

# Planar angles

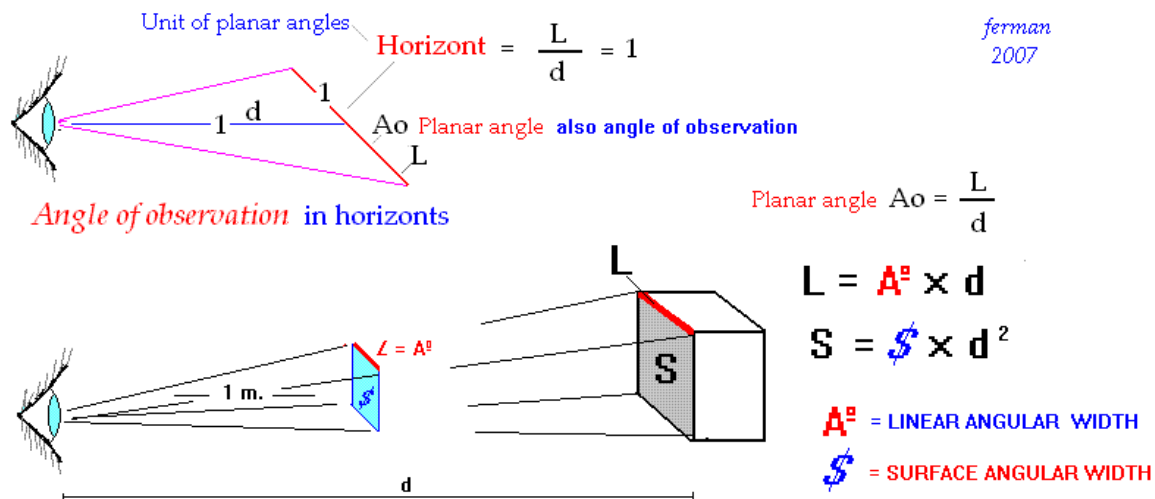
## Angle of Observation Ao

Trimetry, stellar meridian, stellar trimetry.

## Visual parameter: Horizont

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### Horizont as unit of Planar angles in geometry and Angle of observation in vision



## Horizon

### Metric unit of planar angles

$$\text{Horizon} = 1 \text{ m(m)}$$

$$\text{Horizon}^2 = 1 \text{ m}^2(\text{m})$$

We already know some angular measures in their different dimensions, but always referring or taking as reference point to radial centres on which we measure these angles.

In this case always it gives us radial angles that are circumference arch with such units as degrees or radians in longitudinal way or square degrees and steradians in surface form.

But I think we lack the most important centre or reference frame for us, our eyes.

Our field of vision has a width that many estimate around 50 degrees of lateral width.

For our study, adapting us as well the peculiarities of our vision as to the geometric characteristics, we will to apply an amplitude of field of 45 degrees.

To this amplitude of field we would name Angle of Observation (Ao) because of its name indicates we go to use angles for measuring all the observations and all the set of operations on the new field that is Trimetry.

Good, for measuring the whole angle of observation adopted, (of 45 degrees) we go to use as unit to the Horizont due to that angle coincides with the amplitude of the horizon of vision that we have.

Say, observing the horizon of our vision, we can see that the extremes of this horizon form with our eyes an approximate angle of 45 degrees.

Now well, we don't use degrees, but planar angles that are measured by straight metric units.

But why we go to make it in metric units?

Very easy, to avoid all the trigonometry with its angles and tables of application.

Here we don't use tables of trigonometry, but simply straight lineal measurements.

For there, we use the horizon that is an angle of vision or observations corresponding to a 1 meter of longitude (L) en perpendicular way, taken to a 1 meter of distance (d).

So, the horizon will be  $L/d = 1$ , and this way, always that (L) equal to (d),  $L=d$ , this angle will be the unit of Horizon.

The used symbol for the planar angles and observation angle will be  $A^\circ$ .

But for what reason this parameter can serve us and reason we use centimetre instead of degrees?

For the first question, to have a parameter adjusted to our peculiarities of vision.

And to second, we use metric measures instead of angular ones with object of being able to adjust the surface that we observe in metric measures that can serve later to adjust the dimensions of objects.

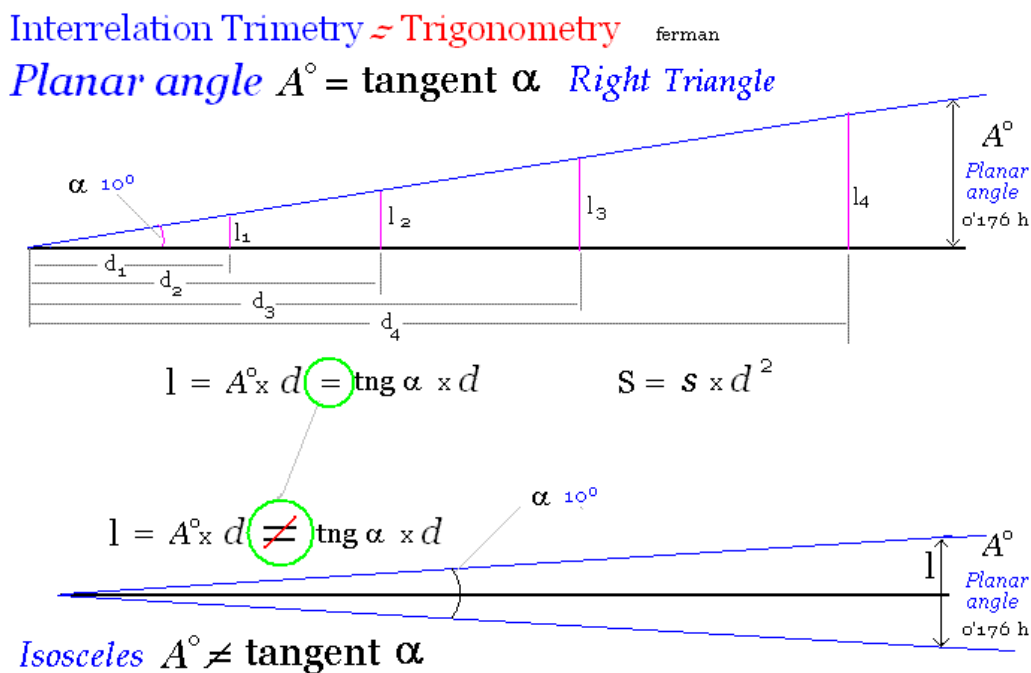
The formulas used with this type of measurements are very easy and the simpler could be:

For longitudinal angles **Horizont = 1 m(m)**       $L = A^\circ \times d$

Where L is the longitude of the piece of horizon (ob object) to measure;  $A^\circ$  is the planar angle of observation; and d is the distance to which the object is situated.

For angles of area or surfeces **Horizont<sup>2</sup> = 1 m<sup>2</sup>(m)**       $S = \alpha \times d^2$

Where S is the area or surface that we need to know of a far object,  $\alpha$  the angular area, and d the distance of the observed object



## Longitudinal planar angles

Although for reason of its visual foundation we have begun seeing the planar angular surface, the planar angular longitude logically also exists.

This would be that plane and lineal width of our horizon of vision with a magnitude of 1 dm to a meter of distance.

Of course their measure unit would be the horizont = 1 m (m).

And the usable formula would be then:  $L = A^\circ \times d$  where

**L** would be the frontal longitude of any observable object.

$A^\circ$  the angular longitude

**d** the distance to that the object is.

Of course, all the considerations on the planar angular surfaces are valid for the longitudinal ones.

## Definition:

Planar angle is an angular geometric structure that is built and defined by lines and planes only, and subjected to metric measures exclusively.

It consists of: ---An **angular vertex** where the lines or planes that form the angle cut themselves.

--- **Sides** are the lines or planes that form the angle.

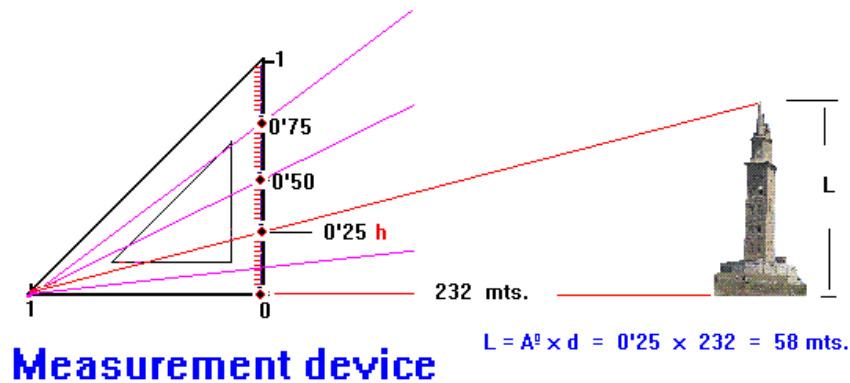
---The **angular horizon** is the line or plane that cuts perpendicularly to the distance **d**, and where the objects to observe are located.

---The distance **d** or bisector of the angle on which the distance units and the distance of the observable objects are measured.

The angular dimensions come determined by the width or opening of the angle and the distance **d** from the angular vertex until the angular horizon where the observable object is situated.

## Measurement of planar angles

### *Ferman* *July 2007* **Trimetry Measure of planar angles** **Unit of planar angles h Horizont (1x1)**



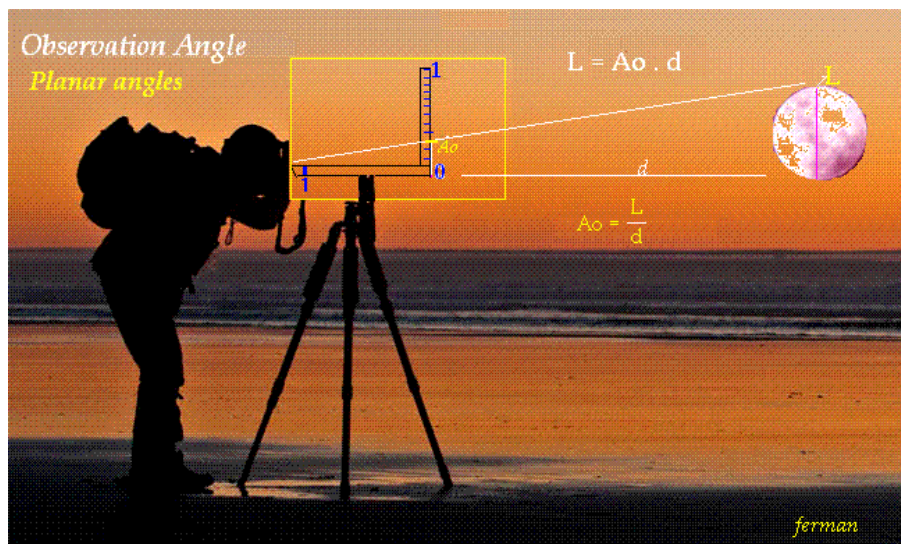
In the following drawing we see as easy is to measure planar angles.

