers	Write down your numbers. Label your numbers by writing down the variables you are setting equal to the numbers.
2. Are your numbers are consistent or inconsistent with the relevant givens?	Label any calculations you do while executing this step.
If your numbers are inconsistent with some or all of the relevant givens: Throw the numbers out and choose new numbers—i.e., go back to step one.	If your numbers are inconsistent with the givens, cross them out. *Never* write down the word "no" to show that your numbers are inconsistent; instead, simply cross them out.
If your numbers are consistent with all the relevant givens: Go on to step three.	If your numbers are consistent with the givens, make a checkmark ("√"). *Never* write down the word "yes" to show that your numbers are consistent; instead, make a check mark.
3. When you choose these numbers, what is the answer the Question?	Label any calculations from this step. Write down the answer to the Question. Label the answer with a question mark ("?"), and circle it.
4. Go back to step one: Choose more numbers, trying to get a *different* answer to the Question.	

When can you stop choosing new numbers?

Once you have found **two** different answers to the Question which are consistent with the all the relevant givens, you can *immediately* stop choosing numbers, because you have *proven* that the statement is insufficient.

If you have chosen many different numbers and have gotten only **one** answer to the Question that is consistent with all the relevant givens, you can *guess* that the statement is sufficient. Unfortunately, the choosing numbers strategy usually can't *prove* that a statement is sufficient. You will have to use your own judgment to determine when to stop choosing new numbers and conclude that the statement is sufficient.

Step one: Choose numbers

Always write down your numbers.

Always label your numbers with the variables you're setting equal to the numbers. For example, don't write down "4, 9". Instead, write down "*x*=4, *y*=9".

In most situations, you will need specific numbers for *all* the variables in the Question and relevant givens.

Therefore, in most situations, you need numbers for all the variables in the givens, even if some of those variables are not mentioned in the Question. The reason is that you can't be sure that your numbers are consistent with the givens unless you've obtained numbers for *all* the variables in the givens. Examples: #7 (tog), #9 (S2), #13 (tog)

Similarly, you usually need numbers for all the variables in the Question, even if some of those variables are not mentioned in the statement. The reason is that you can't answer the Question unless you have numbers for all the variables in the Question. Example: D30

Although you need numbers for all the variables in the Question and relevant givens, you will not necessarily want to *choose* numbers for all of those variables. Instead, it may be convenient to choose numbers for only *some* of the variables. Then, use the numbers you chose, together with the relevant givens, to *determine* numbers for the remaining variables.

Examples: #7, #12, #13, #21 (S2), D30

In some cases, when you choose numbers for *some* of the variables, you will be able to determine whether the remaining variable is consistent with the givens *without* needing to determine a *specific* number for that variable, allowing you to avoid needless calculations. These cases are exceptions to the general rule that you need to obtain specific numbers for all the variables in the givens. Example: D27

Sometimes, a given will put restrictions on a mathematical **expression** that is **based** on a variable, rather than on the variable itself. In such cases, rather than choosing a number for the variable itself, it may be convenient instead to (1) *choose* a number for the mathematical expression that is based on the variable, and then (2) use the number you chose for the mathematical expression to *determine* a number for the variable itself.

For example, in #15, statement two is about $-b$. Therefore, rather than choosing a number for b, it is convenient when evaluating statement two to (1) *choose* a number for $-b$, and then (2) use our number for $-b$ to *determine* a number for b.

Examples: #15 (S2), #17

If you have a free answer from the first statement you evaluated, then, if you choose numbers for the second statement, you should try to get an answer that is *different* from the free answer.

Examples: #3, #4, #9, #14, #20

On data sufficiency, the first number you choose should usually be "0". Examples: D29, D31, #3, #7, #9, #15, #20, #21

You should also be eager to choose "1". Examples: #5, #7, #9, #12, #15, #20

If you pick negative numbers, the first negative number you pick should usually be "-1". Example: #15

(There are certain situations in problem solving where you should avoid picking 0, 1, or -1; however, those situations do *not* apply to data sufficiency.)

