

Problem Sheet PS-04

Note: The problems involve the applications of basic laws on Control Volumes. However, wherever required and the scenario permits, the Bernoulli's equation should be utilized.

1. Figure 1 shows a fire hose nozzle which tapers from a diameter of 2 inch to a diameter of 1 inch at the open end. The nozzle carries water at a volume flow rate of 10 litre / s. Assuming steady, inviscid flow, calculate the force 'F' needed to hold the nozzle.
2. Figure 2 shows a pair of converging streamlines in a two-dimensional incompressible flow. Choose an appropriate control volume and apply mass conservation to show that the fluid between the two streamlines must be accelerating in the flow direction.
3. Consider the incompressible steady flow through a propeller as sketched in figure 3. Upstream of the propeller, the fluid is accelerated from speed V_1 to a higher speed V_p as it passes through the propeller. The acceleration is caused by a drop in pressure from atmospheric level at section 1 to P_{in} in front of the propeller. As the fluid flows through the propeller, the propeller raises the pressure of the fluid to a value P_{out} that is higher than atmospheric pressure. Therefore, in the region downstream of the propeller, the fluid further accelerates to a speed V_2 as the pressure falls to atmospheric value at station 2. As the flow accelerates, the streamlines passing through the tip of the propeller converge (see Prob. 2). The area of flow through the propeller is A_p .
 - a) Select a narrow Control Volume (see fig. 3) surrounding the propeller alone and apply momentum equation to find the magnitude and direction of the force 'F' exerted by the fluid on the propeller.
 - b) Using Bernoulli's theorem on the upstream and the downstream side separately, express the force 'F' obtained in (a) in terms of speeds V_1 and V_2 .
 - c) Choosing an outer control volume bounded by the streamlines (see fig. 3), apply the momentum equation and obtain an expression for speed V_p in terms of speeds V_1 and V_2 .
4. The cylindrical tank of 1 m diameter shown in fig. 4 has a weight of 300 N when empty. The inlet pipe has a diameter of 5 cm while the outlet short pipe has a diameter of 8 cm. The water flow rate from the outlet pipe is so adjusted so that the level in the tank is maintained steady at 80 cm while the weighing scale gives a reading of 7500 N. Neglecting the weight of water in the outlet short pipe, estimate the volume flow rate from the inlet.
5. Consider a rectangular tank of cross-sectional area 'A' filled with water and resting on a surface whose coefficient of static friction is ' μ_s ' as shown in fig. 5. The tank is filled upto a certain height 'H' with water. A small port of area ' A_p ' in the side of the tank is opened and the fluid velocity through the port is given by the Torricelli's equation $V_f = \sqrt{2g(H-h)}$. Obtain the range of values of 'H' if the tank must remain stationary. Further show that for $A_p < \frac{1}{2}\mu_s A$, the tank never moves for any height 'H'.
6. Consider the two-dimensional steady incompressible flow past an oval shaped body as shown in figure 6. The x-direction velocity is measured at a location downstream of the

body and its variation with y is given as $u(y) = U_0(1 - \frac{1}{2}\exp(-y^2))$. The velocity at some distance in front of the body is uniform and is given as $\vec{V} = U_0\hat{i}$. Apply Linear Momentum equation to a suitable CV to obtain an estimate of the force in x -direction exerted by the fluid on the body. The density of the fluid is ' ρ '.