It is enough to use a set-square like in the drawing.

Later you can apply the formula of planar angles to obtain the searched longitude.

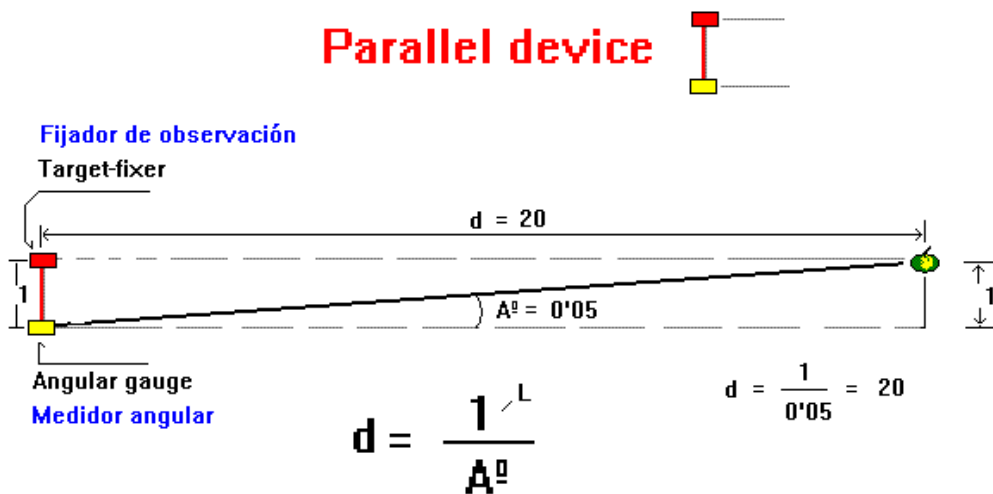
To measure planar surfaces we can use a squared visor that gives us the approximate value of the angular unit of planar surfaces (squared horizont) and later apply the formula of planar surfaces ( $S = \theta \times d^2$ ).

This question will treat later when we build figures of planar surfaces.



## Triangulation for getting distances

### Parallel device



As we can see in the drawing, the triangulation is very simple with angles planares. In such a way that if we have a devise with double viewer (of position and of angularidad) very adjusted, with alone to observe the angle of diphas of the devise we can obtain the distance to the observed object.

In the drawing a simple outline of the device is exposed.

This consists of two observation lens totally aligned in parallel and to a certain unit of distances between these two lenses.

The lens 1 is the one in charge of fixing the point or observed object on its gauging centre.

The lens 2, (when being totally parallel to field of vision to the lens 1) it will mark us a diphas o angular difference between the object and its central point of measure.

This diphas is the angularity  $A^0$  that will be the one that divides to the separation unit between the lenses to find the distance from the observed object, just as you can see in the drawing ( $d = 1 / A^0$ )

Now well, once obtained the distance we can (only with the lens 2) measure the angularity of the observed object and to find its real dimensions.

Therefore, this it is a device to measure distances and dimensions of the distant objects.

### Trimetry, stellar meridian, stellar trimetry.

We already know that trigonometry studies in triangles the relationship between the width of angles and the longitudes of its sides.

These trigonometrical studies are either made with parameters and charts of angular values in degrees or radians, and therefore, under the consideration of radial angles, say, the study of triangles inscribed on circumferences.

However, our measurement parameters are different; say, they are planar angles whose metric is the simple relation between the front plane of observation or horizon and the distance to that plane or horizon, nothing to do with triangles inscribed to circumferences.

Therefore, as our study varies in parameters, charts and characteristic of its components, because we would have to call to these measure methods with another name.

So, I will call it TRIMETRY, if nobody is opposed.

# Horizont, its multiples and dividers

We have checked that the horizon is a unit for the simple observation of our own ocular capacity and for it, this measure unit is designed.

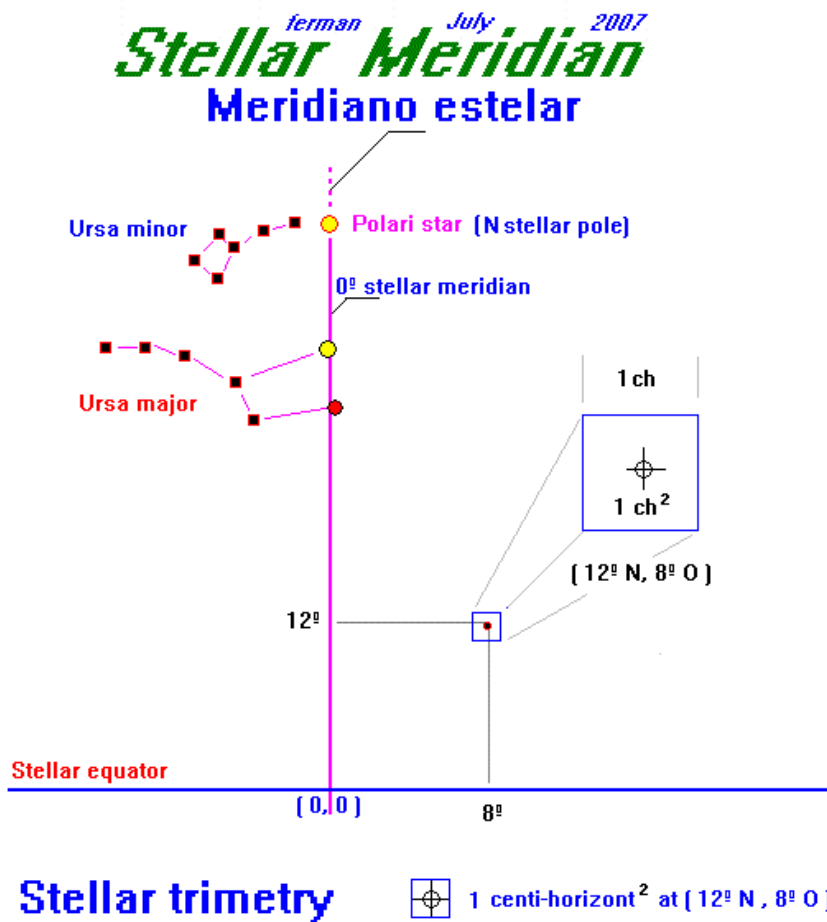
But however many events can exist in that the use of multiples as dividers of this unit (horizont) could be necessary.

This way if we observe some geometric figures as they can be triangles, cones, pyramids, etc., here the ideal would be to use equivalent relative measures, that is to say, not of 1/10 as it is the horizon, but of 1/1 as would be the deca-horizont.

On the other hand in some events such as framing a group of stars of the sky, because it would be more convenient to use a divider of the horizon, since this divider would be better of using.

In this cases we will use the deci-horizont (dh) that would be a relative unit of 1/100, centi-horizont (ch), mili-horizont (mh).

In the following drawing we see an example of this:



## Stellar meridian

In this drawing I also make the proposal of use of a consequent stellar meridian with the structure of the stellar map, without keeping in mind the plane of the ecliptic and the turning plane of earth that are too changing and little locatable.

Well, revised these topics scarcely, we will pass later more thoroughly to revise the trimetry topic of the geometric figures.

## Trimetry in the geometric figures.

As we have said, we will consider trimetry as a small branch of geometry that studies the methods of measures in the planar angles and their triangulation, exclusively supported in metric measure.

When the trigonometry goes exclusively to the triangles rectangles using charts of angular values; trimetry goes to all type of triangles, cones and pyramids (\* and other ) basing its parameters of angular width on the simple ratio among the base (horizon) and the height (distance d) of these geometric figures and on the projection characteristics that have their angles (from the vertex).

This particular relation gives us the specific width for each figure.

So, it is not also necessary to use charts since another relation that the before mentioned doesn't exist.

[--(\* and other ) Beside triangles, cones and pyramids with trimetry of variable angularity we can build all type of figures, similar to when we use Cartesian coordinates.]

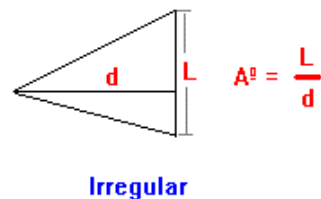
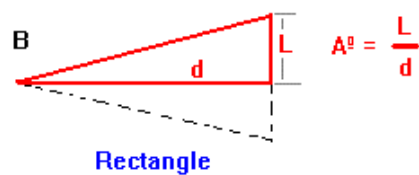
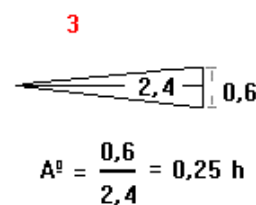
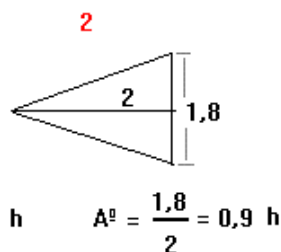
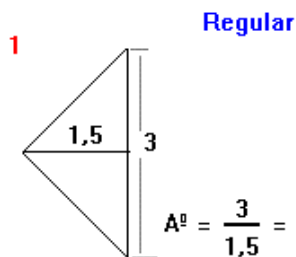
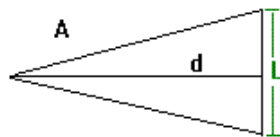
In the following drawings you have some figures where trimetry can be used:

## Trimetry in geometric figures July 2007 Ferman Triangles

Angular planar unit = **horizont**

$A^{\circ}$  = Planar Angles in **horizonts**

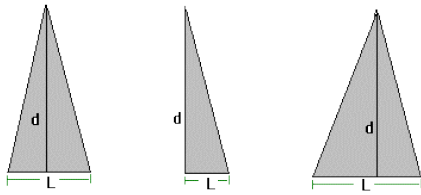
$$A^{\circ} = \frac{L}{d}$$



In this previous drawing the first observation takes us to understand that the ratio among the base L (or horizon) of the triangle and the height (or distance d) gives us the valuation of the planar angle (  $A^{\circ}$  ) of these triangles in horizonts.

