

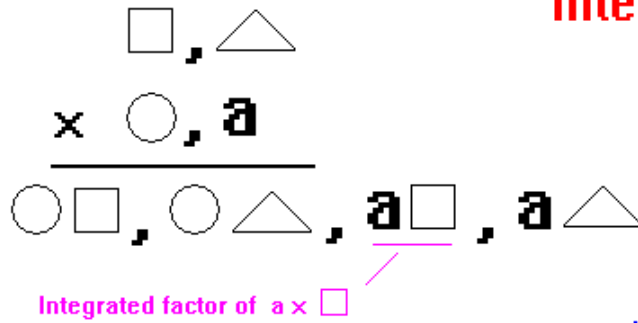
Algebraic product of sets

Ferman: Fernando Mancebo Rodriguez 2002

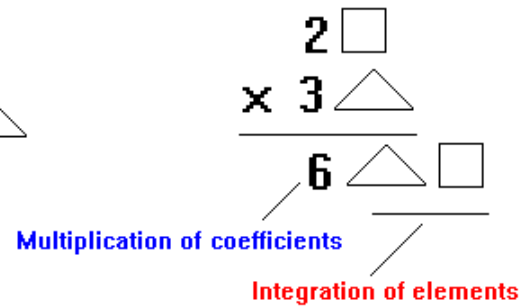
Algebraic product of sets. ---- (Highlights)

Algebraic product of sets.

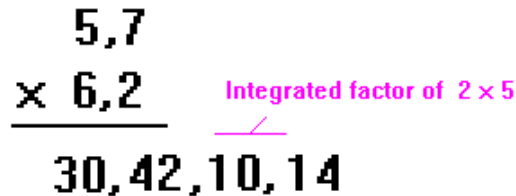
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Integration of factors



With quantities



This theory on the physical and mathematical sets understands that if we want to consider the products of sets of elements, we should come closer to the principles and properties of the multiplication of numbers, but respecting the properties of the elements of each set. For it, I begin with some definition and exposition of the main structural parameters of the algebraic product among sets.

The algebraic product of sets in multiplication of the same ones following the common algebraic rules consists.

When being sets formed by elements of any type and entity, we will consider two types of components in the factors to multiply:

- The proper elements of the set that can be of any type and entity.
(For example: a pencil, a tree, an animal, a sign, a symbol, an idea, a feeling, etc.)
- The multiplier coefficients (or quantity of these previous elements) that can contain each one of the factors to multiply.
(For example: 25 rabbits, being 25 the multiplier coefficient of the elements of the set, say, rabbits)

Now well, as we will see later on in the algebraic product of sets, the coefficients and quantities multiply between them and the elements are integrated (fused) among them.

Algebraic product of sets

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$$\begin{array}{r}
 a + b \\
 \times c + g \\
 \hline
 ca + cb + ga + gb \\
 \times \quad l + m \\
 \hline
 lca + lcb + lga + lgb + mca + mcb + mga + mgb
 \end{array}$$

+ $\langle \implies \rangle$,
Equivalent

$$\begin{array}{r}
 a , b \\
 \times c , g \\
 \hline
 ca , cb , ga , gb \\
 \times \quad l , m \\
 \hline
 lca , lcb , lga , lgb , mca , mcb , mga , mgb
 \end{array}$$

But let us begin to revise the algebraic form of the products of sets.

Given two sets of variable elements A (a, b) and B (x, y) we can multiply them as if they were numbers:

$$A \times B = (a, b) \times (x, y) = AB (xa, xb, ya, yb)$$

And later to this result we can multiply it by another set C (d, e)

$$AB (xa, xb, ya, yb) \times C (d, e) = ABC (dxa, dxb, dya, dyb, exa, exb, eya, eyb)$$

As we see, the product between the set of elements takes INTEGRATION character, when in each operative act, the multiplier factors fuse in a single element, to which we call INTEGRATED FACTOR.

In the previous example when we multiply $y \cdot a$ the integrated factor will be the result ya .

Now then, in the algebraic product of sets, anyone of these variables can be substituted by a number or by an element:

When it is by a number, the operations are already well-known in mathematics:

$$A \times B = (6, 4) \times (3, 2) = AB (3 \times 6, 3 \times 4, 2 \times 6, 2 \times 4) = AB (18, 12, 12, 8)$$

But if the factors of the multiplication are physical elements, then the resulting elements of the INTEGRATION will be formed by the physical union of the elements of each operative act into a compound element.

