

# Properties of division

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Equivalence principle, commutative property, unit referential characteristic.

From zero to infinite

$$0/0 = 1 \text{ \& \; infinite/infinite} = 1$$

## 1.- Equivalence principle of division

"Any number, element, system, structure, idea, etc., that is divided by itself gives us the unit (1)".

### Equivalence principle of division

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$$\frac{x}{x} = 1$$

→ Whenever  $x = x$

$$\frac{0}{0} = 1 \quad \frac{\infty}{\infty} = 1 \quad \frac{a}{a} = 1 \quad \frac{\square}{\square} = 1$$

Where  $x$  is any thing, structure, element, concept, number, etc.  
Donde  $x$  es cualquier cosa, estructura, concepto, número, etc.  
"ius"

To this general sign for any element we can call it "us"

At the moment it is accepted a solution for the division between zeros (0/0) that I think it is not completely correct, which would be:

"If we want to discover the result of the division of 0/0, we should think about the equation":  
 $0/0 = x$  .... giving us .....  $0x = 0$  .....See at the end "operation with empty sets"

And this way **any number multiplied by zero completes this condition**, being therefore UNCERTAIN the solution to this division (0/0)

In this case we say that 0/0 is an INDETERMINATION.

I think this given solution for the division among zeros is incorrect for the reason that it is applied following a unique norm or property of the multiplication, but being 0/0 a division we should also have to apply norms, properties and characteristic of the divisions. And it is not made.

I am referring to a "**structural**" property of the division that establishes a relation of equivalence among its terms, which tell us:

### Equivalence principle:

"In any division the dividend contains N times to the quotient, being N the divider."

So we can conclude that:

"Any number, element, system, idea, etc., that is divided by itself gives us the unit (1)".

In this sense, the current conclusion as for  $0/0$  is uncertain, and so any number could be the result of this division seen to be clearly incorrect. Question that we can see with any example: If were  $0/0 = 7$  then we would have that the dividend ( 0 ) should be seven times superior than the divider ( also 0 ) and this is not this way since they are the same one.

Therefore so that the conditions of the division are fulfilled (in this case equivalence property) it is necessary that  $0/0 = 1$ .

This way if any number that if divided by itself gives us the unit 1 ( $3/3=1$ ;  $8/8=1$ ;  $0'5/0'5= 1$ , etc.), we cannot consider to zero with other properties, neither to the infinite: Zero is an infinitely small number and the infinite is an infinitely big number, but they continue being numbers with the same mathematical properties of the other ones.

If we observe the division between equal numbers, we see that the result is 1. On the other hand if the dividend and divider are different, the quotient will never be 1.

In this case, we can say "the 0 (dividend) contains 1 time (quotient) to the 0 (divider); or that the infinite (dividend) contains 1 time (quotient) to the infinite (divider)".

Now well, this equivalence property or equality between dividend and divider when the quotient is the unit 1, it is possible even to enlarge it to any other physical element, in such a way that if we divide two equal elements, (letter, triangle, tree, etc.) for the same one, it will also give us the unit.

The translation of this property to the physical elements it is because the property of equivalence of the division takes place or affirms equality between dividend and divider.

Then if we have two physical elements as dividend and divider and this division gives us the unit, it tell us that the physical elements that intervene in the division are completely equivalent, and vice versa, if we have two equal (or equivalent) physical elements their division gives us the unit 1.

So, if have two unknown elements # and & but we know they fulfill de equation  $\# / \& = 1$ , then we can know that they are equal o equivalent elements.

But if they fulfill the equation  $\# / \& = 5$ , we know they are not equal (o equivalent) elements.

$$0/0 = 1$$

The operative forms of the multiplication  
Defects of the simplified way.

*Division's Properties*  $\frac{0}{0} = 1$  ferman 20-Sep-2007

$$4 \times 3 = 4 + 4 + 4 \quad \text{Then} \quad 3 = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} \quad \text{So} \quad \frac{4}{4} = 1$$

$$0 \times 4 = 0 + 0 + 0 + 0 \quad \text{Then} \quad 4 = \frac{0}{0} + \frac{0}{0} + \frac{0}{0} + \frac{0}{0} \quad \text{So} \quad \frac{0}{0} = 1$$

Using the faithful method of the multiplication

In the multiplication we can consider two operative ways:

- The faithful, natural or complete method of multiplication and
- The simplified method of multiplication.