Also we see that this property is good for any type of triangles.

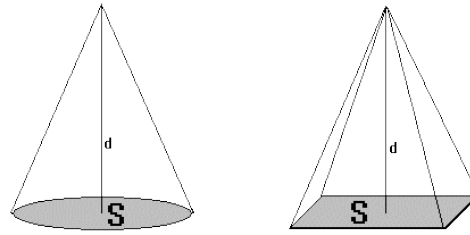
**Trimetry** in geometric figures  
July 2007 Ferman  
**Triangles**



$L = A^\theta \times d$  As the triangle area  $S = \frac{b \times h}{2} = \frac{L \times d}{2}$

Then, the triangle area  $S = \frac{A^\theta \times d^2}{2}$

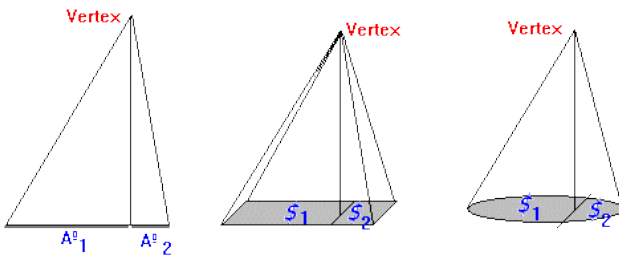
**Trimetry** in geometric figures  
July 2007 Ferman  
**Cubic angles: Cones, pyramids..**



$S = \mathcal{S} \times d^2$  As cone and pyramid volume  $V = \frac{Sb \times h}{3} = \frac{S \times d}{3}$

Then, cone and pyramid volume  $V = \frac{\mathcal{S} \times d^3}{3}$

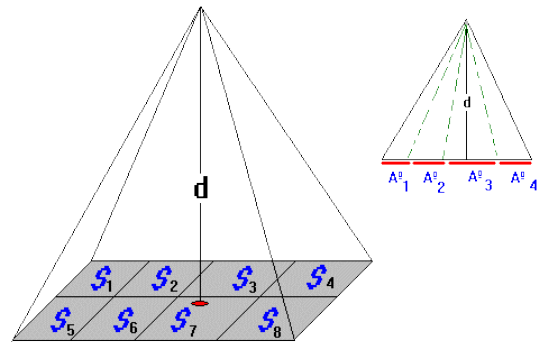
**Trimetry** in geometric figures  
July 2007 Ferman  
**Irregular figures**



$A^\theta = A^\theta_1 + A^\theta_2$

$\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2$

**Trimetry** in geometric figures  
July 2007 Ferman  
**Irregular figures**



$S = \mathcal{S}_1 + \mathcal{S}_2 + \mathcal{S}_3 + \mathcal{S}_4 + \dots$

$A^\theta = A^\theta_1 + A^\theta_2 + \dots$

Where d is common for any S or A<sup>θ</sup>

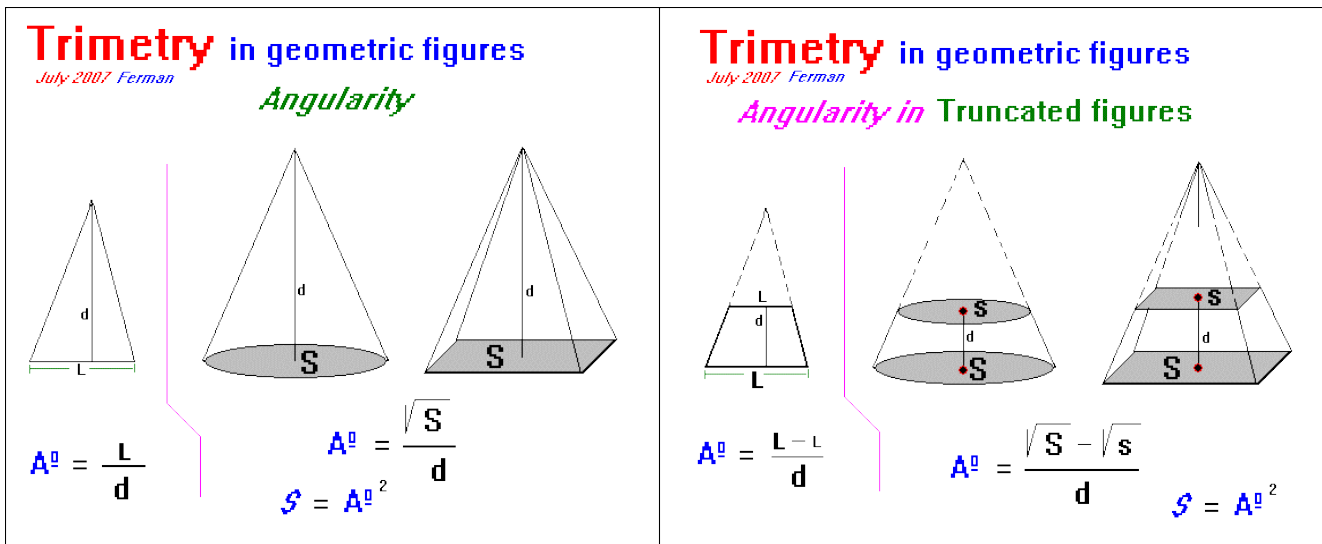
**Angularity**

Angularity is simply the value of the angle of the figure that we are considering. Therefore in the lineal angles or simple angles their angularity ( A<sup>θ</sup> ) is the measure of this angle: A<sup>θ</sup> = L/d.

However in the angular surfaces, (for example in the projection of a square, circle, triangle, stars, or of any complex figure (a flower) their angularities cannot be the measure of the external angle of these figures since these can have different external angles and they can also have holes inside these surfaces.

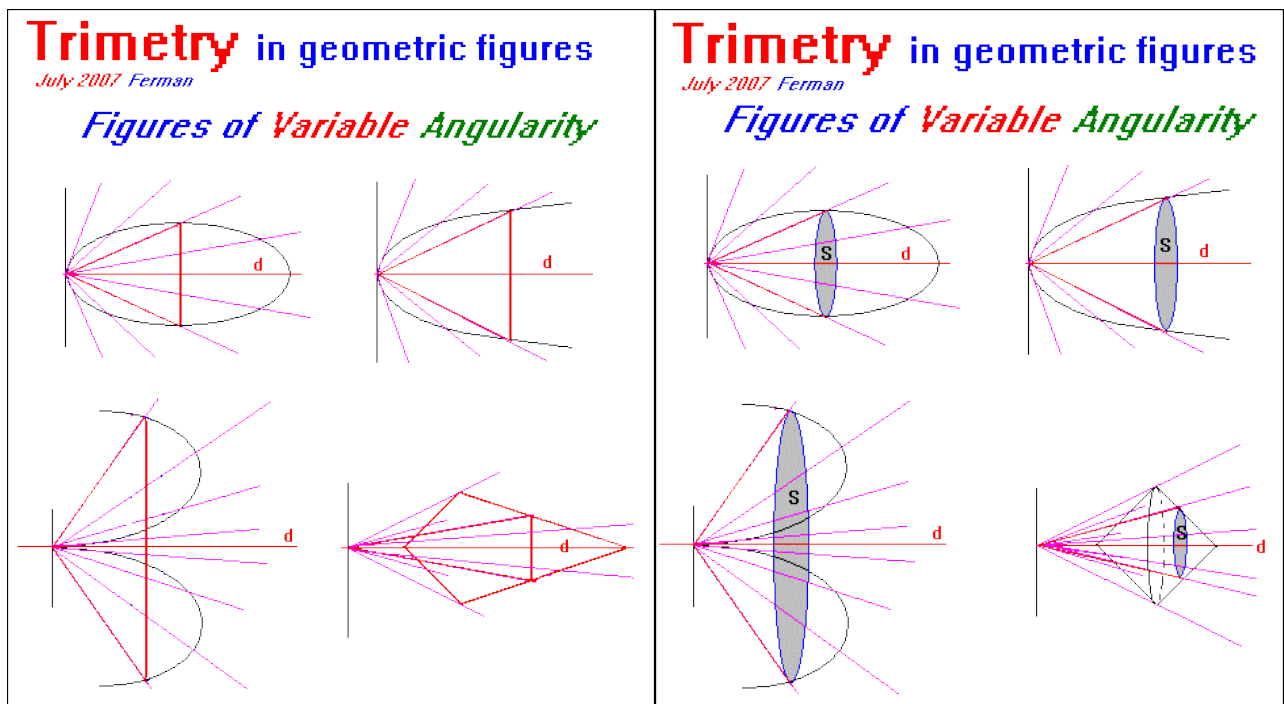
In this case, we have to choose a half angle whose square give us the half angularity \$ of this figure. \$ = S/d<sup>2</sup>.





### Variable angularity

Now well, a used property in trimetry is the application in figures of the variable angularity. This property is when we go changing the angularity of any figure o fields of projection for any value of the distance. With this property we can get a lot of types o figures. So, we can explain the anterior characteristics in the following way:  
 ---Straight angularity is when a figure has the equal angularity for any value of its distance d.  
 ---Variable angularity is when a figure goes changing its angularity for any value of distance d.  
 This question is explained whit their corresponding formulas.



