THE RELATIVISTIC TILTS OF GIZA PYRAMIDS' ENTRANCE-PASSAGES

HOSSAM ABOULFOTOUH

Department of Architecture, Faculty of Fine Arts,
Minia University, Egypt,
e-mail: fotouh@mail.com

ABSTRACT

The tilts of Giza pyramids' entrance-passages have never been considered as if they were the result of relativistic mathematical equations, and never been thought to encode the Earth's obliquity parameters. This paper presents an attempt to retrieve the method of establishing the equations that the pyramids' designer used to quantify the entrance-passages' tilts of these architectonic masterpieces. It proves that the pyramids' designer was able to include the geographic, astronomical and time parameters in one relativistic equation, encoding the date of the design of the Giza pyramids in the tilt of the entrance passage of the great pyramid.

INTRODUCTION

Despite scientific researches to decode the astronomical and architectonic designs of the pyramids of Egypt, particularly the great pyramid, the following two questions were not answered. First, why the designer quantified the tilt of GP's entrance passage equal to 26.51º? Second, why the tilt differed in the other two pyramids in the Giza plateau: 25.92º and 21.40º for the Second pyramid, and 26.05º for the Third pyramid, as well as the tilts of the Red and Bent pyramids at Dahshur? The paper will answer the two questions and proves that the pyramids' designers did not make architectonic design and/or implementation mistakes, taking into consideration the allowable implementation tolerance in megalithic structures. Besides, the above is indirectly related to the issue of the chronology of the pyramids of Egypt, which have come to the forefront of modern scientific and archaeological debates. However, some of the published findings of scientific researches on dating the Egyptian pyramids neither coincide with the architectonic professional norms nor with the published archaeological discoveries during the Roman period in Egypt and the available archaeological evidence. Researchers referred wrongly to inscriptions on the walls of some temples, the so-called Stretching of the Cord Ceremony, as the stellar method for orienting the pyramids (Garde, 2005), while the hieroglyphic text of that inscription says, "we fix vertical posts at the corners of the temple's site, based on the order of A-Gu-ra-q ⟨hieroglyphs⟩, the governor of Qur-Theba, i.e., the city of Theba; it is the queen that Egyptologists pronounce her name as Hatshepsut." Kate Spence, in her article on the astronomical orientation of pyramids (Spence 2000), suggested an empirical, but mid-night, method that pyramids designers might have used in order to orient the Giza pyramids towards the geographic north. She postulated that they had aligned the pyramids towards an imaginary vertical line between the simultaneous transits of two circumpolar stares at the northern meridian of the sky, that is Mizar in Ursae Minor and Kochab in Ursae Major. Accordingly, based on F. R. Stephenson's calculations, she concluded that GP was built in 2478 BC. Since the publication of Spence's paper, the transit method has been accepted partly by commentators as the most probable orientation scenario, because orientation's deviations of eight Egyptian pyramids follow a semi-straight line in the path of time. According to her hypothesis the deviations occurred because pyramids designers were unaware of earth's precession cycle. Her article gives an indirect hint that pyramids-designers lacked some basic astronomical knowledge and their analytic abilities were very limited, e.g., she said: there is no evidence from the period of their history that the Ancient Egyptian measured or understood the concept of latitudes (Spence 2001). Juan Antonio Belmonte followed the same hypothesis but he suggested another horizontal alternative to the transit method, using the stars Phecda and Megrez of the leg of the Bull; accordingly, he concluded that the Great Pyramid was built around 2562BC (Belmonte 2001). The published researches on this subject and their citations (Rawlins *et al.* 2001; Thurston 2003) that confirm partly Spence's basic hypothesis, indicate that researchers and commentators did not take into consideration the simple fact that, today or in the ancient past, architects and site-engineers, never do construction-survey activities at night, i.e., in darkness, which is contrary to the construction norms. This paper, on the contrary, shows that their postulated dates are more recent than the design-date that is encoded in the tilt of the Great Pyramid's entrance-passage, based on retrieving the architectonic design techniques of the Ancient Egyptian Architectonicians.

The paper, therefore, focuses on and endeavors to retrieve the analytic method that pyramids designers used to quantify the tilt of the entrance-passage of the Great Pyramid, and other major Egyptian pyramids, using relativistic mathematical equations. Previous studies (Aboulfotouh 2002 & 2005a) showed that the GP's designer encoded his astronomical knowledge, using geometrical codes, in the design of the great pyramid and the horizon of the Giza pyramids plateau. He created something like a computer-disk that includes many astro-geometrical data-files.

