Problem sheet - PS01

Fluid Statics

1. The temperature in the Earth's atmosphere varies from location to location as well as with the time of the year. The variation of temperature averaged over the U.S continent and in time is found to vary with height above mean sea level in a piecewise linear manner as shown in **Table 1**. This is referred to as the U.S standard atmosphere.

Layer	Height range (km) (z)	Rate of change of temp with height (K / km) (dT/dz)
1	$0 \leq z < 11$	-6.5
2	$11 < z < 20$	0.0
3	$20 < z < 32$	+1.0

Taking the mean sea level (z = 0) pressure and temperature to be 1.01335×10^5 N/m^2 and 288 K respectively, obtain the expressions for the pressure variation with height for the different layers.

2. Figure 1 shows a gasoline tank of an automobile with a pressure gauge at the bottom caliberated to read the gasoline level in the tank. The owner of the automobile takes the vehicle to a filling station and orders to fill the tank completely. The filling is stopped when the gauge reads 'full'. After the filling, the owner realized that in order to maximize profits the filling stations often indulge in the malpractice of mixing kerosene in small amounts with pure gasoline. In order to investigate this, the owner checks the gasoline level in the tank using a dip gauge (a graduated bar) and to his dismay finds that the level is 28 cms. Estimate the percentage by volume of kerosene in the adulterated gasoline. ($SG_{gasoline}$= 0.68, $SG_{kerosene}$= 0.804). (Assume uniform mixing).

3. In fig. 2 containers (a) and (b) are cylindrical and conditions are such that $p_a = p_b$. Derive a formula for the pressure difference ($p_a - p_b$) when the oil-water interface on the right rises a distance $\Delta h < h$ for a) $d << D$ and b) $d = 0.2$ D.

4. A pump slowly introduces mercury into the bottom of the closed tank in fig. 3. At the instant shown the air pressure p_B = 80 kPa. The pump stops when the air pressure rises to 110 kPa. All fluids remain at 20^o C. What will be the manometer reading 'h' at that time, in cm, if it is connected to standard sea-level ambient air p_{atm}.

5. A reservoir of water is closed at one end by a barrier. At the top of the barrier is an L-shaped channel that further restrains the water when its level is higher than the barrier top by the amount 'h' as shown in fig. 4. The channel is hinged at the corner point O, so that it can rotate in the counter-clockwise direction but not in a clockwise direction. Calculate the minimum value of the ratio h / H that will just prevent the channel from rotating.

6. A rectangular gate (fig. 5) of negligible thickness hinged at it's top edge and of width 'b', separates two tanks in which there is the same liquid of density 'ρ'. It is required that the gate shall open when the left hand tank falls below a distance 'H' from the hinge. The level in the right hand tank remains constant at a height 'y' above the hinge. Derive an expression for the weight of the gate in terms of H, Y, y, b and g.

7. Figure 6 shows a long, square wooden block pivoted along one edge. The block is in equilibrium when immersed in water to the depth shown. Evaluate the specific gravity of the wood if friction in the pivot is negligible.

8. A Gate of mass 2000 kg is mounted on a frictionless hinge along the lower edge as shown in fig. 7. The length of the reservoir and gate (perpendicular to the plane of view) is 8 m. For the equilibrium conditions shown, compute the width 'b' of the gate.

9. Gate AB in fig. 8 is semi-circular, hinged at B, and held by a horizontal force P at A. What force P is required for equilibrium. Take sp. gravity of oil as 0.8.

10. Gate AB in fig. 9 is a quarter circle 10 ft wide into the paper and hinged at B. Find the force F just sufficient to keep the gate from opening. The gate is uniform and weighs 3000 lbf. (1 ft = 0.3048 m, 1 lbf = 4.45 N).

11. The 4-ft-diameter log (Sp. Gravity = 0.8) in fig. 10 is 8 ft long into the paper and holds water on either side as shown. Compute the net reaction at C.

12. The uniform body A in fig. 11 has width 'b' into the paper and is in static equilibrium when pivoted about hinge O. What is the specific gravity of this body if (a) $h = 0$ and (b) $h = R$.

13. The face of a dam, as shown in fig. 12, is curved according to the relation $y = x^2 / 2.4$, where y and x are in meters. Calculate the resultant force due to water acting on unit width of the dam, and determine the position of the point B at which the line of action of this force cuts the horizontal plane through A.

14. The valve at the bottom of a tank consists of a sphere of radius R and negligible mass that closes off a circular opening in the bottom of the tank as shown in fig. 13. The line of contact between the sphere and the opening subtends an angle 'ϕ' from the vertical. The air / water interface is at a distance 'h' above the top of the sphere. The water pressure force on the sphere will hold the sphere in place unless the opening is too small.

    a) Derive an expression for the net downward force 'F' on the sphere due to water pressure as a function of the angle 'ϕ'.
    b) Estimate the minimum value of h / R that will make $F \geq 0$ for a given angle 'ϕ'.