### Formulas for building figures of variable angularity

We already know the basic formulas of trimetry, so much for lineal angles ( $L = A_o \times d$ ) as surfaces angles ( $S = A \times d^2$ ). Nevertheless, when we use variable angles to build figures, we need to substitute these parameters for algebraic functions to make this angles go changing according to the applied variables. The different possibilities of substitution of parameters and of obtaining different figures are numerous, and with time maybe we can see many of them. At the moment I will choose any of them to build geometric figures.



Nevertheless, we will have first to begin to propose use bases in trimetry and maybe one of them (perhaps it is changed in the future) would be the one of considering that as much lineal angles as surfaces angles would not should have negative values. In logic it is considered that an angle or a surface will always be positive. Therefore, a formula that builds a geometric figure will be considered alone in the tract on which its resulting values are positive.

Next, we have some formulas for figures of variable angularity:

<b>Trimetry</b> Some formulas on <small>July 2007 Ferman</small> figures of <i>Variable Angularity</i>	<b>Trimetry</b> Some formulas on <small>July 2007 Ferman</small> figures of <i>Variable Angularity</i>
<p><b>Linear angles</b> <math>L = A^{\circ} \times d</math></p> <p><math>A^{\circ} = f(x)</math>  <math>d = f'(x)</math></p> <div style="border: 1px solid red; padding: 5px; display: inline-block;"> <math>L = f(x) \cdot f'(x)</math> </div>	<p><math>L = f(x)</math>  <math>d = f'(x)</math></p> <p><math>A^{\circ} = \frac{L}{d} = \frac{f(x)}{f'(x)}</math></p> <hr/> <p><math>L = f(x)</math>  <math>A^{\circ} = f'(x)</math></p> <p><math>d = \frac{L}{A^{\circ}} = \frac{f(x)}{f'(x)}</math></p>
<p><b>Surface angles</b> <math>S = \mathcal{S} \times d^2</math></p> <p><math>\mathcal{S} = f(x)</math>  <math>d = f'(x)</math></p> <div style="border: 1px solid red; padding: 5px; display: inline-block;"> <math>S = f(x) \cdot [f'(x)]^2</math> </div>	<p><math>S = f(x)</math>  <math>d = f'(x)</math></p> <p><math>\mathcal{S} = \frac{S}{d^2} = \frac{f(x)}{[f'(x)]^2}</math></p> <hr/> <p><math>S = f(x)</math>  <math>\mathcal{S} = f'(x)</math></p> <p><math>d = \sqrt{\frac{S}{\mathcal{S}}} = \sqrt{\frac{f(x)}{f'(x)}}</math></p>

### Application of angularity according to the triangulation of figures.

As we see in the previous and following drawings, the planar angles can be observed with central perspective, that is to say, when the plane to observe or measure is located in the centre of vision or consideration of the same one.

In this case, the perpendicular of observation coincides with the centre of the plane or figure to consider, and therefore, the plane to observe represents the base of an isosceles triangle that is observed from its superior vertex. (See drawings)

But we can also consider (or observe) a figure, line or plane in a not centred way, that is to say, our perpendicular with the plane of this figure coincides with the outermost or exterior of their sides (observation in right angle) or it is located in any part of the plane but not in the centre or end of the same ones (irregular observation).

--In the first case, when being centred the observation on the centre of the plane, then to each side of this centre we will have the same angularidad, that is to say,  $A_0/2$  on the superior angle and  $A_0/2$  on the inferior angle.

--In the second case, or in rectangular observation, the whole angularidad  $A$  will be on the superior side (or inferior side if we decide so).

--In the third case, or irregular observation, it will be necessary to know the angle percentage that will be applied to the superior part and the inferior one.

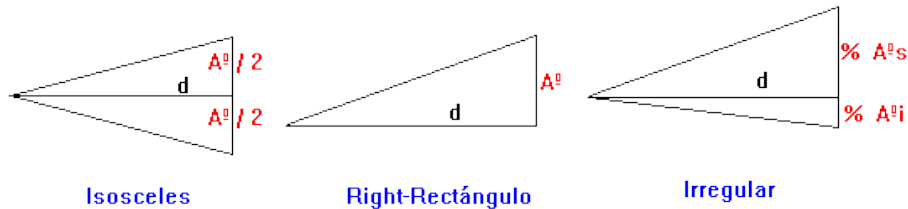
All this is explained in the drawings.

# Trimetry

July 2007 Ferman

$$L = \frac{A^{\theta}}{f(x)} \cdot \frac{d}{f'(x)}$$

## Application of angularity in different types of triangulation



### Some examples of construction of figures

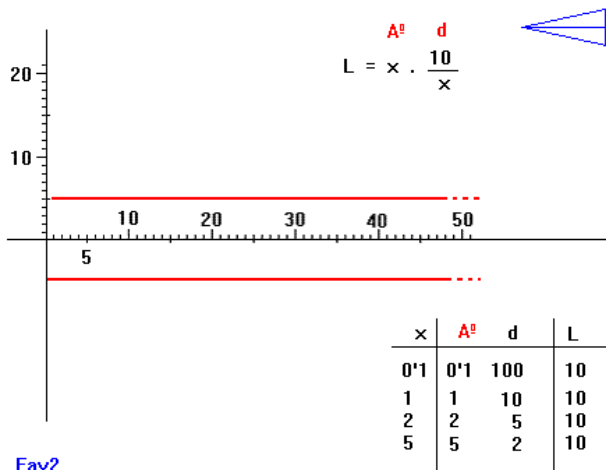
In the following examples, we can see how we can build figures of variable angularity. In them we see the three types of triangulation, which is expressed in the drawings. Firstly we have the lineal angles. Later we already see angles of surfaces.

# Trimetry

July 2007 Ferman

$$L = \frac{A^{\theta}}{x} \cdot \frac{d}{10}$$

## Figures of variable angularity



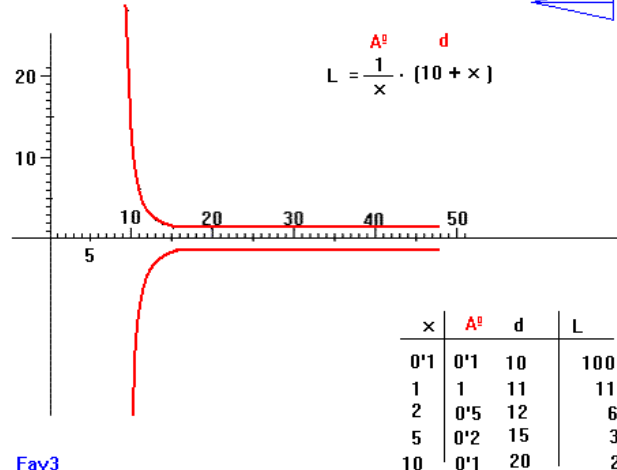
Fav2

# Trimetry

July 2007 Ferman

$$L = \frac{A^{\theta}}{\frac{1}{x}} \cdot \frac{d}{10+x}$$

## Figures of variable angularity



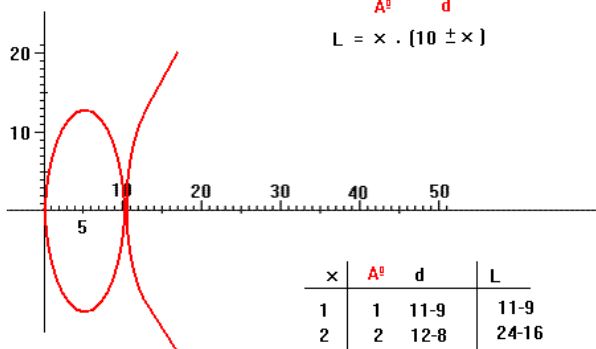
Fav3

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity



$$L = x \cdot (10 \pm x)$$

x	A <sup>d</sup>	d	L
1	1	11-9	11-9
2	2	12-8	24-16
5	5	15-5	75-25
8	8	18-2	144-16
10	10	20-0	200-0

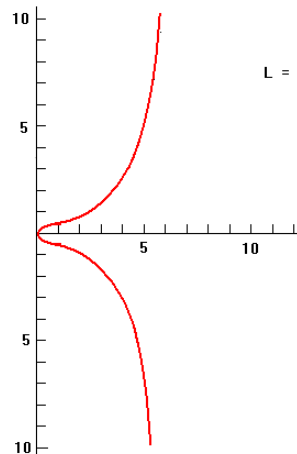
Fav4

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity



$$L = \frac{20-x+(x/2)^3}{10} \cdot x$$

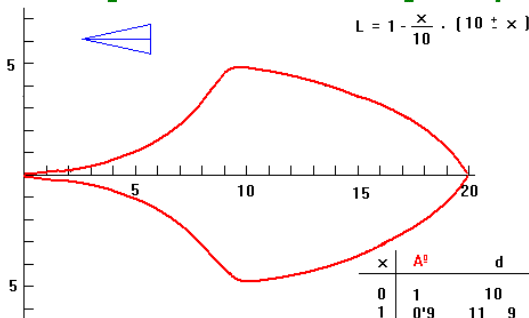
x	A <sup>d</sup>	d	L
0	2	0	0
1	1'5	1	1'5
2	1'1	2	2'2
3	0'82	3	2'4
4	0'8	4	3'2
5	1	5	5
6	1'7	6	10'2
8	4'4	8	35'2

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity



$$L = 1 - \frac{x}{10} \cdot (10 \pm x)$$

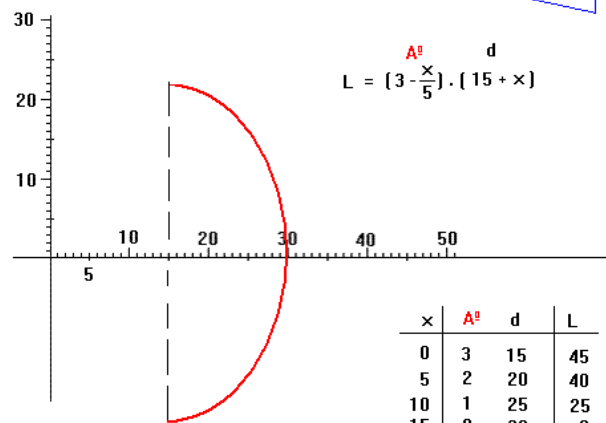
x	A <sup>d</sup>	d	L
0	1	10	10 10
1	0'9	11 9	9'9 8'1
2	0'8	12 8	9'6 6'4
3	0'7	13 7	9'1 4'9
4	0'6	14 6	8'4 3'6
5	0'5	15 5	7'5 2'5
6	0'4	16 4	6'4 1'6
7	0'3	17 3	5'1 0'9
8	0'2	18 2	3'6 0'4
9	0'1	19 1	1'9 0'1
10	0	20 0	0 0