--- The faithful or complete method of the multiplication is the one that complete the structural rule of the multiplication:

"To multiply is to add the multiplicand so many times as units has the multiplier."

E.g.  $4 \times 3 = 4 + 4 + 4$  with final value of 12.

--- The simplified multiplication is an operative method for which we use some tables that we know usually by memory, which tell us the final result without necessity of making the summands one after one.

E.g.  $4 \times 3 = 12$

But of course, in this procedure of simplification some information and operative fidelity is lost, as we see next.

And the worst thing of everything in mathematic is that this way of multiplying is the one that we use to make many mathematical deductions and postulates that are corrupted or wrong in some cases.

And the theme that occupies us belongs to them,  $0/0 = 1$ .

Let us see the faithful or complete procedure by means of an example:

$$4 \times 0 = 0 + 0 + 0 + 0$$

Clearing the 4 we have:

$$4 = 0/0 + 0/0 + 0/0 + 0/0$$

What demonstrate us that  $0/0 = 1$ .

However if we use the simplified form of the multiplication, we see that it gives us a result  $0/0 =$  indeterminate, since it has infinite solutions already when  $n \times 0 = 0$  and  $0/0 = n$ , and the division  $0/0$  with this operative way would give us any result.

And to what is this owed to?

Because due to the loss of information, fidelity and accuracy that this simplified form of the multiplication has.

This operative method jumps from the exhibition of the operation ( $4 \times 3$ ) to the final value of the multiplication (12) without taking in mind the complete operative process, with which, we lose the intermediate properties that all multiplication should have, those which are necessary for demonstrations and proposal of postulates that use to the multiplication as base.

Therefore, and to my understanding,  $0/0 = 1$ , question that can be demonstrated as previously we have seen.

## 2.- The unit, as reference of division.

The division is an operation of partition that takes as reference to the unit 1.

This means it doesn't care if the dividend to distribute is superior or inferior to the unit, we will always take the result as "quantity that would correspond to the unit."

Let us see some examples:

If we divide 60 euros among 5 people, we will give 12 euros to each one (12 euros per 1). And this we understand that it is completely logical and real.

But if we divide  $1/2$  cakes between  $1/2$  rooms of children, the division result is 1. But this not means that the  $1/2$  room of children have to get 1 cake, but rather the resulting quotient is for each unit, and therefore, what tells us the division is that to 1 complete room of children will correspond 1 complete cake.

In the same way if we divide 4 among  $0'5$  ( $4/0'5 = 8$ ) it doesn't mean it is to  $0'5$  that correspond 8, but to the unit 1 (8 per unit 1).

Now well, that is also what means  $0/0=1$ . It is not to the divider (0) that corresponds to the unit 1, but rather when being equal dividend and divider, to each unit of "the receiving thing" would corresponds a unit of "the distributed thing".

This way if we distribute 0 euros among 0 people ( $0/0 = 1$ ), it doesn't mean that to zero people we give 1 euro, but to the reference unit (a person) it is what we would give 1 euro. Therefore we see that the divisions of rational numbers are percentages to the unit: So much per unit. And clear in this case  $0/0 = 1$  is also a percentage to the unit, one for each unit.

### 3.- Commutative property of division:

#### Equivalence principle of division

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$$\frac{x}{x} = 1$$

$$\frac{0}{0} = 1 \quad \frac{\infty}{\infty} = 1 \quad \frac{a}{a} = 1 \quad \frac{\square}{\square} = 1$$

#### Commutative property of division

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$$\frac{3}{3} = 1 \quad \frac{0}{0} = 1 \quad \frac{\infty}{\infty} = 1 \quad \frac{a}{a} = 1 \quad \frac{\square}{\square} = 1$$

$$\frac{3}{1} = 3 \quad \frac{0}{1} = 0 \quad \frac{\infty}{1} = \infty \quad \frac{a}{1} = a \quad \frac{\square}{1} = \square$$

Another reason that induces us to enlarge the equivalence principle of division to any elements, when it gives the unit 1 as a result, it is the one that provides us the Commutative property of division.