Of course, you should avoid choosing 0, 1, or -1 if those numbers are inconsistent with the givens!

When you are given a boundary, you should usually start by choosing a number that is close to the boundary; then pick a number that is far from the boundary. For example, suppose you're given "$x>7$". A good first number to pick would be $x=8$, since it's close to the boundary. A good second number to pick would be "$x=1$ million", since it's far from the boundary. For another example, if x is a percentage, then $0\% \leq x \leq 100\%$, so we should start by choosing $x=0\%$ and then choose $x=100\%$, since both numbers are at boundaries. Examples: D29, D31, #4, #7, #15, #16, #18

If you can't find *any* numbers that are consistent with the relevant givens, does that mean that the statement is *sufficient* or *insufficient*?

Neither. It means you just haven't found the right numbers yet. Remember that there is always at least one answer to the Question that is consistent with all the relevant givens. Therefore, there is always at least one set of numbers that is consistent with all the relevant givens.

If you have to choose numbers for more than one variable, it is usually a good idea to start by choosing the same number for all the variables. For example, if you have to choose numbers for x and y, it would be good to start by choosing $x=0$ and $y=0$. Of course, if you are given that x and y are distinct, then you should *not* choose the same number for both, as that would be inconsistent with the given. But, unless the givens *say* that the variables have to be distinct, they don't have to be distinct. Examples: #5, #12

(There are certain situations in problem solving where you should avoid choosing the same number for multiple variables, but those situations do not apply to data sufficiency.)

Reuse your numbers. Try to use the same numbers for the second statement that you evaluate as you used for the first statement. Try to use the same numbers for evaluating the statements together as you used to evaluate the statements separately. Examples: D25, D26, D30, D31, #5, #18, #22

Exceptions: (1) Of course, if your old numbers are not consistent with the new statement, you will have to throw them out and try new numbers. Examples: D26, D30, D31, #22. (2) If you got a free answer from the previous statement, then there is no point reusing your numbers from that statement since they will simply give you that same free answer again. Instead, in this situation, you should be trying to choose numbers that will give you a *different* answer than you got from the previous statement. Examples: D33, #3, #16

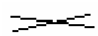
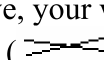
Step two: Are your numbers <u>consistent</u> or <u>inconsistent</u> with the relevant givens?

Do **not** go on to step three unless your numbers are consistent with all the relevant givens. If your numbers are **not** consistent with the relevant givens, throw the numbers out and return to step one—i.e., choose new numbers.

Do **not** go on to step three unless your numbers are consistent with ***<u>all</u>*** the relevant givens. If your numbers are <u>inconsistent</u> with *any* of the relevant givens, you must throw them out, even if the numbers are consistent with the other givens.
Example: #5 (S2)

A *common mistake* is to check only **some** of the relevant givens. For example, sometimes students check whether their numbers are consistent with the statement, but forget to check whether the numbers are consistent with the initial given.

Executing step two may involve some calculations. Label the work you do while executing these calculations.
Examples: D33 (S1), #3 (S1), #9, #12 (S1), #20

If your numbers are *consistent* with the relevant givens, do **not** write down the word "yes". **Instead**, make a checkmark: √
If your numbers are *inconsistent* with the relevant givens, do **not** write down the word "no". **Instead**, cross the numbers out: 
Note that, if you use good notation as recommended above, your work from step two should **always** include **either** a checkmark (√) **or** a crossout (). Make a habit of checking to make sure that your notation for each set of numbers you choose includes either a checkmark or crossout. If it includes neither, you know you have made a mistake.

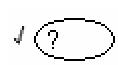
A *common mistake* on yes/no Questions is "treating the given like a Question".

When your numbers are *<u>consistent</u> with the **givens***, it is tempting but WRONG to think that that means that *the answer to the **Question*** is "yes". "Consistent" is a positive-sounding word, so it is tempting (but wrong) to think that consistent numbers guarantee an answer of "yes". Actually, when your numbers are consistent with the givens, it is possible that the answer to the Question will be "yes", but it is equally possible that the answer to the Question will be "no".