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity



$$L = \left(3 - \frac{x}{5}\right) \cdot (15 + x)$$

x	A <sup>d</sup>	d	L
0	3	15	45
5	2	20	40
10	1	25	25
15	0	30	0

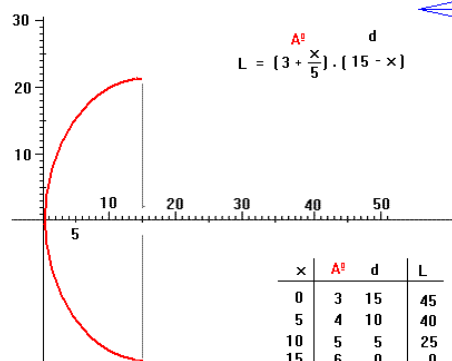
Fav5

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity



$$L = \left(3 + \frac{x}{5}\right) \cdot (15 - x)$$

x	A <sup>d</sup>	d	L
0	3	15	45
5	4	10	40
10	5	5	25
15	6	0	0

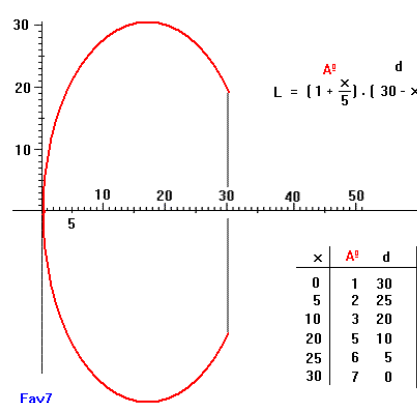
Fav6

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity



$$L = \left(1 + \frac{x}{5}\right) \cdot (30 - x)$$

x	A <sup>d</sup>	d	L
0	1	30	30
5	2	25	50
10	3	20	60
20	5	10	50
25	6	5	30
30	7	0	0

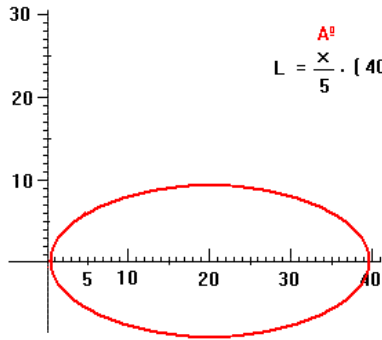
Fav7

# Trimetry

July 2007 Ferman

$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

## Figures of variable angularity



Fav8

$$L = \frac{x}{5} \cdot (40 - 4x)$$

x	A <sup>d</sup>	d	L
0	0	40	0
1	0'2	36	7'2
2	0'4	32	12'8
5	1	20	20
8	1'6	8	12'8
9	1'8	4	7'2
10	2	0	0

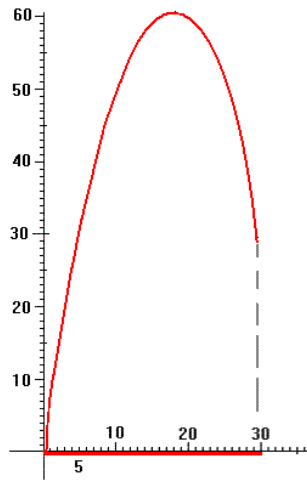


# Trimetry

July 2007 Ferman

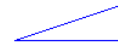
$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

## Figures of variable angularity



$$L = \left(1 + \frac{x}{5}\right) \cdot (30 - x)$$

x	A <sup>d</sup>	d	L
0	1	30	30
5	2	25	50
10	3	20	60
20	5	10	50
25	6	5	30
30	7	0	0



Right of Fav7

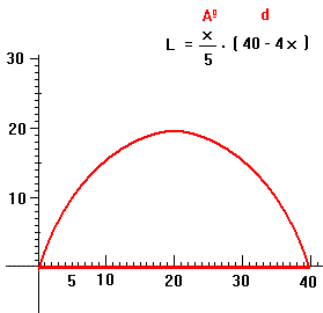
Fav9

# Trimetry

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$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

## Figures of variable angularity



Fav10

$$L = \frac{x}{5} \cdot (40 - 4x)$$

x	A <sup>d</sup>	d	L
0	0	40	0
1	0'2	36	7'2
2	0'4	32	12'8
5	1	20	20
8	1'6	8	12'8
9	1'8	4	7'2
10	2	0	0



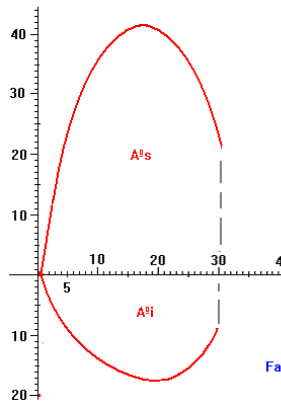
Right of Fav8

# Trimetry

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$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

## Figures of variable angularity



$$L = \frac{A^s + A^i}{(0'7 - 0'3)} \cdot \frac{d}{f'(x)}$$

x	A <sup>d</sup>	d	L
0	1	30	30
5	2	25	50
10	3	20	60
20	5	10	50
25	6	5	30
30	7	0	0



Irregular of Fav7

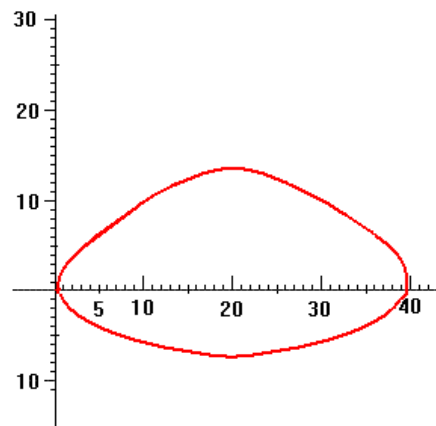
Fav11

# Trimetry

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$$L = \frac{A^d}{f(x)} \cdot \frac{d}{f'(x)}$$

## Figures of variable angularity



Fav12

Irregular of Fav8

$$L = \frac{A^s + A^i}{(0'7 - 0'3)} \cdot \frac{d}{f'(x)}$$

x	A <sup>d</sup>	d	L
0	0	40	0
1	0'2	36	7'2
2	0'4	32	12'8
5	1	20	20
8	1'6	8	12'8
9	1'8	4	7'2
10	2	0	0

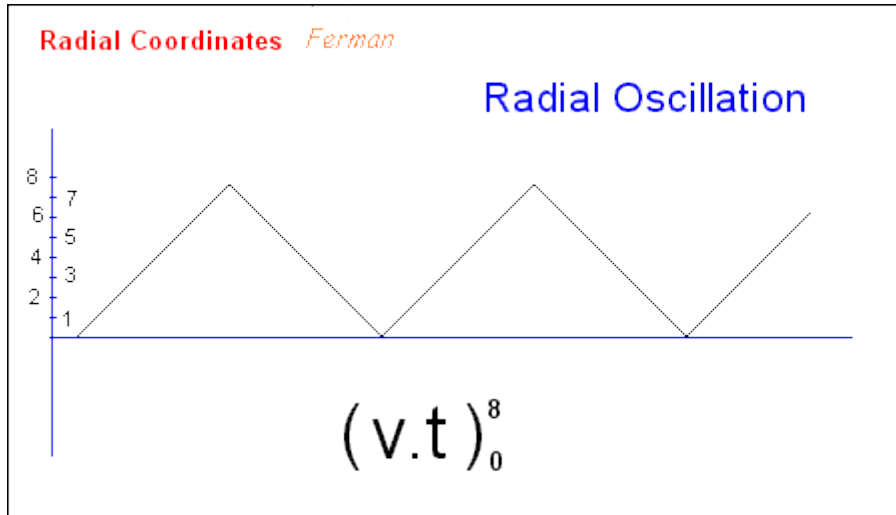


# Oscillatory intervals

The same as we have seen in radial coordinates, the oscillatory interval can be applied in the trimetry formulas to get some figures, as for example rhombuses and rhomboid figures.

Let us remember that the oscillatory intervals consist on the application to a variable (x) of oscillatory values between n and m.

If we have a oscillatory expression (x) 0/5 (see drawings better) this mean that x goes taking values from 0 to 5 and from 5 at 0 continuously (0,1,2,3,4,5,4,3,2,1,0,1,2,3,4,5,4,3,2,1,0,1 ... etc.)