There are no written papyri that show the mathematical knowledge in the old kingdom, like for example the so-called papyrus Moscow, dating from the beginning of the second millennium BC (Pedersen 1993). However, Giza Pyramids are such an outstanding architectonic evidence that no one could be able to design and construct them, even today, using the primitive tools, without studying the basic architectonic and civil engineering subjects for constructing megalithic structures. In the construction field, descriptive geometry and mathematics are the essential studies for any architectonic designer, in order to be able to imagine the projections of the geometric forms, as well as to calculate many things, e.g., dimensions, volumes, loads, slopes, bills of quantities, etc. Contrary to the criteria of the field of archaeology, in the architectonic field, even if there are on written mathematical evidences, e.g., the so-called the notebook of the building's design calculations, the architectonic product or the structure itself is the evidence, which indicates the geometric and mathematical ability of its team of designers. Besides, usually the designers of mega-structures do not make decision concerning any single line in these important national structures without studying it. We can observe the basic laws of the descriptive geometry even in the structure of the Hieroglyphic writings, i.e., drawing the figures in plans, side-views and elevations. Accordingly, since there are no written papyri or stelae that explain and prove how the designers quantified the tilts of the pyramids' entrances, and the pyramids as hard evidence confirm indirectly that the designers did not lack the basic mathematical knowledge and the geometric imagination, this paper will follow the opposite hypothesis that they may have quantified these tilts using such mathematical theory, and perhaps these tilts encode some of the astro-mathematical knowledge of the pyramids designers.

The methodology to test this hypothesis will follow, therefore, a reverse process, by showing first how the designers established the mathematical equations, i.e., showing only the modern formats of the ancient multiple processes of manual calculations; the ancient representation of each equation may take three or four pages in hieroglyphic writing. Then, showing how they postulated, the basic astronomical data by applying a simple analytic technique, particularly the data concerning the minimum and the maximum values of earth's obliquity range.

In short, this paper presents part of the ancient astro-geometrical sciences that concern the Earth's, precession and, obliquity cycles as it was understood by the designers of all pyramids models found in Egypt. Consequently it shows one of the arrays of basic methods in pyramids design
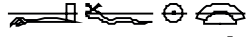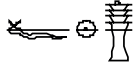
philosophy and particularly how to encode the (design/implementation) date in architectonic designs.

The hidden message of this paper is that perhaps the mental and analytic ability of the human beings do not change along time, but their knowledge reaches its climax prior to the major cataclysmic events of the earth, which may occur on uniform time intervals. However, the engineering and architectonic applications of the same basic knowledge differ, between two intervals, due to the accumulation and the pace of advancements of the industrial researches. Here we show the ancient architectonic application of the Special Theory of Relatively of Albert Einstein in its primitive format that was understood by a team of designers who lived on earth nearly 5000 years ago.

HISTORIC AND ARCHEOLOGICAL REFERENCES

Al-Maqrizi (1364-1442 AD), the best Egyptian historian, mentioned in his book that was first published in 1436AD (Al-Maqrizi 1849), in the pyramids' chapter, a story from older Arab historians that had been translated from old Coptic sources. It is about an old papyrus that was found in one of the tombs in the Abou-Hermis area (probably Al-Lahoun near Al-Fayoum), which mentions that the ancient Egyptians built the pyramids based on knowing the time intervals of Earth's cataclysmic and extinction cycles. They thought they will die and their sciences will be lost, thus, they decided to encode them in mega-geometric forms, in order to save their knowledge for the future generations, after the cataclysmic event. The introductory part of the papyrus says: "this was written during the first reign year of King Deuclidianos, which was copied from another papyrus that was written during the first reign year of King Philip (the father of Alexander). The latter was a translation of an ancient Egyptian golden plate that was written in 1372 before King Philip's 1st reign year; its hieroglyphic text was translated to King Philip by the best two Egyptian translators of his days: *Ilow* and *Yrtha*. The first written date of the ancient original text was 1785 years older than it, i.e, 3157 B.Philip." The translator of the Coptic version of the papyrus into Arabic added together the dates-of-writing the texts that were mentioned in it to his day (225 AH = 840 AD). He found that the original ancient text was 4321 years old, i.e., that it went back to 3481 BC. Then he concluded that the original text was 400 years earlier than the great deluge (Tsunami of the Mediterranean), that he said, it happened in 2741 years earlier than the first reign year of King Philip, i.e., the great deluge occurred in 3081 BC.

The most important part of the original golden text says: "the master Egyptian priests (were 130) said: based on observing the motions of the celestial bodies, we identified the time intervals of Earth's cataclysmic and extinction cycles (e.g., inundations, fires from meteorites, extinction and re-origination); accordingly, we asked our King A-so-ra-eid (the *Taurus*) the son of So-ra-g (*Scorpion*) to give his order to build the pyramids, based on informing him about the results of our studies. Firstly, the coming great deluge will happen when the star Regulus of the Leo (in the sidereal zodiac) enters the first minute of Cancer (in the tropical zodiac); then they mentioned the positions of planets, moon and comet during this event. Secondly, the coming great fire, from meteorites, will happen when Regulus enters the last minute of the fifteenth degree of Leo, i.e., moved 45º of arc from the first minute of Cancer; then they mentioned the positions of planets, moon and comet during this event. Thirdly, Earth's extinction, due to celestial disturbance, occurs when Regulus completes 2/3 and 1/6 of its cycle, i.e., moved 300º of arc, and when it completes its cycle, re-origination starts again with its new cycle, i.e., for each cycle of Regulus in the tropical zodiac, Earth remain 4320 years without live beings equivalent to the precession motion of 60º of arc."

