KING ABDULAZIZ UNIVERSITY
Faculty of Environmental Design
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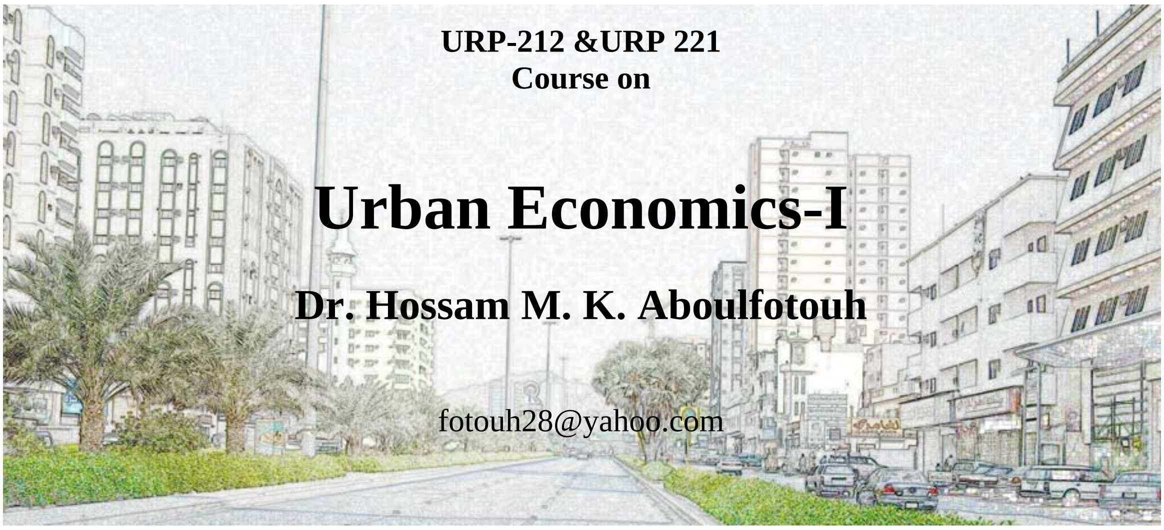

URP-212 &URP 221
Course on

Urban Economics-I

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<u>Lecture note#6: Using the Logarithms in Reckonings.</u>

Finding the logarithm of any number as (*A*), implies counting how many time (*n*) another number as (*B*) is repeated in the multiplication to get the number (*A*).

If *A*= 9 and *B*=3, we say *B* is repeated *n* times to get *A*; and in this case *n*=2; usually we write this in the following form: $3^2 = 9$, which could be denoted by the following general form.

$B^n = A$

In case we know both *A* and *B*; the value of *n* could be reckoned using the following equation.

$Log_B\ A = n$

In short, finding the logarithm of a number means finding the value of the exponent for specific *B* value. Here *B* is the base of the logarithm. In most calculators, *B*=10, which is the base of the common logarithms and is denoted by the **Log** key; also, ***B***=***e***=2.7182818.., which is the base of the natural logarithms, and is denoted by the **Ln** key, where ***e*** is the Euler's number. One can also use other values for *B* particularly in Excel spread sheets, e.g., *B*= 2.

For example,
Log 10=1,
Ln 10 =2.302...,
And
Log_2 10=3.321…

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And if we divided any of these values by the other, we can get the transformation module between any two logarithms as follows:

Log 10/ Ln 10 = 0.4342944819

Log 10/ Log_2 10 = 0.301029996

Ln 10/ Log_2 10 = 0.693147181

Application:

The equation of the demand curve is the following,

$Y = T * X^{-E}$

If T =50, X = 20, and E=0.5 then Y = 11.18034

This means

$Y/T = 1/X^E$

$X^E = T/Y$

$E \text{ Log } X = \text{Log } (T/Y)$

And,
$E = \text{Log}(T/Y) / \text{Log } X$

In Excel sheet we write it as follows:
1- Using base 10, E = SUM((LOG(T/Y))/(LOG(X)))
2- Using base ***e***, E = SUM((Ln(T/Y))/(Ln(X)))
Also,
3- Using base 2, E = SUM((LOG(T/Y,2))/(LOG(X,2)))

Now, if we want to reckon: after how many years ***N*** the population ***Pl_o*** will be doubled, i.e., (**$Pl_N = 2\, Pl_o$**), using the annual growth rate ***R***? We can do it as follows:

$$Pl_N = Pl_o\, (1+R)^N$$

$$Pl_N / Pl_o = (1+R)^N$$

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$$Log(Pl_N / Pl_o) = N\ Log(1+R)$$

$$N = Log(Pl_N / Pl_o) / Log(1+R)$$

Similarly, if we want to reckon: after how many years N the saved money SV_o in a bank will be doubled, i.e. ($SV_N = 2\ SV_o$), using the annual interest rate R; we can do it using the following equation.

$$N = Log(SV_N / SV_o) / Log(1+R)$$

If $R = 0.05$, then;

$$N = Log(2) / Log(1.05) = 14.2 \text{ years}$$

Assignment:

If the annual interest rate R_1 in the Egyptian local banks = 0.09, and after three years it was increased to R_2=0.12, while the annual inflation rate Pi was 0.04, and remained constant, after how many years N, the real value of the saved money SV_o was doubled? The following equation represents this case.

$$SV_N = SV_o * [(1+R_1)*(1-Pi)]^3 * [(1+R_2)*(1-Pi)]^{N-3}$$

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