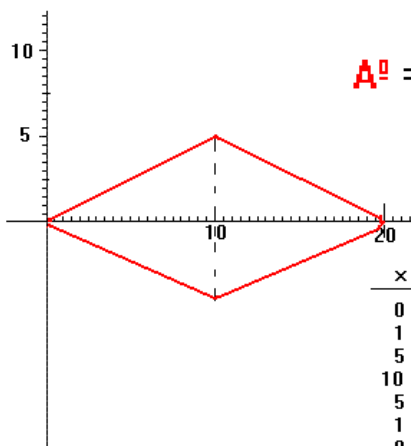


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*Figures of variable angularity*

Oscillatory intervals  $(x)_m^n$

$$A^q = \frac{L}{d} = \frac{(x)_0^{10}}{(x')_0^{20}}$$



x	x'	L	d	A <sup>q</sup>
0	0	0	0	0
1	1	1	1	1
5	5	5	5	1
10	10	10	10	1
5	15	5	15	0'333
1	19	1	19	0'053
0	20	0	20	0

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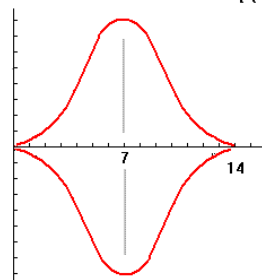
$$f(x) = A^q \cdot f'(x) \frac{d}{L}$$

*Figures of variable angularity*

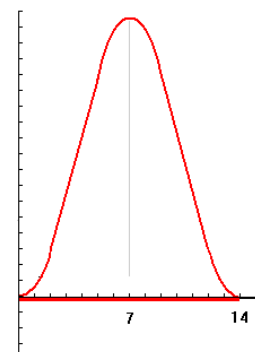


Isosceles

$$\frac{L}{[(x)_0^7]^2} = \frac{[(x)_0^7]^3}{10} = A^q \cdot x$$



x	A <sup>q</sup>	d	L
0	0	0	0
1	0'9	1	0'9
2	1'6	2	3'2
3	2'1	3	6'3
4	2'4	4	9'6
5	2'5	5	12'5
6	2'4	6	14'4
7	2'1	7	14'7
10	0'96	10	9'6
12	0'27	12	3'2
14	0	14	0



Right-Rectángulo

## Use of trigonometric parameters.

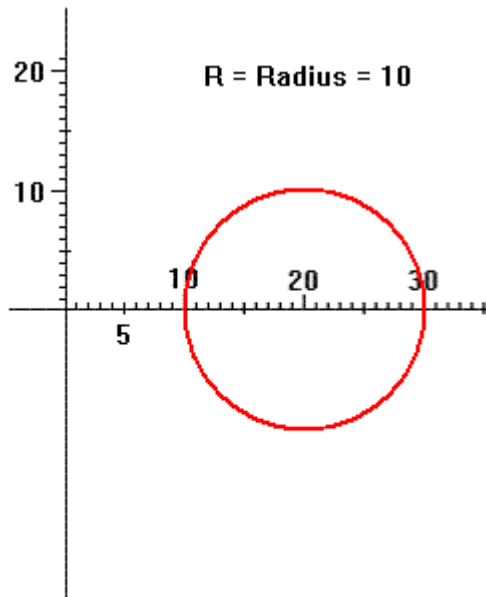
**Trimetry**  
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$$\frac{L}{f(x)} = A^{\alpha} \cdot \frac{d}{f'(x)}$$

*Figures of variable angularity* 

$$L = A^{\alpha} \cdot d$$

$$2 R \cdot \sin \alpha = A^{\alpha} \cdot (20 + R \cdot \cos \alpha)$$



$\alpha$	$A^{\alpha}$	d	L
0	0	30	0
15	0'174	29'66	5'18
30	0'348	28'66	10
45	0'522	27'07	14'14
60	0'692	25	17'32
75	0'854	22'59	19'32
90	1	20	20
105	1'108	17'41	19'32
120	1'154	15	17'32
135	1'092	12'93	14'14
150	0'882	11'34	10
165	0'5	10'34	5'18
180	0	10	0

In these examples we are using the trimetry formulas but including parameters of trigonometry with object of studying the possibilities that give us these trigonometric parameters.

In the previous drawing we see how we build a circumference (in isosceles triangulation).

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In the following drawing we see how we can build an entire range of curves with trigonometric parameters.

1.- When we apply exponentials:

--With variable exponent (x) to sine and cosines we obtain curves (toward the interior) that go from the semi-circumference when we apply  $x=1$ ; straight line (or rhombus) when we apply  $x=2$ ; and curves with more and more degree of curvature until getting a double right angle with  $x=\infty$ .

2.- When we apply roots:

--Roots with variable exponent (x) to sine and cosines we obtain curves (toward the exterior) with more and more curvature until ending up building a rectangle when  $x=\infty$ .

# Trimetry

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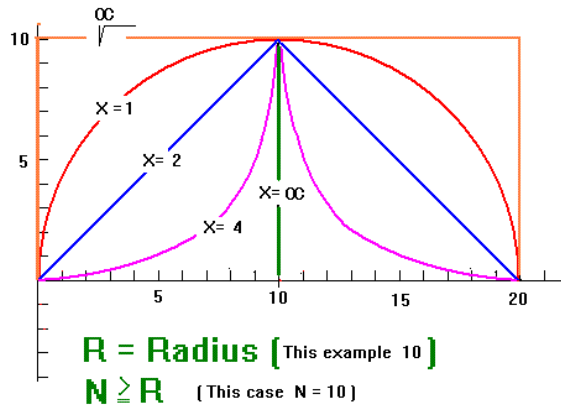
$$f(x) = \frac{L}{A^0} \cdot \frac{d}{f'(x)}$$

Figures of variable angularity

Trigonometric curves



$$R \cdot (\sin \alpha)^x = A^0 \cdot N + R \cdot (\cos \alpha)^x$$



Constants.

As we see in the following drawings, with variable angularidad we can obtain different types of geometric figures if we make constant any one of their parameters.

---If we make constant the horizon L, we will obtain squares and rectangles in longitudinal angles. Also cubes, cylinders, etc., in angles of planar surfaces .

---If we make constant the planar angles A<sub>0</sub>, we will obtain triangles and trapeziums in longitudinal angles and pyramids, cones and projections in surfaces planares.

---If we make constant the distance d, we will obtain horizons or perpendicular lines in longitudinal angles and square horizons or perpendicular plane surfaces (screens) in surfaces planares.

# Trimetry

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$$L = \frac{A^0}{f(x)} \cdot \frac{d}{f'(x)}$$

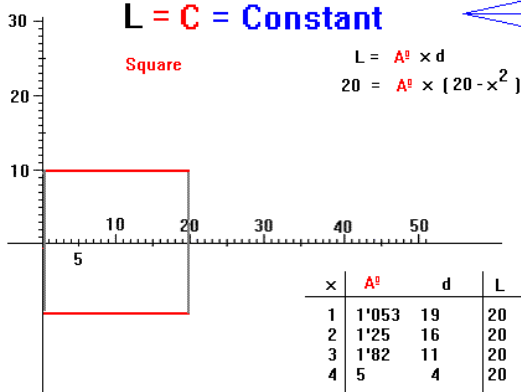
Figures of variable angularity

L = C = Constant

Square

$$L = A^0 \cdot d$$

$$20 = A^0 \cdot (20 - x^2)$$



# Trimetry

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$$L = \frac{A^0}{f(x)} \cdot \frac{d}{f'(x)}$$

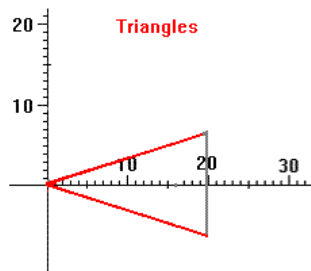
Figures of variable angularity

A<sup>0</sup> = C = Constant

Triangles

$$L = A^0 \cdot d$$

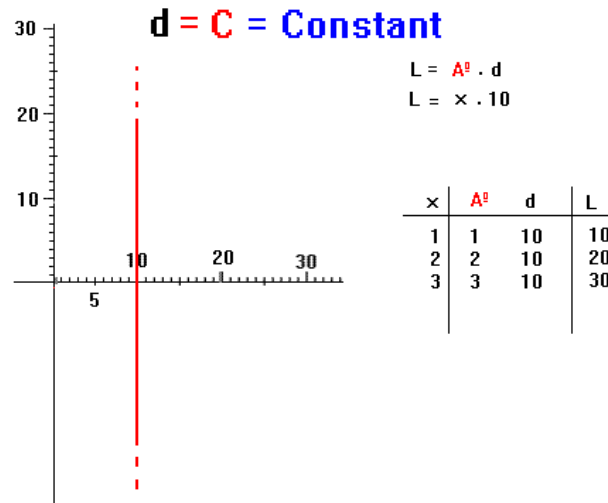
$$L = 0'8 \cdot (x)_0^{20}$$





$$L = \frac{A^{\theta}}{f(x)} \cdot \frac{d}{f'(x)}$$

*Figures of variable angularity* 



## Shapes diversity in planar surfaces: Fields and images of projection.

We have seen as to planar surfaces you can consider as projection figures that extends along a distance ( d ) or simply as visual fields that also extend along certain distance.

This projection character makes possible the representation of any figure type, from a simple square or circle until the projection of complicate figures as any figure of number, any flower, an animal, etc.

This enormous field of possibilities also makes difficult the correspondence between the planar surfaces and their simple longitudinal angles.

This way can be easy and clear the correspondence, adjustment and representation of a square surface with the lineal angle that would give us any side.

But we cannot find a representative lineal angle of a complex figure as it can be the projection of the figure of an animal.

However (as we have seen in previous drawings) there is a parameter that has correspondence between lineal angles and surfaces angles that is its angularity, that is to say, the "half or middle angle" of the surface. This angularity is simply the square root of the figure surface, which as we have said, it corresponds with the side of a square surface.

And if we are alone considering a field of observation, this case the angularity will be de square root of this field.

Therefore, this angularity is the unit of angular surface \$ of each figure or field of observation.

So as the angularity have correspondence between linear angles and surface angles, because we would have that the square of the unit of lineal angle Ao (Ao<sup>2</sup>) would give us the unit of surface angle \$. (Ao<sup>2</sup> = \$)

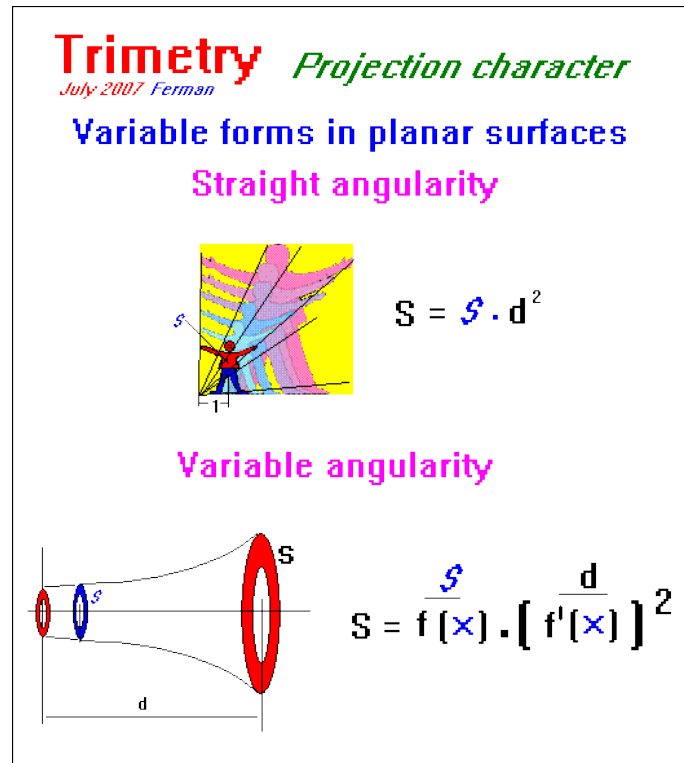
Therefore of the above-mentioned we can reach the following conclusions:

- 1.- The parameters and formulas of the planar surfaces don't define entirely the structure of these surface, but they measure, manage, project and transform to these surfaces.
- 2.- The planar surfaces contain, beside these parameters and formulas that we are describing, a model, pattern of TEMPLATE that it is the one that is transformed, measured and projected with the described parameters.