The information mentioned in this papyrus is not part of modern archeological discoveries, because it remained for centuries in an Arabic text book without English translation. It gives a hint to that, contrary to current belief, the Egyptian priests, at the time of building the pyramids, did understand earth's precession cycle, and were capable enough to measure the minutes of arc. Besides, it is unlikely that its translators mistranslated the various versions of the papyrus or added the astronomical knowledge of their days, on the earth's precision cycle, to the text of the original golden plate, because the mentioned information about earth's cataclysmic and extinction cycles never was mentioned in any book on the ancient astronomy of any civilization, and in the Arabic context contradicts with the religious beliefs.

Decoding the information of Denderah's Zodiac will support this historic note that concerns the likely scientific ability of the pyramids designers, since it provides real hard evidence from the same period (3480 BC). An ancient Egyptian inscription in Denderah temple shows that the oldest cult building on this site was constructed during the lifetime of Khufu in the Old Kingdom (Kieth Grenville; Cauville 1990), i.e., the current temple was build on the ruins of an older temple from the old kingdom. Denderah's zodiac was placed in the ceiling of one of the upper eastern rooms of the current temple. According to Cauville, the zodiac was dated to 50BC, by astronomical scientists, because two eclipses are represented in it, and that were occurred in that date (Cauville 1997); they imagined wrongly that the sun is represented, by a circle below Pisces, in a zodiac that displays only the night sky.

As shown in fig-1, if one carefully studied Denderah's zodiac, one may observe that the precession cycle of the earth is recorded in precise way, i.e., both the tropical and sidereal zodiacs are presented. It shows that the point of the vernal equinox was in the middle of Taurus constellation, represented by the *Qor*, i.e., a vertical pole in the middle of a horizon, presented in a side view, and means "*the premises of*". It records the date of 3430 BC, approximately. The beginning of Libra in the tropical zodiac (outman equinox) is shown by the *Gadad or Geila*, i.e., an instrument like funnel to weight the seeds, which still used till this moment in rural areas in Egypt, presented in side view but upside down, and means weight or judgment. Egyptologists thought it is a feather, and indicates what is right and truth and pronounced it as *Shu* or *Maat* (Budge 1910), which also equates with the scales.

The key of the orientation of Denderah's zodiac is presented by a circle, close to Scorpion in the tropical zodiac, that within which eight men seating on their knees and put their hands behind their backs, imaging that they are carrying the zodiac on their backs. It is not the sign that indicates a captive, since their legs are not tied with their hands. Also, fourteen of these men, putting their hands in front of them, were put around the moon in another astronomical presentation in the ceiling of
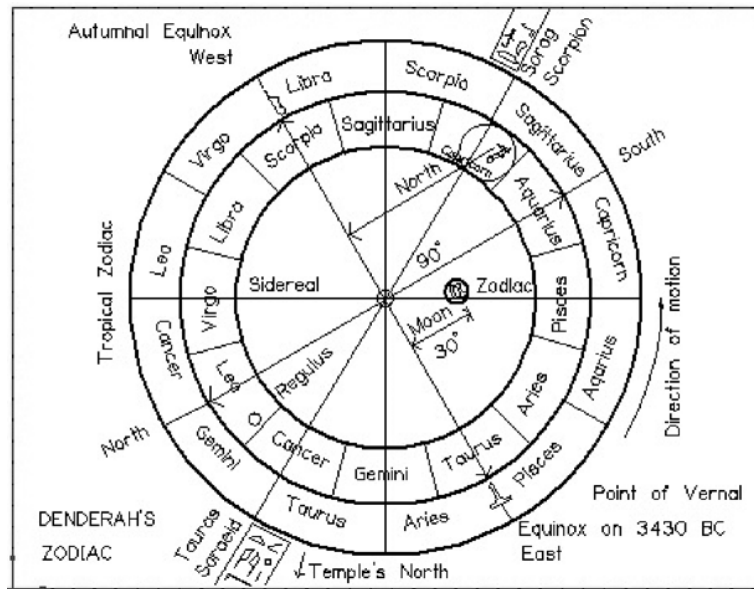
Fig.1: Diagrammatic presentation of Denderah's Zodiac, based on plate 21 in Description de l'Egypt (1809-1828); it shows the sidereal zodiac in the middle surrounded by the tropical zodiac. And the point of vernal equinox was in the middle of Taurus, in 3430 BC approximately, the inclination of the spines of men in the circular key of the zodiac is the direction of the meridian of the place of the observer where their heads point towards north. The full moon is located close to the eastern side of the tropical meridian at 30°, south of the east-west coordinate. And the north of the temple's site does not match the north of the tropical zodiac; it follows the north of the sidereal zodiac.

the first floor of Denderah temple, indicating the full moon of the night of the fourteenth day (plate 19 in *Description de l'Egypt* 1809-1828). The inclinations of the spines of the eight men indicate the north-south direction, where their heads point to north; then, when one observes the zodiac from below and looks towards north, the point of vernal equinox is upon the right (east) side. Further, the inner circle of the zodiac is the spread out of the sky dome, i.e., its radius is 90° instead of 57.29°, and its center is the projection of the azimuth of the place of the observer. The full moon ⊙ is shown in the zodiac, in the southern part of the sky, close to the designed tropical meridian upon the eastern side, at a north-

south distance equals 30°, from the east-west coordinate of the tropical zodiac. The zodiac shows the full moon during the vernal equinox when it passed the equatorial plane of the earth, as was seen from the latitude of 30° north, in their days, while the latitude of the temple is 26.16° north.

