Experiment No. 2: Pipe Flow

Theory:

A well-behaved state of flow is called laminar and a random chaotic state of flow is called turbulent. Conventionally both branches of flow are separately studied assuming the flow to be laminar or turbulent before hand. But experimentally you don't know whether the flow would be laminar or turbulent. A fully developed laminar or turbulent flow would be that kind of flow where irrespective of the inlet velocity profile, after a certain entrance length (L_e), the velocity distribution is the characteristic of the flow. For a laminar flow velocity distribution would be different from what we obtain for the turbulent flow.

The flow becomes fully developed after a certain length, which is known as entrance length. Consider a uniform flow at the inlet of the duct. At the walls the velocity would be zero (No slip boundary condition) but after a certain length in the wall normal direction (r) the velocity would be same as the inlet velocity. This region is called the boundary layer. It is assumed that the viscous effects are confined to this layer and the region is of inviscid nature (Figure 1)

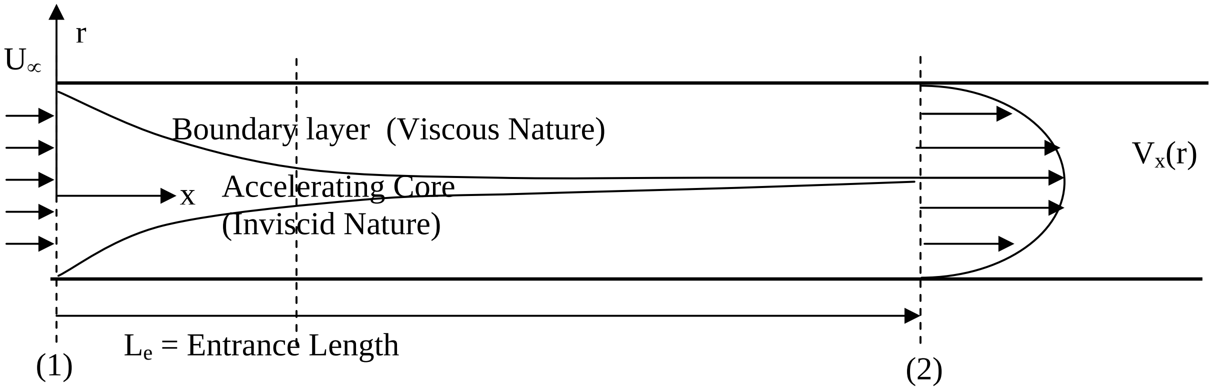

Figure 1 Schematic diagram of developing flow in a pipe

After the entrance length the fluid has a velocity profile, v(r), independent of stream-wise direction (x). Due to the growth of boundary layer, the inviscid core accelerates to have a mass balance. When the boundary layer merges into each other the acceleration is not stopped but continues till flow becomes fully developed i.e $\frac{\partial v}{\partial x} = 0$. The entrance lengyh is given by

L_e / Re ~ 0.05, Re = $U_\infty D/\nu$

Determination of Shear Stress distribution and velocity profile:

Performing momentum balance on a fluid element as shown in Figure 2.

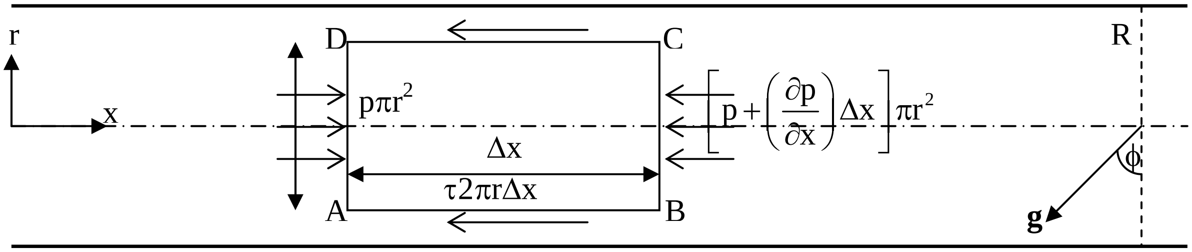

Figure 2 Schematic diagram of surface force on an element in a fully developed pipe flow.

As there is no acceleration in the fully developed region, from second law of motion it follows that

$$\tau(r) = -\frac{r}{2}\left(\frac{\partial p}{\partial x} + \rho g \sin\phi\right) = -\frac{r}{2}\left(\frac{\partial p}{\partial x} + \rho g \frac{\partial z}{\partial x}\right)$$

where, z is coordinate along gravity

$$\Rightarrow \frac{2\tau(r)}{r} = -\frac{\partial \hat{p}}{\partial x} \quad \ldots(1)$$

where peizometric pressure $(\hat{p}) = p + \rho g z$

It may be observed that in equation (1), LHS is a function of r and RHS is a function of x only. Therefore the two sides can only be matched if both are equal to a *same constant.* **Therefore it is deduced that peizometric pressure *decreases linearly* along x (flow direction).** Further, shear stress also varies linearly along pipe radius. It is worth noting that the equation (1) applies for both steady laminar and a stationary turbulent* flow. In the case of turbulent flow, τ is total stress (sum of molecular and turbulent fluctuation contributions).