When your numbers are *<u>inconsistent</u> with the **givens***, it is tempting but WRONG to think that that means that the *answer to the **Question*** is "no". "Inconsistent" is a negative-sounding word, so it is tempting but wrong to think that inconsistent numbers generate an answer of "no". Actually, when your numbers are inconsistent with the givens, you never get any answer to the Question at all—when your numbers are inconsistent with the givens, instead of going on to step three, you just throw your numbers out and go back to step one.

Here are some habits that will help you avoid the mistake of treating the given like a Question:

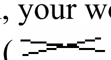
- You should usually write down the givens. Label each given with a *G* to remind yourself that it's a *given*, not a Question.
- When you are executing step two, use the precise wording provided in the systematic approach. Do *not* ask yourself, "Is my number <u>consistent</u> with the relevant givens?" This is dangerous phrasing because it prompts you to respond with the words "yes" or "no", which can tempt you into thinking that you have discovered that the **answer to the Question** is "yes" or "no". *Instead*, ask yourself, "Is my number <u>consistent</u> or <u>inconsistent</u> with the relevant givens?" This is better phrasing because it prompts you to respond with the words "consistent" or "inconsistent", which will remind you that you are still on step two, not step three.
- Use good notation. **Never** write down the word "yes" to indicate that a number is *<u>consistent</u> with the givens*, since that will tempt you to think that you have obtained an *answer to the Question* of "yes". Instead, write down a checkmark ("√") to indicate that the number is consistent with the givens. Similarly, **never** write down the word "no" to indicate that a number is *<u>inconsistent</u> with the givens*, since that will tempt you to think that you have obtained an *answer to the Question* of "no". Instead, just cross the numbers out to indicate that they are inconsistent with the givens. If you use good notation, your notes for each set of numbers you choose should either

look like this (if the numbers are <u>consistent</u> with the givens): 

or like this (if the numbers are <u>inconsistent</u> with the givens): 

The advantage of using this systematic notation is that, when your notes look like neither of these patterns, you will be alerted that you have made a mistake—probably, treating the given like a Question.

Examples: D32, #4, #9, #12, #14, #20 (S1)

Again, please note carefully that, if you use good notation, your work from step two should **always** include **either** a checkmark (√) **or** a crossout (). If your work from step two includes neither of these notations, you must have made a mistake—likely, treating the given like a Question.

Step three: When you choose these numbers, what is the answer to the Question?

Use this wording when you are thinking about step three: "When I choose these numbers, what is the answer to the Question?"

Don't say that step three is: "Do these numbers answer the Question?"—this is a pointless, misleading, and dangerous thing to say. Your numbers will **always** answer the Question. The issue is not **whether** you can answer the Question, but **how many** answers there are.

Remember, there is always at least one answer to the Question—the only issue is whether there is *more than one* answer to the Question.

An even more accurate way to describe step three is, "Figure out what the answer to the Question is when the variables are equal to the numbers you have chosen."

Executing step three may involve some calculations. Label the work you do while executing these calculations.

Examples: D33, #3, #12, #17, #21

Use good notation. Write down the answer to the Question, label it with a question mark ("?") and circle it.

When choosing numbers, it is usually important to calculate a precise answer to the Question when executing step three. In some cases, however, it may be possible to figure out whether you can obtain two different answers *without* actually calculating precise answers. In such cases, you can save time by avoiding needless calculations. Examples: D26, D28, D29, #5

When the Question is asking for the value of a variable, you may end up choosing a number for that variable.

For example, consider the Question, "What is the value of *z*?" Suppose you end up choosing *z*=8. When you choose *z*=8, what is the answer to the Question?

When you choose *z*=8, the answer to the Question is "8".