Besides, the zodiac was wrongly placed in the Denderah temple; the north-south coordinate of the temple's site is inclined by 41.5° to the east of the designed north-south axis of the zodiac, which follows the north of its sidereal zodiac, and not the north of its tropical zodiac. We can also observe this mistake, if we notice the direction of the eye of the full moon. If we stand under the center of the zodiac and

looked towards its designed south direction and then look up, we can see the right east-west position of the horizontal axis of the moon's eye as shown in the above text, but if we looked towards the south of the temple's site, the horizontal axis of the moon's eye will be inclined by 41.5°. This mistake, and the date of its point of vernal equinox (3430 BC), indicate that this zodiac was not carved for that temple, and it was placed in it in wrong orientation.

The information included in the above mentioned papyrus and that was shown in Denderah's Zodiac are correlated; they hint to the same period (3430BC approximately), and show part of the excellent astronomical knowledge of the Master priests (*Jedars* or the architectonicians) of that period. And even if this zodiac was carved during the Hellenistic period, it may was a copy from an older zodiac, either it was a ceiling of any other temple or only it was part of an ancient astronomical papyrus, or golden plate, from the days of the pyramids builders. Besides, the scenario of rewriting King Soraeid's papyrus/plate in different times, languages and places, hints to that it was one of the important documents in the Bibliothecas of the ancient civilizations. However, according to the current knowledge (www.winshop.com.au), Regulus marked the first minute of Cancer in 2345 BC, which is later than the known date of the great deluge in 3081BC (Al Maqrizi 1894). Commentators thought the great deluge of Egypt, or the tsunami of the Mediterranean as a myth story, however, lately, scientific studies (Pareschi *et al* 2006) show that similar tsunami happened in the Mediterranean in 8000 years ago. The date of the Egyptian great deluge that was mentioned by Al Maqrizi is close to the encoded date (3055 BC) in the design of the site plan of Giza pyramids plateau (Aboulfotouh 2002, 2005a). If we take the

above papyrus as reliable source, the difference between the papyrus's postulated date of the great deluge (2345 BC) and the encoded design date of the site plan of Giza plateau (3055 BC) may be due to that the pyramids designer had built Giza pyramids after they corrected their previous postulations. Even if that papyrus was not reliable from the archeological point of view, it hints to very interesting information. The Egyptian priests estimated the intervals of the cataclysmic cycles based only on the constellations' intervals and mid-intervals in the precession, i.e., every 30° and 45° of arc for four times that make 300° or four Hor-Mageddon's cycles (Aboulfotouh 2005b), which gives 21600 years approximately. Perhaps identifying the correct date of the great deluge, in the site plan of the pyramids plateau, was based on correcting the spans of Hor-Mageddon's cycles from 75° to 70° and its breakdowns (form 30° and 45° to 19° and 51°); and the total life span of the four Hor-Mageddon's cycles in each precession cycle is 280°, i.e., equals 2/3 and 1/9 of the precession cycle, which gives 20160 years approximately. The intervals 19° and 51°, means if the great deluge was occurred in 3055 BC, as encoded in the Giza pyramids' site plane, after moving 19° degrees of arc from the first minute of Gemini, the second cataclysmic event from fire would have been occurred after moving 51° of arc, i.e., after 3672 years, or around the beginning of the seventh century AD. Recent scientific findings show that natural disasters were occurred close to that date (Hirschfeld 2006).

Moreover, concerning the names of kings, similar to other mis-pronounced names of the Egyptian kings, which do not match the list of Egyptian Kings mentioned by Al-Maqrizi based on the original Coptic sources, the names of King Soraeid and his

father Sorag are pronounced wrongly as Khafra and Jedefra, respectively. This was due to missing the order of pronouncing the letters in the vertical writings (human, then bird/animal, then inanimate) following anti-clockwise motion, and some letters were given wrong sounds, due to misunderstanding the rule of assigning a letter-sound to each figure as the first letter of the ancient Egyptian name of this figure (Aboulfotouh, 2005b). Also, according to Al-Maqrizi the correct pronunciation of the name of the king that Egyptologist call Khufu is Sa-wo-g 𓆓𓅱𓎼, the grandfather of King Soraied.

THE GEOMETRY OF COSMIC CIRCLES

In basic geometry, the relation between the radii r_1 and r_2 of two stationary circles – when unifying their centers at C_0 - could be represented by an angle θ, where $r_1 > r_2$ and cosine $\theta = r_2/r_1$; fig-2 shows this case. We can also imagine that one of these circles shrinks or expands, which may be observed as the domain of the event, where it matches at specific point in time any one of the array of the transit positions of circles within the possible range of motion; while the stationary circle may be observed as the frame of reference.