7. The jet of fluid having density ' ρ ' issues out of a nozzle at a velocity V_j as shown in fig. 7. The jet strikes a cart moving at a uniform velocity ' U ' towards right on a level surface as shown in fig. 7. If the nozzle exit area is ' A_N ' and the magnitude of the relative velocity entering and leaving the cavity is same: Estimate the force needed to restrain / prevent the cart from accelerating.
8. For the scenario of Problem 7, the cart is allowed to accelerate from a state of rest ($U = 0$). Taking mass of the cart as ' M ' and neglecting the small mass of water in the cavity in comparison to the mass of the cart, obtain:
 - a) the differential equation governing the evolution of the velocity of the cart in time
 - b) Solve the differential equation to obtain the cart velocity U as a function of time.
 - c) What is the 'theoretical upper limit' to the speed attainable by the cart.
9. Consider a rocket with an initial mass ' $m+M$ ' (fuel+rocket) fired vertically from the surface of the earth into outer space. If the rocket engine / motor burns fuel at the rate ' m_f ' Kg / s and the burnt gases are ejected from the nozzle at a relative velocity ' V_e ' and a pressure ' P_e ' that is different from the outside ambient pressure ' P_a ' (fig. 8). If the nozzle exit area is ' A_e ' obtain the expression for the vertical acceleration of the rocket ' a ' as a function of time ' t ' and altitude ' h ' of the rocket above the surface of the earth. (Hint: the outside ambient pressure and gravity in general are functions of altitude of the rocket)
10. A cylinder equipped with a frictionless piston of area ' A_p ' and filled with a fluid of density ' ρ ' is mounted on a wheeled cart as shown in fig. 9. A person standing on the cart exerts a force ' F ' on the piston, causing a jet of fluid to be ejected into the surrounding air through the opening having an area ' A_o ' on the right end of the cylinder. The cart is restrained from moving by a rope as shown in figure 9. Assuming steady motion of the piston and inviscid, uniform flow in the cylinder, employ the deforming CV shown to derive expressions for the velocity ' V_j ' of the jet and tension ' T ' in the rope in terms of ρ , F , A_j and A_p .
11. Consider two identical circular disks of radii ' R ' initially separated by a gap ' H ' as shown in fig. 10. The upper disk is pushed downward at $t = 0$ at a steady velocity V . The movement of the upper disk squeezes out the fluid of density ' ρ ' between the two disks into the atmosphere. Assuming radial, inviscid flow between the disks, employ a suitable deforming CV to obtain
 - a) The expression for the exit radial velocity $U(r = R)$. Is the flow steady ?
 - b) The expression for radial velocity at any radius $r < R$.
 - c) The expression for pressure at any radius $r < R$ by considering an annular CV of inner radius r and outer radius R and applying momentum equation in the radial direction.
 - d) Would the answers obtained in b) and c) apply near the center of the disks ($r = 0$). If not why ?

12. Figure 11 shows a steady flow branching taking place horizontally through a Y-Junction involving three circular pipes carrying a fluid of density ' ρ '. The flow velocities and gauge pressures at various sections are indicated in figure 11. The force F_y on the Junction exerted by the fluid is found to be zero. Neglecting the effects of viscosity, obtain:
- The general relation between the downstream flow velocities (V_2, V_3), the upstream flow conditions (P_1, V_1) and the geometric parameters. (Hint: Apply y-momentum equation to a suitable CV).
 - Show that if the areas of the downstream pipes A_2, A_3 , satisfy the relation $A_2 \sin \theta_1 = A_3 \sin \theta_2$ then $V_2 = V_3 = Q_1 / (A_2 + A_3)$ and $Q_2 / Q_1 = A_2 / (A_2 + A_3)$, $Q_3 / Q_1 = A_3 / (A_2 + A_3)$.
13. It is possible for a water flow in an open channel without an obstruction to undergo a spontaneous increase in level as shown in fig. 12. The region of transition from the uniform flow upstream to another uniform flow downstream is irregular and random, being a turbulent flow with small wavelets on the surface that appear to break. While we cannot describe the flow in this transition region, we can relate the upstream and the downstream conditions by utilizing the mass conservation and linear momentum equations. Apply these laws to the CV shown in figure 12 and obtain the expressions separately for V_1 and V_2 in terms of h_1, h_2 and gravity g . Neglect the effects of viscosity and assume hydrostatic pressure variation with depth on the uniform upstream and downstream states of flow.