In this type of situation, some students find the idea of "answering the Question" to be confusing. Therefore, it may be helpful to remember the most accurate way of describing step three: "Figure out what the answer to the Question is when the variables are equal to the numbers you have chosen." For example, when we choose *z*=8, the answer to the Question ("What is the value of *z*?") is "8".

The very fact that step three is so easy to execute in this situation can sometimes make students uncomfortable—but it shouldn't! You should follow the same exact choosing numbers method in this type of situation as in any other.

Examples: D26, #13, #22

Step four: Choose more numbers, trying to get a *different* answer to the Question

Remember that "insufficient" means "at least two answers". If you only choose one set of numbers, then you can't possibly get more than one answer. The only way to give yourself a fair chance to get two answers is to choose at least two sets of numbers.

On problem solving, you can usually choose just one set of numbers. For the reason we have just explained, however, on data sufficiency it is not enough to choose just one set of numbers—you have to choose at least **two** sets of numbers.

A *common mistake* on data sufficiency is to choose just **one** set of numbers. People make this mistake when they use poor wording to describe step three. Remember that the correct wording for step three is, "When I choose these numbers, what is the answer the Question?" If, instead, you wrongly think of step three as, "Do these numbers answer the Question?", then, when your numbers *do* give you an answer to the Question, you are likely to wrongly think that that means that the statement is sufficient. Remember, your numbers will always give you an answer. The issue is how many answers you can get, not whether you can get an answer. (There is always at least one answer to the Question.)

Examples of the mistake of choosing only one set of numbers: D28, #5, #12, #13 (tog), #16, #22

There are a few exceptional cases where it's OK to choose only one set of numbers for data sufficiency. These exceptions are will be discussed later in this section.

Say you choose one set of numbers, and the answer to the Question for those numbers is "yes". Does this mean that the statement is sufficient or insufficient?

Neither. If you have only chosen **one** set of numbers, then you can't tell yet whether the statement is sufficient or insufficient. To have a fair chance of getting two different answers to the Question, you have to choose at least **two** sets of numbers.

A *common mistake* is to think that getting one answer of "yes" means "sufficient". This mistake is tempting because "yes" is a positive-sounding word, and "sufficient" is also a positive-sounding word, so it seems like "yes" means "sufficient".

Example: #12

Say you choose one set of numbers, and the answer to the Question for those numbers is "no". Does this mean that the statement is sufficient or insufficient?

Neither. If you have only chosen **one** set of numbers, then you have no way of telling yet whether the statement is sufficient or insufficient. To give yourself a fair chance of getting two different answers to the Question, you have to choose at least **two** sets of numbers.

A *common mistake* is to think that getting one answer of "no" means "insufficient". This mistake is tempting because "no" is a negative-sounding word, and "insufficient" is also a negative-sounding word, so it seems like "no" means "insufficient".

Example: #16

When you choose more numbers, try to get a *different* answer to the Question. Unless you have actually *tried* to get a different answer to the Question, you cannot be confident that there is only one answer that is the consistent with the relevant givens.

When you are trying to get different answers to the Question, it makes sense to choose different *kinds* of numbers—if you keep choosing the same kind of number, you are likely to keep getting the same answer to the Question.

For example, in some cases it is important to try choosing nonintegers—i.e., fractional numbers. Remember that variables do not have to be integers unless the givens *say* they have to be integers. Example: #18

In some cases, you can use your intuition to guess which numbers are most likely to give you a different answer to the Question. In other cases, it's best to just use systematic trial and error to check all the possibilities. For example, you might systematically test x=0, x=1, x=2, x=3, x=4, x=5, x=6, x=7, x=8, etc..

As mentioned above, there are some exceptions to step four—i.e., there are some situations where you do not have to choose more than one set of numbers:

1. Sometimes the process of choosing one set of numbers will make it obvious to you that it will be impossible to get a different answer to the Question that is consistent with the relevant givens. In that case, you can conclude that there is only answer to the Question, and that the statement is sufficient, without having to choose any more numbers.