In both cases, the value of θ indicates the geometrical deviation, from the matching state when θ equals zero degree; i.e., the value of θ indicates the transitory or the permanent value of the radius of the shrinking or expanding circle. Similarly, if r_2 r_3, and r_n are the radii of shrinking circles, then θ_2, θ_3 and θ_n indicate, respectively, their geometrical deviations, at specific point in time, from the matching state with their frame of reference that r_1 is its radius.
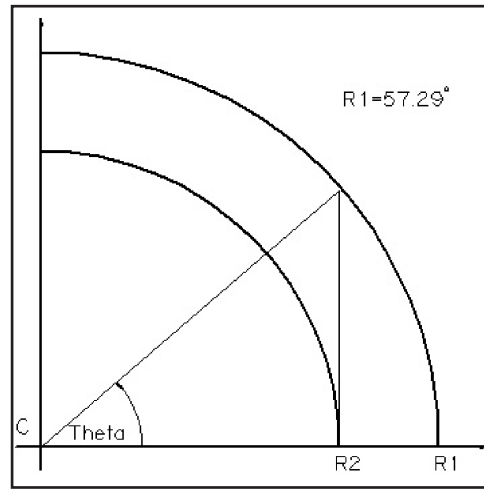

Fig. 2: It shows two quarters of two cosmic circles of the radii r_1 and r_2, where cosine θ indicates the geometrical deviation between them at specific point in time; either taking any of them as the permanent frame for referencing the shrinking or expanding motion of the other.
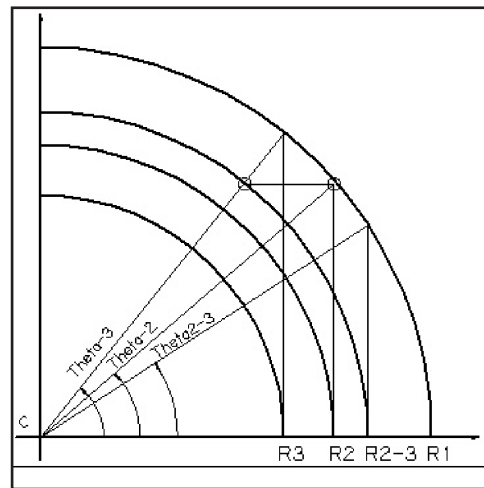

Fig.3: quantifying the resultant geometrical deviation of two shrinking circles of the radii r_2 and r_3; the deviation angle of the imaginary circle of r_{2-3} is the resultant.

As shown in fig-3, spouse that r_2 and r_3 are the radii of two shrinking circles within the frame of reference that r_1 is its radius, the resultant geometrical deviation of these two circles, could be indicated by the angle θ_{2-3}, where (cosine $\theta_{2-3} = sin\theta_2/sin\theta_3$) and where $\theta_2 < \theta_3$. The angle θ_{2-3} indicates the geometrical deviation of a fourth imaginary shrinking circle, where r_{2-3} is its radius.

Moreover, the circles in fig-3 are in one plane and have the same center. The case will be also valid if the circles of r_2 and r_3 were in different planes with two frames of references, either having the same center or not. Besides, as the deviation of any shrinking circle from its frame of reference is indicated by sine of an angle, one can get the resultant of any two shrinking circles in two isolated planes, no mater the radii of their frame of references were equivalent or not, i.e., in all cases the radius of the outer circle equals 57.29°, either it was or wasn't the frame of reference.

ESTABLISHING THE ANKH DIAGRAM

The diagram, in fig-2, could be used to indicate the relation between the perihelion and the aphelion radii (r_p & r_a) of any celestial orbit, where $Cos\,\theta = r_p/r_a$. Call this the diagram of the orbital deviation from the circular state when θ equals zero degree. Similarly, the minimum and maximum values of earth's obliquity range could be represented linearly in degrees by two radii O_i and O_x respectively in other deviation diagram.

As shown in fig-4, if we put half of the obliquity deviation diagram on top of the orbital deviation diagram, where the center C_2 of the first tangs the outer circle of the orbital diagram and rotate the radii O_i and O_x until meeting the equivalent arc-distances and then maintain the two lines till meeting the center C_1 of the orbital
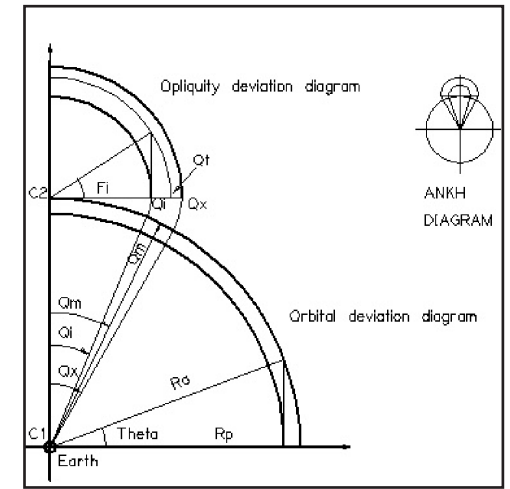

Fig. 4: The Ankh diagram that links the orbital and the obliquity deviation diagrams, where the lengths of r_a and r_p of the earth's orbit are represented in degrees and conserved the apparent deviation of θ, i.e., r_a equals 57.29°. Despite that both O_i and O_x are angles; they are represented as radii, imagining that the cape zone of their corresponding sphere is being spread out, in order to identify the geometrical deviation in 2D plane like that of the orbit.

diagram, they generate the compound Ankh ☥ (or Naga) diagram of the pyramids designer(s), the diagram of cosmic motions that was the symbol of cosmic life.