This property shows us that if we apply this property to the division between equal elements, the resulting equality is also fulfilled with any type of elements, although they are not numbers.

#### Definition:

The Commutative property would tell us that if in an equality ( $a / b = c$ ) we change the divider (b) for the quotient (c), this new equality is also fulfilled ( $a / c = b$ ).

#### Revised properties:

Of this study we can see as main properties and characteristic of division to the following ones:

#### Equivalence principles:

"In any division the dividend contains N times to the quotient, being N the divider."

### Commutive property:

"In any division if we change the dividend for the quotient the new resulting equality it is also fulfilled."

### Referential property:

"In all division, the quotient takes as partition or allotment reference to the unit 1 "

### Operation with empty sets.

The empty set is a set without elements that can be represented by the zero ( 0 ), but it continues being a set that can be subjected to the operations of set. Therefore we can say "zero (0) is essentially the empty set."

### Empty set: Operations $0 \times = 0$ September 2007 *ferman*

$$0 \times = 0$$

$$3 \cdot 0 = 0 \quad \text{Is partially incorrect}$$

$$0 + 0 + 0 = 0 \quad \text{Is partially incorrect}$$

Three EMPTY set = one EMPTY set

$$\square \square \square = \square \quad \text{Is partially incorrect}$$

Three empty glasses = One empty glass

$$\frac{\square \square \square}{3} = \square, \text{ but not indetermined}$$

$$\frac{\square}{1} = \square \quad \text{and} \quad \frac{\square}{\square} = 1 \quad \text{are correct}$$

When we operate with empty sets, we usually look on (simple and exclusively) the result of their component elements to which we date as zero when having none.

But we forget something essential, and it is the number of empty sets with which we are operating.

If, as in the drawing, we take an empty glass to which we multiply by 3, the real result will be we have 3 empty glasses, but the partial result will be we have zero elements in these 3 empty glasses.

So, in this case we adjust as result ALONE THEIR ELEMENTS, but we forget we are USING A SERIES OF SETS.

Although this operation method is of great importance due to we later use this property as principle, base and justification of other operations, as can be in division.

And clear, when taking as principle and explanation to a partial result and not to the total result of the operation, because we end up accepting indetermination principles that are not correct.

For example, if we put  $1 \times 0 = 4 \times 0$  we are accepting that both terms are identical, when they are not because in the first term there is alone an empty set and the second term there are four empty sets, although the number of component elements is same in both term of the equality.

This way, when we operate ( $3 \times 0 = 0$ ) we should accept that we are operating PARTIALLY and alone with relation to the elements of the empty sets that we are using.

In the same way we should accept that this operation is PARTIALLY UNCERTAIN, since three empty sets cannot be the same thing that an empty set.

For this same reason we cannot use this type of postulates to conclude that  $0/0$  are an uncertain operation, since their solution is  $0/0=1$  abiding to the properties of the division.

## *Representation of* Empty set:

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### In pure mathematics

$$4 \times 0 = 0$$

$$6 \times 0 + 0 + 2 \times 0 = 0$$

$$21 \times 0 - 6 \times 0 = 0$$

$$\frac{21 \times 0}{3} = 0$$

### In mathematics of sets

$$4 \times 0 = 4.\bar{0}$$

Four empty sets

$$6 \times 0 + 0 + 2 \times 0 = 9.\bar{0}$$

Nine empty sets

$$21 \times 0 - 6 \times 0 = 15.\bar{0}$$

Fifteen empty sets

$$\frac{21 \times 0}{3} = 7.\bar{0}$$

Seven empty sets

## Operations and *Representation of* Empty set

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$$\frac{3 \times 0}{6} = \frac{1}{2}.\bar{0}$$

$$\frac{4 \times 0}{6} = \frac{2}{3}.\bar{0}$$

$$\frac{1}{2}.\bar{0} \times 4 = 2.\bar{0}$$

$$\frac{2}{3}.\bar{0} \times 12 = 8.\bar{0}$$

Es incorrecto

$2.\bar{0} \times 0$  It is not correct

In operation with set

It is not possible to multiply set by set; alone set by numbers. And  $\bar{0}$  is an empty set.

Es imposible multiplicar conjuntos por conjuntos, solo números por conjuntos y  $\bar{0}$  es un conjunto vacío.