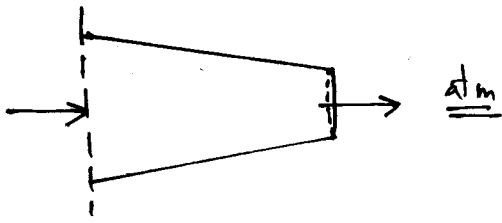


FIG. 1

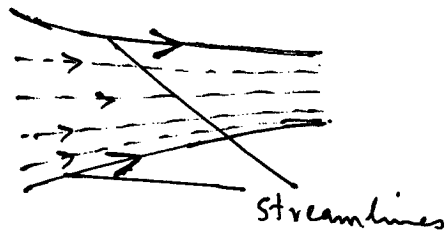


FIG. 2

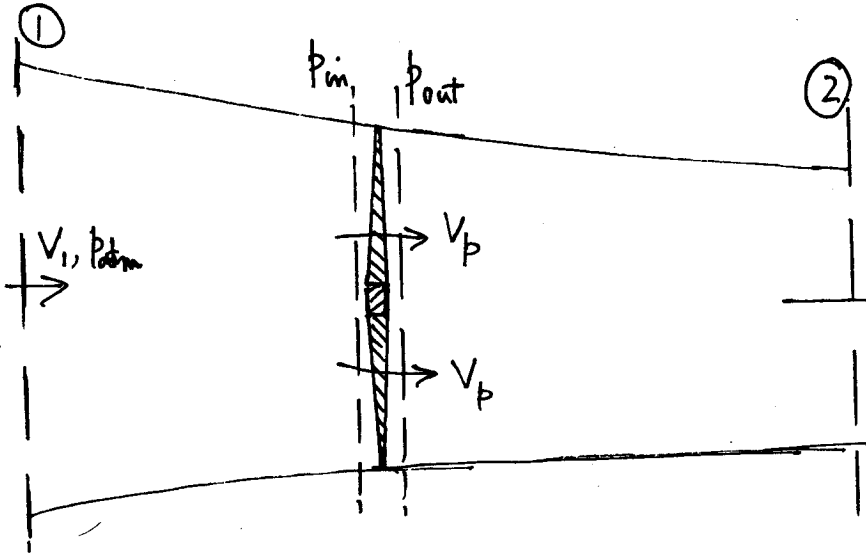


FIG 3

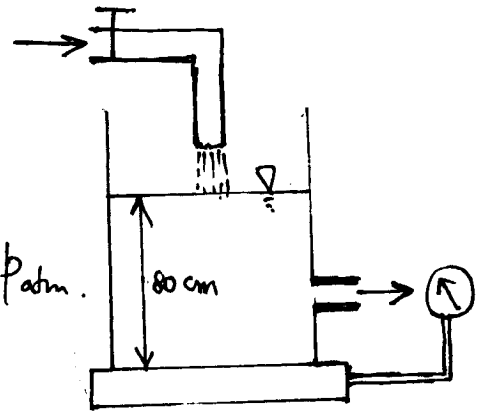


FIG 4

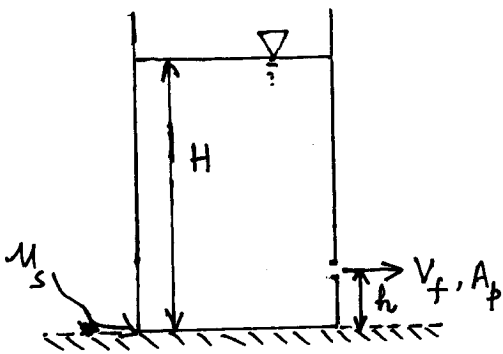


FIG 5

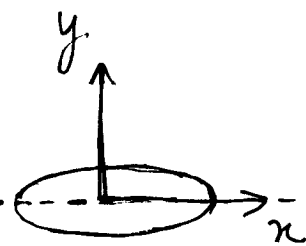
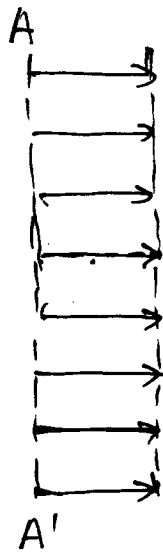