The orbital part of the Ankh-diagram shows two presentations. The first is a projection-plan of an imaginary orb of the earth's orbit, where its center C_1 is the center of the sun. The second is a cross-section in a coordinate system, perpendicular to earth's orbital plane, where its center C_1 is the center of the earth; i.e., the point C_1 represents the earth and the sun in the same time. Besides, the two zones between the two circles of r_a and r_p; and of O_x and O_i in the two sub-diagrams that form the Ankh diagram could be observed as the geometrical domains wherein both orbital and obliquity motions take place.

In the orbital part of the Ankh diagram, since earth's axis is inclined to the plane of its orbit, the median obliquity angle O_m indicates the mean quantity of the vertical deviation of the obliquity range. Similarly, in the obliquity part, if the circle of the minimum obliquity O_i was taken as a frame of reference for the circle-of-event at such obliquity value O_t within the obliquity range, the angle ϕ between the circles of the two radii O_t and O_i indicates the quantity of the horizontal deviation at that specific point in time. These two types of vertical and horizontal deviations correspond to time; the former indicates which obliquity range, and the latter indicates which moment in the obliquity range. When dividing sine the former by sine the latter, the result is cosine ω, where ω is the resultant deviation angle of time, as shown in Eq-1.

$$\cos \omega = \frac{\sin O_m}{\sin \phi} \qquad (1)$$

When taking into consideration that $(cos\ \phi = O_i/O_t)$ and $(sin^2\ \phi = 1 - cos^2\ \phi)$, we could put Eq-1 in the form of Eq-2, as follows:

$$\cos \omega = \frac{\sin O_m}{\sqrt[2]{1 - \dfrac{O_i^2}{O_T^2}}} \qquad (2)$$

Further, a previous study (Aboulfotouh, 2002) showed that the Giza pyramids designer was able to identify the value of the geographic latitude of the place by recording the daily shadow of a vertical post (like obelisk), during a year time. Based on the above geometric technique concerning one of multiple uses of the Ankh diagram, by the pyramids designer(s), for understanding spherical astronomy, the analysis for reverse-designing the GP showed that its designer had observed that

if he set the tilt of the entrance of the GP equivalent to the latitude angle of the place (30°) he would encode only the geographic location of the GP and its horizon without encoding the component of time, or the event. Thus, he thought to get the relativistic but geographic tilt that corresponds to the deviation of the component of time by using the principle of equivalence, i.e., he thought it is the best geometric technique in order to inform the future generations about his sciences and about the date of the cataclysmic event that may wipe out his civilization. He set the ratio between sine the angle of the relativistic tilt α of GP's entrance passage and sine the angle of the latitude of the place λ equivalent to cosine ω, and established the relativistic Eq-3,

$$\sin \alpha = \frac{\sin \lambda * \sin O_m}{\sqrt[2]{1 - \dfrac{O_i^2}{O_T^2}}} \qquad (3)$$

The unknowns in Eq-3 are O_m, O_i and O_t; the latter is the obliquity of the event that was known or could be measured by the designer. Before mentioning the analysis that the GP's designer might have carried out in order to identify the first two unknowns of the equation, the following are some primary notes, and based on the published survey data (Petrie 1883; Edwards 1961). The Giza pyramids' designer encoded what he thought it is the value of earth's O_i in the tilt of the lower entrance of the second pyramid that equals 21.672°; and encoded the angel of the horizontal deviation of time ϕ in the tilt of the upper entrance of the same pyramid that equals 25.93°; taking into consideration the tolerance in megalithic structures. In a previous study (Aboulfotouh 2002), based on investigating the design of the site plan of Giza pyramids,

it was found that the obliquity angle of event O_t that the layout of the horizon of Giza pyramids indicates, was 24.10° (the earth's tilt at 3055 BC approximately); since (cosine $\phi = O_i / O_t$), then cosine 25.93° = 21.672° / 24.10°. The same work shows that pyramids designer did encode O_x equal to 24.30°, as he designed the horizon of Giza pyramids like a cross-section in a coordinate system similar to the lower part of Ankh diagram. Then, according to his knowledge the mean obliquity angle O_m equals 22.986°. Using the assumed values of the three unknowns in Eq-3, if the latitude angle λ was 30°, the relativistic tilt of GP's entrance passage would be 26.51°. Concerning the above speculated values of earth's obliquity range: O_i and O_x, it is unlikely that a designer living on earth five millennia ago was able to quantify them, using any experimental technique. The only logical answer is that he used only his mental ability to figure out the likely numbers. The 2/n table of the so-called papyrus (Rhidn) Ahmes (Noisefactory 2007) shows the level of the numeric calculations of the Ancient Egyptians during the New Kingdom, while the architectonic designs of the buildings from this period were not as sophisticated as the architectonic design of the Giza pyramids, which supports the hypothesis that the numeric calculation during the Old kingdom was at least equivalent to that of the new kingdom if it was not better than it. After long process of trial and error investigating on the ancient approach for quantifying the obliquity ranges, it is found that GP's designer might subdivided the arc of 90° into harmonic ranges; he started the arc by 2^{-12}, which is the ratio between the frequencies of two succeeding musical half-tones; and since he neither used zero nor the decimal system, he put it as 1 +(1/16)=1.0625. Thus, the arc starts from

