**Experiment No. 1: Bernoulli’s Theorem**

**Object:**

To verify Bernoulli's theorem for a viscous and incompressible fluid.

**Theory:**

In our daily lives we consume a lot of fluid for various reasons. This fluid is delivered through a network of pipes and fittings of different sizes from an overhead tank. The estimation of losses in these networks can be done with the help of this equation which is essentially principle of conservation of mechanical energy.

**Formal Statement:**

Bernoulli's Principle is essentially a work energy conservation principle which states that for an ideal fluid or for situations where effects of viscosity are neglected, with no work being performed on the fluid, total energy remains constant. Bernoulli's Principle is named in honour of Daniel Bernoulli. This principle is a simplification of Bernoulli's equation, which states that the sum of all forms of energy in a fluid flowing along an enclosed path (a streamline) is the same at any two points in that path.

**Mathematical Description:**

![Figure 1 Forces and Fluxes for Bernoulli’s Equation for frictionless flow along a streamline.](image)

Applying the conservation of mass and momentum equation yields the following equation

\[ \rho + \frac{\partial \rho}{\partial t} + \mathbf{V} \cdot \nabla \rho = 0 \]

\[ P + \frac{\partial P}{\partial t} + \mathbf{V} \cdot \nabla P = 0 \]

\[ \mathbf{U} \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla P + \mathbf{F} + \nabla \mathbf{V} \cdot \mathbf{U} \]

\[ 0 = \mathbf{F} \cdot \mathbf{n} + \rho \mathbf{U} \cdot \nabla \mathbf{U} \cdot \mathbf{n} + \frac{1}{2} \rho (\mathbf{U} \cdot \mathbf{n})^2 + \rho \mathbf{U} \cdot \mathbf{F} \cdot \mathbf{n} \]
Equation (1.1) is the unsteady frictionless flow along a streamline. Integrating, the above equation between any two points 1 and 2 on a streamline.

\[
\int_{1}^{2} \frac{\partial V}{\partial t} ds + \int_{1}^{2} \frac{dP}{\rho} + \int_{1}^{2} \frac{1}{2} dV^2 + \int_{1}^{2} gdz = 0
\]

For any steady incompressible flow, equation 1.2 reduces to

\[
\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0
\]

or

\[
\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{Cons tan t}
\]

or

\[
\frac{P}{\rho g} + \frac{V^2}{2g} + z = H \text{ (Total Head)}
\]

\[
h = \frac{P}{\rho g} + z
\]

Figure 2 Block Diagram of the set up.

Equation 1.4 is widely known as the Bernoulli’s Equation for a steady incompressible, frictionless flow. In equation 1.5, ‘h’ is known as the peizometric head. While \(V^2/2g\) is known as the Kinetic head and \(H\) is the Total Head.

Practically, application of Bernoulli’s equation is difficult so following approach is used. It has been observed that same set of streamlines pass through all cross sections. Thus,
Avg BE Constant = \( \frac{1}{A} \int_{0}^{A(x)} (BE \text{ Constant})dA \)

Avg BE Constant = \( \frac{1}{A} \int_{0}^{A(x)} \left( \frac{P}{\rho g} + \frac{V^2}{2g} + z \right) dA \)  

(1.5)

Avg BE Constant = \( \frac{1}{A} \int_{0}^{A(x)} \left( \frac{P}{\rho g} \right) dA + \frac{1}{A} \int_{0}^{A(x)} \frac{V^2}{2g} dA + \frac{1}{A} \int_{0}^{A(x)} z dA \)  

(1.6)

or \( \left( \frac{P}{\rho g} \right)_{\text{avg}} + \left( \frac{V^2}{2g} \right)_{\text{avg}} + (z)_{\text{avg}} = \text{Avg BE Constant} \)  

(1.7)

or \( h_{\text{avg}} + \left( \frac{V_{\text{avg}}^2}{2g} \right) = H \)  

(1.8)

where \( L(x) \) represent the height of the converging-diverging duct.

Loss Coefficient (LC) is defined as

\[ LC = \frac{H_o - H}{H_o} \]

where \( H_o \) is the total supply head and \( H \) is the sum of average piezometric and kinetic head.

**Given Data**

1. Tank Area (A) = \( m^2 \)

**Observations:**

1. Supply Head (\( H_o \)) =

2. For Flow Rate (Q)

\[ \text{Time} (\Delta T \text{ in sec}) \text{ for } 10\text{cm rise } \Delta T_1 = \ldots \Delta T_2 = \ldots \Delta T_3 = \ldots \]

**Observation Table**

<table>
<thead>
<tr>
<th>Tube No.</th>
<th>Distance from the tank X (m)</th>
<th>Cross Sectional Area((A_r) \ m^2)</th>
<th>Peizometric Head (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2…13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Result Table**

<table>
<thead>
<tr>
<th>Tube No.</th>
<th>Distance(X)</th>
<th>Peizometric Head (h)</th>
<th>Flow Velocity ((V_{\text{avg}}=Q/A))</th>
<th>Kinetic Energy</th>
<th>Total Energy Head (H)</th>
<th>Loss Coefficient (LC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2…13</td>
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BournoulliEquation
**Aim:**
To evaluate the different components of mechanical energy possessed by a fluid element at different sections of a converging diverging duct and thus appreciate the limitations of Bernoulli’s equation applied to incompressible flows.

**Graphs to be plotted**
- 1. Peizometric Head along the duct.
- 2. Kinetic Head along the duct.
- 3. Total energy head (H) along the duct.
- 4. Loss Coefficient along the duct.

**Note:** All plots have to be drawn on the same graph paper.

**Discussion**
- 1. Discuss the physical significance of the experiment?
- 2. Explain the limitations of BE?
- 3. Explain the dynamics of
  - a. Golf Ball
  - b. Base Ball
  - c. Ping Pong Ball
  - d. Lawn Tennis Ball
- 4. Explain why is a piece of paper is dragged towards a fast moving train when it enters a station?