FIG. 6

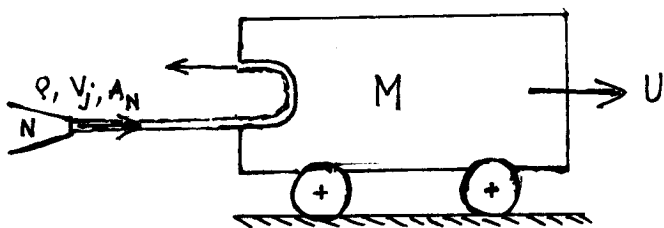


FIG. 7

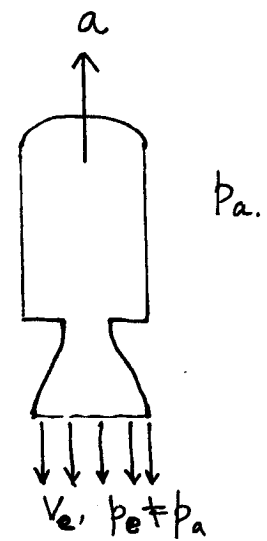


FIG. 8

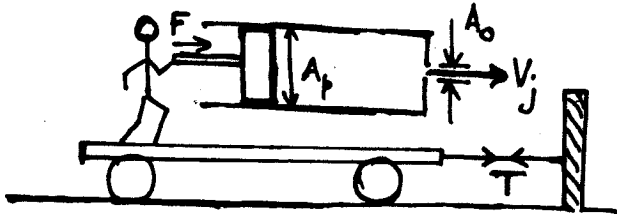


FIG. 9

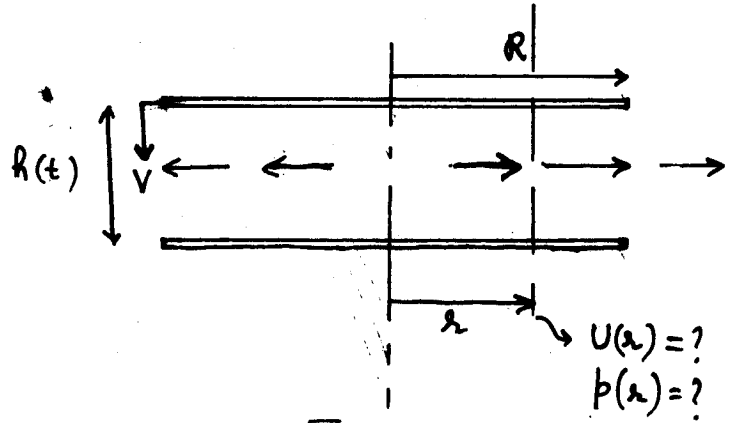


FIG. 10

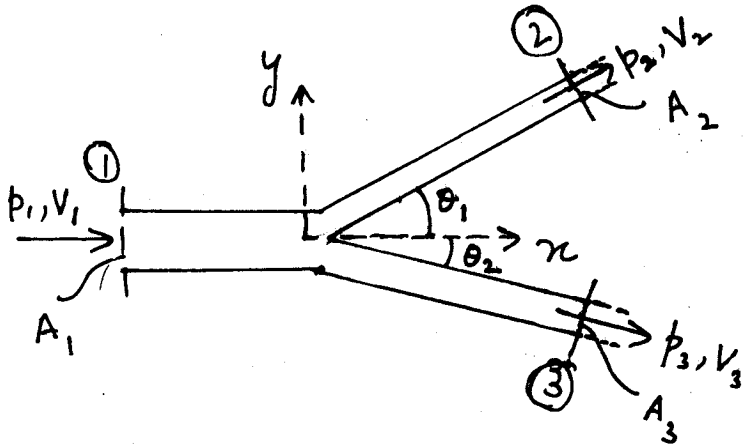


FIG. 11

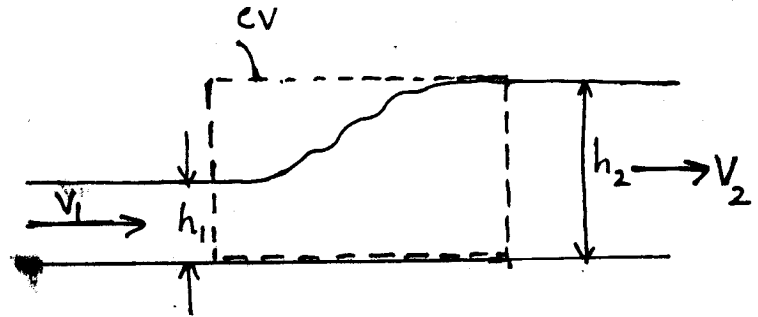


FIG. 12