1.0625° and ends by 91.0625°. Besides, the obliquity diagram in fig-4 shows, when taking O_x as the circle of the event, then $(O_x * Cos\ \phi = O_i)$ for each obliquity range; accordingly, one can calculate the harmonic breakdown of the obliquity intervals, starting from 91.0625 °, and based on the harmonic but regressive exponential series of $Cos\ \phi$, using the fraction system as follows:

91.0625° * 1/[2+(1/32)]   = 44.83°
44.83° * 1/[1+1/(2+(1/16))]   = 30.19°
30.19° * 1/[1+1/(4+(1/8))]   = **24.30°**
24.30° * 1/[1+1/(8+(1/4))]   = **21.672°**
21.672° * 1/[1+1/(16+(1/2))] = 20.43°
    And so on.

Further, previous study showed the experimental field technique that pyramids designer might have done in order to identify the earth's obliquity in his days; and its encoded value in the design of the Giza Pyramids' site plan was 24.10° (Aboulfotouh 2002). Accordingly, if the GP's designer found that earth's obliquity in his days was 24.10° and his ideological believe in the harmonic structure of the cosmos was the key for getting the hypothetical breakdown of the obliquity intervals; he would conclude that earth's obliquity range is between 21.672° and 24.30°. But one might ask, what made him think that there is an obliquity range; the answer might be he realized that the observed motion of the sun and the moon have ranges, e.g., the range of sunrise and sunset angles, and perhaps he investigated the earth's obliquity mechanism, like what *Ibn Qura* did in the theory of trepidation in ninth's century AD (Pedersen 1993), and end up with that conclusion. This mere mental conclusion of the pyramids designer do not match the two values of the earth's

obliquity range that was postulated by the twentieth century scientists, e.g., 22.1° and 24.5°, based on *Milankovitch* 's theory (*Milankovitch 1941*). Discussing the theoretical hypotheses that upon which earth's obliquity range was postulated by modern scholars is outside the scope of this paper. On the contrary, after mentioning the equations of other important Egyptian pyramids, the paper will show some notes on the trigonometric relation between θ and φ of earth's system that may support the postulation of GP's designer, using the same ancient geometrical technique of Ankh diagram.

By analyzing the design of all major pyramids in Egypt and reviewing the published survey results (Petrie 1883; Edwards 1961; Dorner 1986), it is found that there are similar equations used for quantifying the tilts of their entrance-passages, in order to encode geographic, astronomical, and time information. The equations of the major pyramids of the fourth dynasty will be taken as examples. In each pyramids horizon, or site, that contains more than one pyramid, the greater of them, has two design models: the model that encodes the time O_t plus the geographic and obliquity information, and the model that encodes only geographic and obliquity information like the Red Pyramid at Dahshur. In this regard, it should be mentioned that quantifying the surface inclination of the pyramids of the time-model has been discussed in detail in previous paper (Aboulfotouh 2005a), based on the value of O_x of the earth and the latitude angle λ of the place, e.g., the case of GP. But, for the second model like the case of the red pyramid, cosine its surface angle equals O_i /λ, i.e., 21.672°/29.808° = cosine 43.36°. The surface angle of this pyramid model is the deviation angle between two

latitude angles: the latitude of the tropic of cancer at the time of minimum obliquity and the latitude where the pyramid stands (29.808° based on Google earth information).

Now, the equation for quantifying the tilt of the third pyramid's entrance passage in Giza plateau is as follows, where it gives $α$=26.05°, for $λ$=30°:

$$\sin \alpha = \sin \lambda * \frac{\sqrt[2]{1 - \frac{O_m^2}{O_x^2}}}{\sin O_i} \qquad (4)$$

Eq-4 implies that the Giza pyramids' designer set the tilt of the third pyramid's entrance passage in order to encode the minimum, maiden and maximum values of earth's obliquity range, that according to his knowledge were equal to 21.672°, 22.986° and 24.30° respectively.

In Dahshur, the following Eq-5 quantifies the tilt of the northern entrance-passage of the bent pyramid; where it gives $α$ = 25.41° for $λ$=29.808° and the same obliquity values as mentioned above.

$$\sin \alpha = \frac{\sin \lambda * \sin O_m}{\sqrt[2]{1 - \frac{O_i^2}{O_x^2}}} \qquad (5)$$

While, the dual tilt of the western entrance passage of the bent pyramid (Dorner 1986) encodes two information of the earth's obliquity range. The first is the value of O_x = 24.30°, and the second is the resultant deviation angle ω for the whole obliquity range, i.e., which could be quantified by Eq-2; when putting O_t =24.30°, then, ω = 30.29°.