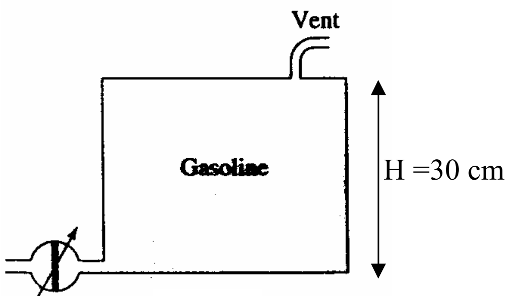

Fig .1

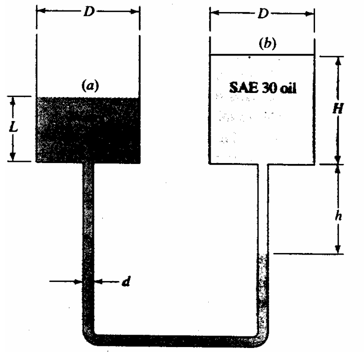

Fig .2

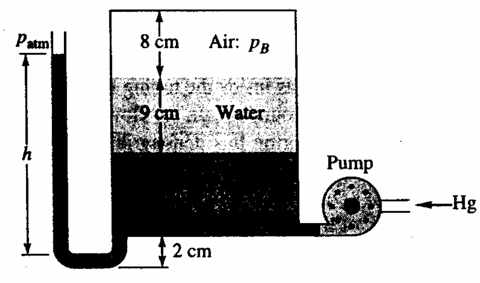

Fig .3

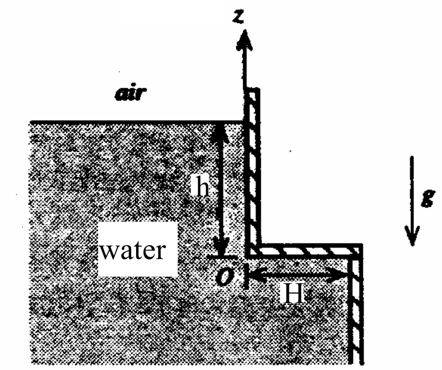

Fig .4

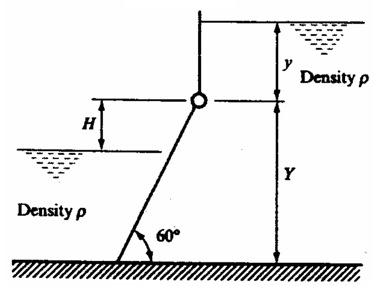

Fig .5

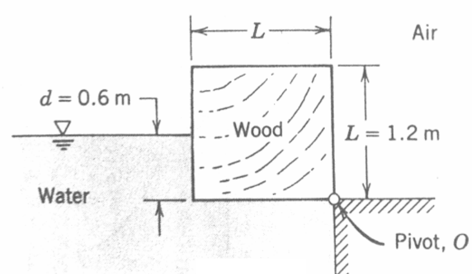

Fig .6

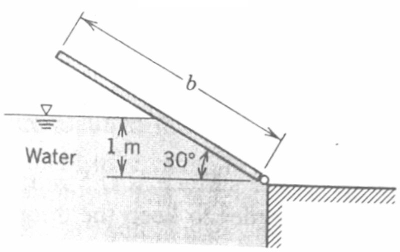

Fig .7

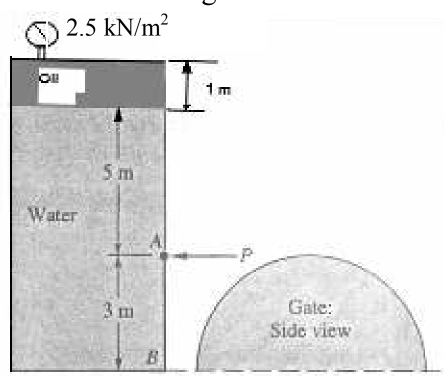

Fig .8

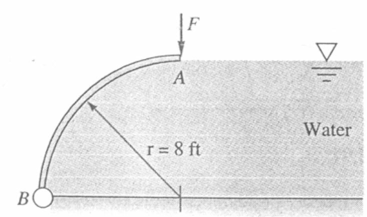

Fig .9

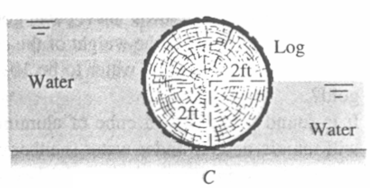

Fig .10

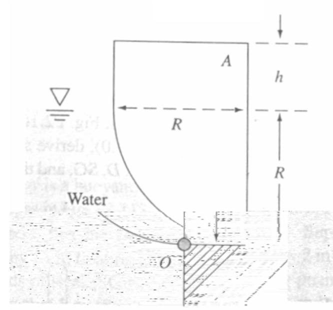

Fig .11

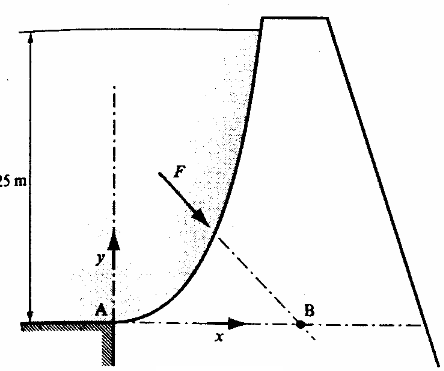

Fig .12

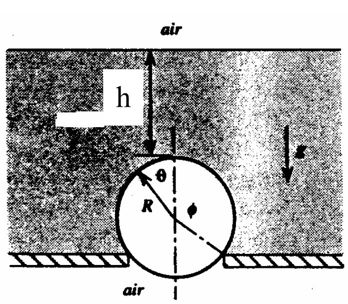

Fig .13