Equation (1) can be written at the wall,

$$\frac{2\tau(r = R)}{R} = -\frac{d\hat{p}}{dx} \cdots\cdots(2)$$

Equation (2) also proves that as the flow rate is increased in the pipe, wall shear stress $\tau(R)$ increases resulting in higher-pressure drop.

$$\tau_w = -\frac{D}{4}\frac{d\hat{p}}{dx} \cdots\cdots (3)$$

Therefore, $f = -\frac{2D}{\rho U_m^2}\frac{d\hat{p}}{dx}$

Relationship of Pressure drop with Flow rate and other parameters:

For any engineer it is imperative to know the pressure drop as a function of flow rate and geometry. Because for a given flow rate (dictated by design considerations), a piping system with least pressure gradient has to be designed. Thus for any engineer it is very vital to know the relationship between geometry, flow rate and pressure gradient. To explore this two ways are possible. The first method is analytical and the other is through dimensional analysis and experiments.

Dimensional analysis and experiments

In general, Δp or $\tau_w = \text{Function}(D, \rho, \mu, U_m, \varepsilon)$

Applying Buckingham pie theorem, we arrive at following pie groups

$f = \frac{8\tau_w}{\rho U^2_m}$, $Re_D = (\rho/\mu)U_m D$ and $\varepsilon_p = \varepsilon / D$

Here, 'f' is friction factor, Re_D is Reynolds number based on pipe diameter and ε_p is non-dimensional roughness ratio.

$$f = \Phi\left(Re_D, \varepsilon_p\right)$$

By measuring the friction characteristics we want to know the function 'Φ'.

By carrying out experiments various forms of Φ are established in transitional and turbulent flow regimes.

Colebrook and White:

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left[2\left(\frac{\varepsilon_p}{D} + \frac{18.7}{Re\sqrt{f}}\right)\right]$$ **Hydraulically rough**

To estimate this friction factor 'f' Colebrook-White suggested an implicit equation in 'f' valid for entire range of Reynold's number. This equation being implicit was required to be solved by trial and error (±1%).

VonKarman:

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left[2\left(\frac{\varepsilon_p}{D}\right)\right]$$ **Hydraulically smooth**

Lewis Moody:

Gave an approximate Explicit equation as which gave the result within ± 5% variation of Colebrook-White equation.

$$f = 0.0055\left\{1 + \left[2000\frac{\varepsilon_p}{D} + \frac{10^6}{Re}\right]^{\frac{1}{3}}\right\}$$

Unified equation (Churchill):

$$f = 8\left[\left[\frac{8}{Re}\right]^{12} + \left[\frac{1}{(A+B)^{1.5}}\right]\right]^{\frac{1}{12}}$$

where

$$A = \left\{2.45\ln\left[\frac{1}{\left(\frac{7}{Re}\right)^{0.4} + \left(\frac{\varepsilon_p}{D}0.27\right)}\right]^{16}\right\}$$

$$B = \left\{\frac{37530}{Re}\right\}^{16}$$

Swami-Jain:

Swami-Jain further gave an accurate explicit equation within ± 1% accuracy of Colebrook's equation.

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left[2\left(\frac{\varepsilon_p}{D} + \frac{21.25}{Re^{0.9}}\right)\right]$$

Experimental results are documented in **Moody's Chart.**

Analytical

From the solutions of Navier Stokes equation for a circular pipe, we have

$$u = \frac{-dp/dx}{4\mu}\left(r_o^2 - r^2\right)$$

Evaluating U_m and wall shear stress from the above relationship, we have

$f = 64 / Re_D \ldots(4)$

The expression in equation (4) has been validated experimentally for fully developed ***laminar*** flows. It is seen to be valid for Re < 2000 and is insensitive to the roughness characteristics of the pipe. Further for the total pipe (Entrance length and further), the pressure drop is larger than the predicted by equation (4) hence this is the lower limit of pressure drop.

The piezometric pressure gradient in equation (3) can be expressed in terms of head loss coefficient h_f using modified Bernoulli's. Thus equation (3) can be transformed as:

$$h_f = f \frac{LU_{av}^2}{2gD} \ldots(5)$$

This equation (5) is used to predict gradient and head loss correctly 'f' has to be measured correctly. This equation is also called Darcy-Weisbach equation.

Aim:

1. To experimentally measure the frictional characteristics (friction factor) of a circular pipe in fully developed region of flow.
2. To determine 'f' from standard Moody diagram, Moody equation and Swami-Jain equation and to compare the value of 'f ' obtained from these equations with the value obtained from Colebrook-White Equation..

Graphs to be plotted

1. f vs Re
2. log f vs log Re ----------Obtain coefficients
3. log h_f vs log Re
4. Wall Shear (τ_w) vs Re

Given Data

1. Fluid flowing through pipe – Water

    a. Density of water (ρ) = 1000 kg/m3

    b. kinematic viscosity of water at ------^{o}C (ν) = 0.0085 .

2. Pipe diameter (D)= 19.5 mm

3. Pipe length (L) = 9.15 m.

4. Pipe material = Galvanized iron (G.I.) with equivalent sand grain roughness ()= 0.15mm

Observation and Result Table

S.No.	Manometer Reading		Head loss (h_l)	Level rise in collecting tank	Actual volume flow rate	Velocity of flow	Reynolds number	f = h_l $2gD/LV^2$
	X_1	X_2						

Discussion

1. Discuss the physical significance of the experiment?
2. Comment on why ‘f’ becomes independent of Re for very large Re?