This question can be clearly observed in the projection of movies, where the projector with their peculiarities and characteristic alone emits or projects the slides of the movie, but it doesn't build this slide, but rather we give them for their projection.

In the planar surfaces this template can be simple as a projected square, which gives us a square pyramid; a projected circle that gives us a cone; or a complex figure that gives us a projection of complex figure.

In the following drawing we see examples of projection of figures in not variable angularity and also in variable angularity.



### Construction of figures of planar surfaces.

Several forms of contemplating and to study the planar surfaces can exist.

However, and following the initial line of considering to the planar surfaces as fields or frames of visual observation, my way of studying them will be the framing of any planar surface (as any geometric figure, any type of objects or figures of the nature) inside a visual field.

So, it will be to this visual field or observation frame to which we will subject in their entirety to the formulas and considerations that we make on the planar surfaces.

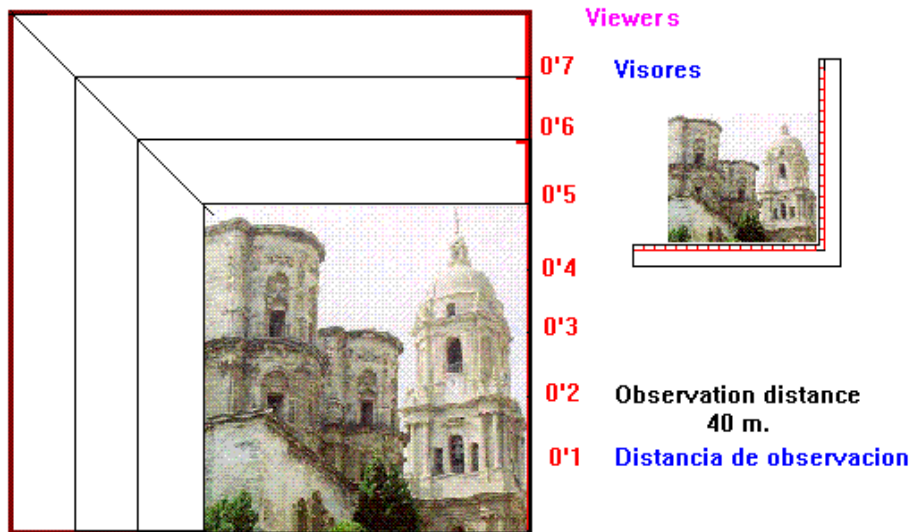
As we see in the following drawing, we will apply the planar formulas to the whole observation frame and not alone to the represented figure inside this frame.

The reason is very clear: it is the simplest way to manage the planar formulas to measure with more easiness, conserving the relation of angularity among the different parts of the figure without distorting this figure when we apply the mentioned transformation formulas.

# Trimetry Measure of planar surfaces

July 2007 Ferman

## In $h^2$



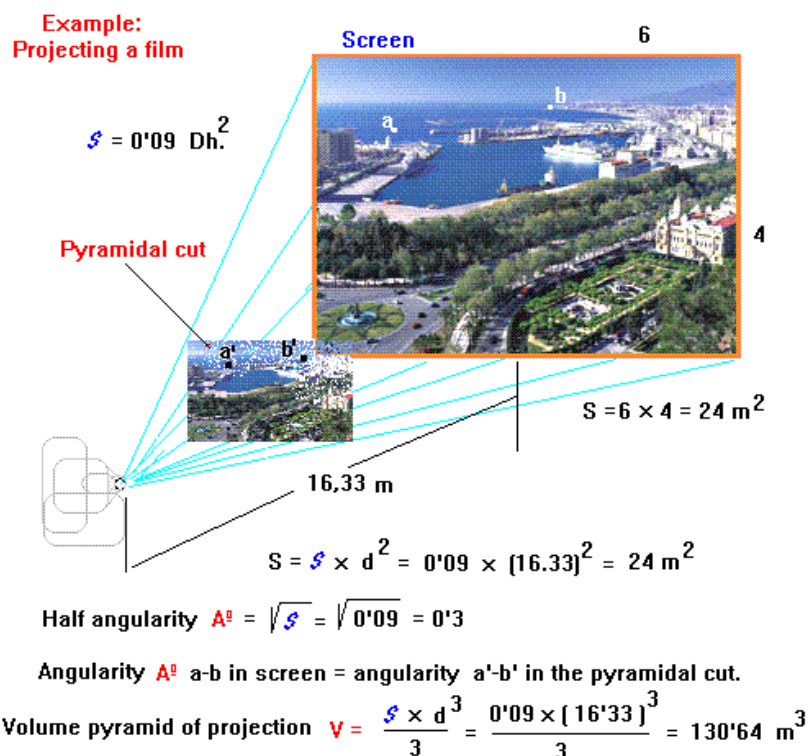
$$S = \mathcal{S} \times d^2 = 0'25 \times (40)^2 = 400 \text{ m}^2$$

In the first drawing we have simple instruments for measuring planar surfaces as can be any simple set-square (or any type of viewer) locate at the appropriate distance to proceed to measure the angular unit of surface. (to 1 decimetre when the set-square have also 1 decimetre)

# Trimetry Measure of planar surfaces

July 2007 Ferman

## Projection: Invariable angle



In this previous drawing we already contemplate an example of the parameters that we can see in any projection of planar surfaces.

In this case, when being a projection a planar composition of invariable angularity, its unit of angular surface \$ remains equal along the whole projection.

---We see in the first place that the whole focus of the projection of this movie provides us a pyramidal structure with base in the screen and vertex in the focus of emission of the movie.

This pyramid or entire luminous focus of emission has a volume of 130'64 cubic meters, of which you can see its adjustment in the drawing with arrangement to the formula that we saw previously.

---We also observe that if, between the screen and the vertex or emission focus, we cut this focus with another smaller screen, we also obtain the projected figure with the same angularity proportions in all and each one of their points. (Angularity of a-b = a'-b' angularity).

---In the same way, we see that if the own projection machine already took adjusted its emission angularity (\*\*), we could know with accuracy the dimensions that would have the movie square of the screen in anyone of the different distances to that you could locate this screen using the formula of planar surfaces that is in the drawing. [ $S = \$. d^2 = 0'09 \times (16'33)^2 = 24 \text{ m}^3$ ].

\*\* If we don't know the angularity of the projection machine, is it enough making a test of projection from 1 meter of distance and measuring the surface that we obtain in square meters. (We will obtain square Decahorizonts "decas")

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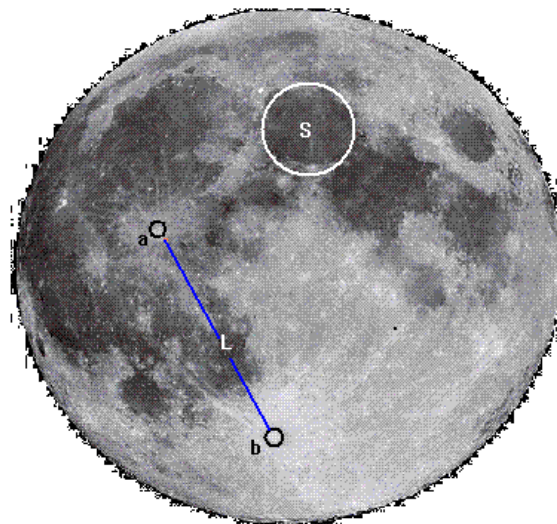
In the following drawing we see (with a practical example as our moon) as we can study all and each one of the elements of a distant surface -if we know its distance- and their relationship among them with alone to measure their angles with simple instruments as it can be a set-square.

**Trimetry** *Planar angles and surfaces*  
*July 2007 Ferman*

$$L = A^\circ \cdot d$$

$$S = \$. d^2$$

$$d = 384.000 \text{ kms.}$$



$$L = A^\circ \cdot d = 0'00417 \text{ Dh} \times 384.000 \text{ km} = 1602 \text{ km.}$$

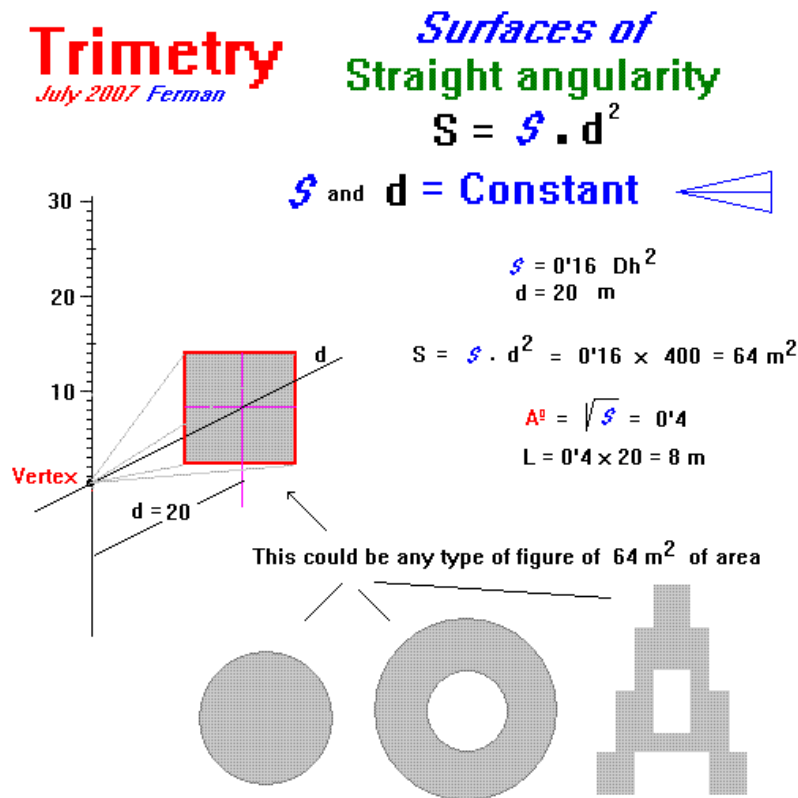
$$S = \$. d^2 = 0'0000018 \text{ Dh}^2 \times [384.000]^2 = 265.420 \text{ Km.}^2$$

Now well, as the surface angularity that we are measuring is very small, then we can name it with metric parameters only.