2. With a free answer, you may be able to prove that a statement is insufficient by choosing only one set of numbers. For example, suppose your free answer is "4". And suppose you choose one set of numbers, and get an answer of "7". Since you now know that there are at least two answers to the question that are consistent with all the relevant givens (namely, "4" and "7"), you can be sure that statement is insufficient, and there is no need to choose a second set of numbers. Examples: #3, #4, #14

A *common mistake* that can occur on yes/no Questions is "treating the Question like a given". For example, suppose the Question is "Is $x \geq 0$?" Some students might avoid choosing negative numbers for x because they seem "inconsistent with the Question". But if you choose only nonnegative numbers for x, then the only answer to the Question you can get will be "yes", and the statement will seem sufficient. The only way to test whether the answer can be "no" is to choose some negative numbers.

Treating the Question like a given is a mistake because, although we **do** have to choose numbers that are consistent with the givens, we do **not** have to choose numbers which are "consistent with the Question". In fact, "consistent with the Question" is a meaningless phrase which you should avoid using. In our example, the Question is not **telling** us that x is nonnegative—the Question is **asking** us **whether** x is nonnegative.

Here are some habits and ideas that will help you to avoid the mistake of treating the Question like a given:

- Write down the Question. Label it with a *?* to help yourself remember that it's a Question, not a given.
- For yes/no Questions, include the verb (usually "is" or "does") when you write down the Question. For our example we would write down

?: Is $x \geq 0$?

rather than

?: $x \geq 0$?

You can see that writing down the verb "is" makes the Question look more like a question and less like a given.

- Use good vocabulary. **Never** use the phrases "consistent with the Question" or "inconsistent with the Question"—these are **meaningless** phrases that tempt you to treat the Question like a given. On our example, if someone mistakenly says that x=5 is "consistent with the Question", what they really mean, and what they should really say, is "when I choose x=5, the answer to the Question is 'yes'". If someone mistakenly says that x = -4 is "inconsistent with the Question", what they really mean, and what they should really say, is "when I choose x = -4, the answer to the Question is 'no'".
- Realize that there is nothing "bad" about getting an answer to the Question of "no". An answer of "no" is just as useful as an answer of "yes".
- Remember that, when you execute step four, you should be *trying to get a different answer to the Question*. Therefore, if your first answer to the Question was "yes", when you choose new numbers you should be *trying* to get an answer of "no".

Examples: #4, #14, #16, #20

Vocabulary

The following table summarizes good and bad vocabulary for choosing numbers. Using good vocabulary will help you to avoid some of the common errors mentioned above. Using bad vocabulary will make you susceptible to committing those errors.

good vocabulary	**bad vocabulary**
Step two: "Are my numbers consistent or inconsistent with the givens?"	Step two: "Are my numbers consistent with the givens?"
The number is *consistent* with the givens. The number is *inconsistent* with the givens.	The number is *sufficient*. The number is *insufficient*.
Step three: "When I choose these numbers, what is the answer to the Question?"	Step three: "Do these numbers answer the Question?"
The answer to the Question is "8". The answer to the Question is "-27.3". The answer to the Question is "yes". The answer to the Question is "no".	The answer to the Question is "sufficient". The answer to the Question is "insufficient". The number is consistent with the Question. The number is inconsistent with the Question.
The statement is *sufficient*. The statement is *insufficient*.	The statement is *consistent*. The statement is *inconsistent*.

Summary of common mistakes for choosing numbers

- Treating the given like a Question
- Treating the Question like a given
- Choosing only one set of numbers

Examples

Examples of choosing numbers: D25, D26, D28, D29, D30, D31, D32, D33, #3, #4, #5, #7, #9, #12, #13, #14, #15, #16, #17

Choosing numbers is a versatile strategy. You can choose numbers, not just for variables, but also for a "list" or "set". Example: #5

You can choose numbers for geometry problems. Examples: D28, #13, #15

Examples of choosing numbers for evaluating the statements together: D25, D26, D28, D29, #5, #13, #18

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