Besides, for the red pyramid in Dahshur, Eq-6 quantifies the tilt of its entrance-passage; where it gives $α$ = 27.45° for $λ$=29.808°.

$$\sin \alpha = \frac{\sin \lambda * \sqrt[2]{1 - \frac{O_m^2}{\lambda^2}}}{\sqrt[2]{1 - \frac{O_i^2}{\lambda^2}}} \qquad (6)$$

Eq-6 is based on that, for this specific model of pyramids, the resultant deviation angle ω is quantified only on latitude bases; despite O_i and O_m are obliquity angles, they have been treated in the equation as latitudes, i.e., the transit positions of the tropic of cancer at the time of O_i and at the time of O_m. Thus the value of cosine ω in Eq-6 is the output of dividing the sine(s) of the two deviation angles between the latitude of where the pyramids stands and the latitudes of the two transit positions of the tropic of cancer.

The above shows that the relativistic tilts of the pyramids' entrance passages were quantified by the pyramids designers of the old kingdom based on the same astro-geometric theory; thus, mathematically, the known eight values of the tilts that mentioned above would never mean something else. Not only that, if one wishes to apply the above theory on the pyramids of the fifth dynasty, he will find similar equations.

TESTING THE TRIGONOMETRIC RELATION BETWEEN θ AND φ

Since earth is dynamically stable system, there may be a trigonometric relation between earth's orbital deviation and its obliquity deviation, i.e., between θ and φ. Taking into consideration that the values of φ, O_i and O_x are the above mentioned hypotheses of the pyramids designer, and the value of θ is based on the modern data (Williams 2002) of earth's

aphelion r_a and perihelion r_p, by reviewing all possibilities of the trigonometric ratios of both angles, it is found that for this specific hypothesis case ($tan^2 \phi = sin \theta$) and that could be written as follows:

$$\frac{O_x^2 - O_i^2}{O_i^2} = \sqrt[2]{1 - \frac{r_p^2}{r_a^2}} \qquad (7)$$

Eq-7 applies only to the case of Earth and Mars. If we followed the hypothesis of the pyramids designer concerning the harmonic obliquity ranges, according to NASA's fact sheets (Williams 2002) Mars's current obliquity (axial tilt) is 25.20° and its r_p and r_a are 206.6 MK and 249.2 MK respectively, its obliquity range would be between 24.30° and 30.19°, which qualifies Eq-7, similar to earth. However, the modern postulation (Head *et al.* 2003) of Mars's obliquity range was between 14° and 48°. One can suspect that ($cos \phi$) of the obliquity ranges follow exponential series, as the postulation of the pyramids designer. Besides, there is no hard evidence that shows that pyramids designer did postulate the value of ($cos \theta$) of the earth' orbit. However, if we investigated the quantities of ($cos \theta$) of the planets of our solar system we will find that it follows exponential series too. Based on the orbital data of the plants in our solar system (Williams 2002), it is observed that the ratio (r_a / r_p) between the aphelion r_a and perihelion r_p of the orbits of the known planets, i.e., (1/ cos θ, or Acos θ) in the orbital diagram, follows the following progressive series, using the system of fractions:…, 1+(1/3⁴), 1+(1/3³⁺¹ᐟ²), 1+(1/3³), 1+(1/3²⁺¹ᐟ²), 1+(1/3²), 1+(1/3¹⁺¹ᐟ²), **1+(1/3)**, 1+(1/3¹ᐟ⁽¹⁺¹ᐟ²⁾), 1+(1/3¹ᐟ²), 1+(1/(3¹ᐟ⁽² ⁺¹ᐟ²⁾), 1+(1/3¹ᐟ³), 1+(1/3¹ᐟ⁽³⁺¹ᐟ²⁾),… etc. The following is the same series in decimal system with the name(s) of plant(s) that its orbital deviation matches, or is very close, to the term of the series: …, Venous-1.0123,

Nepton-1.0213, Earth-1.037, 1.064, Uranus/ Saturn /Jupiter- 1.111, Mars-1.1924, **1.333**, Mercury-1.48, 1.577, Ploto-1.644, 1.69,… etc. Accordingly if the values of (1/$cos\ \theta$) follows a series, the postulation of the pyramids designer that the values of (cos ϕ) follows a series may be correct.

CONCLUSIONS

The ancient approach for understanding spherical astronomy was based only on trigonometric ratios. It seems that the pyramid designers were the first to investigate the correlation between these ratios, and invented the first geometrical language that one could read it in the designs, forms, and angles of their megalithic structures. The addition of this paper is very minor in comparison with the whole spectrum of the pyramids design philosophy. This paper showed that pyramids designers did study spherical astronomy and formulated their own theories on the subject. The secondary finding of this paper is that it confirms the identification of the time of event of the GP and its horizon, when earth's tilt was 24.10º, in 3055 BC approximately; if the earth's tilt was diminishing with a flat rate equal to 0.47 second of arc per year, which is the date of the great deluge, or the tsunami of the Mediterranean.

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