This case we can say the angularity \$ of the surface S is of 1'8 square milimetres.

Some examples of construction of figures of planar surfaces:

I will begin with a simple figure with which I can explain some of the parameters that we have seen before.



This is a figure of constant angularity and also at predetermined distance (20 meters) that produces us a planar surface on this distance.

At first, we see that this figure is a square or screen of 64 square meters and located to 20 meters from the vertex or point of observation and measure.

But as we said before, this figure could have any form and content, (even to be an advertising poster), provided that it is located to twenty meters and it has a surface of 64 square meters, which is the dimensions that gives us the planar parameters.

Perhaps firstly, this lack of definition of the interior characteristics of the planar surface can seem negative for the aspirations and expectations that we request to the theory of planar angles. But contrarily, it can be an advantage when it allows us embrace to all type of surfaces from a simple square until the most complicated drawing or scene.

This way if we are observing a landscape of nature, we can frame it and to study all and each one of their angles; all and each one of the surfaces of their internal figures; all and each one of their points.

If what we seek is to build (mathematically) a surface or scene at certain distance, it is enough with providing us of a template or model, projecting it to certain distance by means of the simple trimetric rules that we are seeing.

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With this second example we enlarge concepts and can contemplate more properties of the planar angles and on their trimetric measures.

In this case we have built a square pyramid and we have exposed the trimetric formula of volume ( $V = (\mathcal{S} \cdot d^3) / 3$ ) to analyze it.

But observing this formula, we see as the pyramid is built and at the same time we can calculate the parameters and values of this pyramid.

That is to say, it is not simply a formula of description of a geometric figure but rather at the same time it takes matched the calculation of the same one for the different positions that we want to give to the variable x (variable distance).

Therefore, when we choose a vertex, let us give an angle  $s$  ( $0.16 Dh^2$ ) and we choose a distance  $d$  (also direction) with variable values of  $x$  (from 0 to 20), these parameters build and describe us a pyramid with a maximum of 426'66 square meters.

If we give different values to  $x$  (distances or height of the pyramid) we go obtaining different values of the pyramidal cuts that we have with these variable values of  $x$ .

**Trimetry**  
July 2007 Ferman

### Surfaces of Straight angularity

$$V = \frac{s \cdot d^3}{3}$$

$d = \text{metres}$

$$s = \text{Constant}$$

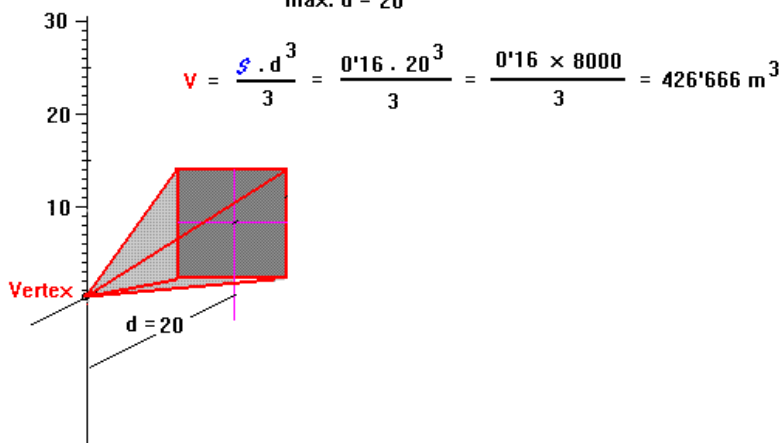
$$s = 0.16 Dh^2$$



$$d = [x]_0^{20}$$

$$V = \frac{s \cdot [(x)_0^{20}]^3}{3}$$

max.  $d = 20$



### Surfaces of variable angularity.

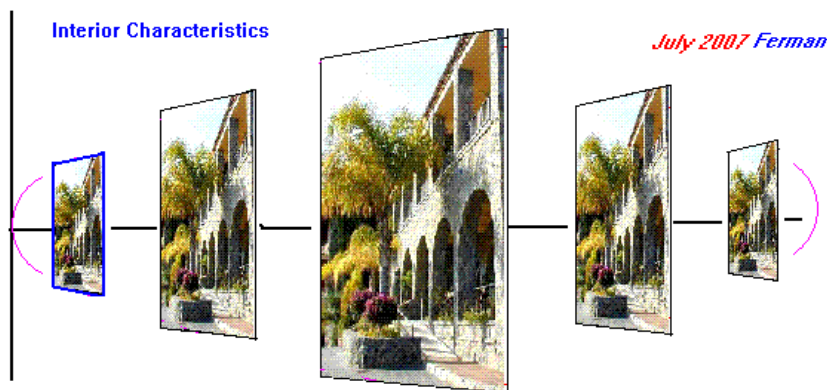
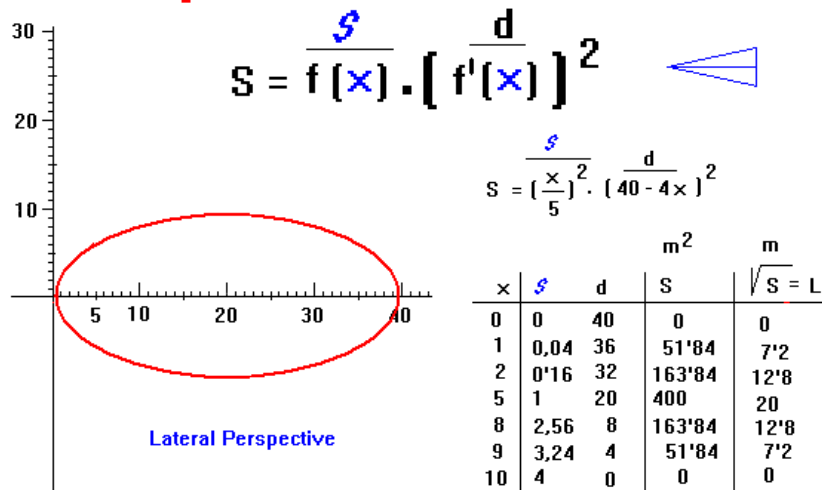
In following drawing we have an example of construction of figures of variable angularity.

In it we see as we can build and find the planar surface of these figures when applying the corresponding formula.

For this it is enough when we give different values to the variables.

Let us remember that the interior structure of these geometric bodies can be compact or to contain any kinds of consistency and forms, as it is the case of the drawing that is a projection of variable angularity.

# Trimetry Measure of planar surfaces $h^2$



## Ubiquity parameter

Looking at the easy formula of the observation angle ( $A_o = L / d$ ) this can represent a wide definition of the Cosmos structure because it can measure and contain the two space dimension (plane and exponential), situating us in a determined point of them.

Let me expose two examples for explaining it.

1.- Plane dimension of space:

If we apply the formula ( $A_o = L / d$ ) on a planet (1.000 km. diameter) belonging a far star to 100.000 years-light from us.

$k = 1.000.000. / 1.000.000.000.000.000.000.000 = 0,000000000000001, (10^{-15}).$

It gives us a very small angle  $A_o$  ( $A_o = 10^{-15}$ ) and so, impossible for the planet observation.

2.- Exponential dimension

But if now we want to apply the formula on an electron, we need to divide the electron dimension by the distance:

$A_o = 0,000000000000001 / 1 = 0,000000000000001, (10^{-15}).$

Obtaining similar value ( $10^{-15}$ ) and so, also impossible for the observation.

As you can see, both values are very small what says us that we are very far from the situation of the planet and electron, but while the planet is far from us in lineal distance; the electron is far from us in exponential distance.

In both cases we are far from them, but with distinct type or concept of dimensional distance.

2b.- But let me apply now the formula on the known universe, which supposedly measures about  $5 \times 10^{26}$  m.



$$A_o = 5 \times 10^{26} / 1 = 5 \times 10^{26}$$

\* Here the distance (d) is 1 already we are inside the universe.

This case the angular value ( $A_o$ ) result is infinitely big ( $5 \times 10^{26}$ ) what says us that we are situated inside de Universe, in an infinitesimal part of it.

Then, when the angle is good for the better observation?

Of course with value 1 (1 horizont), because then we are situated in front of the object.

For example, observing a vase of 40 cmts tall from 40 cmts of distance; a bush of 1 m. tall from 1 m. of distance; a house of 6 ms. tall from 6 ms. of distance; the earth of 12.000 km of diameter observed from 12.000 km of distance, etc.

So the  $A_o$  value (planar angle or observation angle) can measure us our situation regarding any other object inside the Cosmos, either in the plane dimension as in the exponential one, being this way very important this observation angle ( $A_o$ ) in our cosmic situation.

In this sense, when the observation angle (planar angle) measures the unit (1 horizont) it is when we have the better position for observing and appreciating the object in its whole extension.

If the observation angle is very little, then dimensionally, we are very far from the object and in bad site to appreciate its qualities.

Contrarily, if the observation angle is very large, then we are situated alone on a part of the object or inside it, and so, we have little possibilities of observing and studying it in its whole extension.