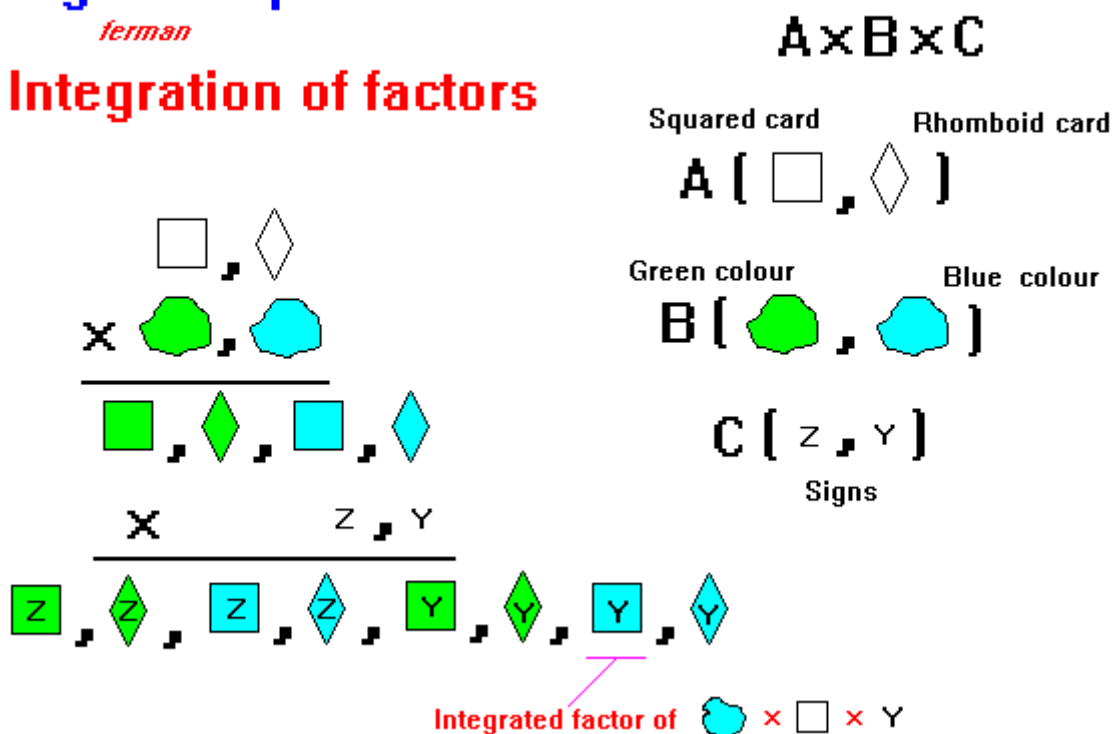
For example:

$$A \times B = (@, \&) \times (\#, \%) = AB (\#@, \#\&, \%@, \% \&)$$

Algebraic product of sets.

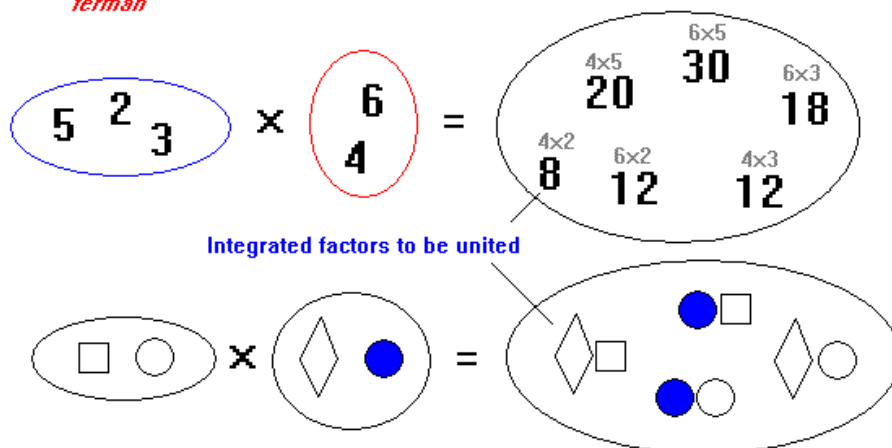
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Integration of factors



Algebraic product of sets

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Coefficients and elements

In the multiplication of sets of elements we should distinguish between the physical (or symbolic, metaphysical, etc.) elements and the multiplier coefficients of these elements. In a set A (5 rabbits, 4 bottles) can have physical elements (rabbits and bottles) and coefficients that express the quantity of these elements (5 and 4). Although, the coefficients always multiply in each operative act as well their own elements, as to the elements of the other factors.

For example:

A (5 rabbits, 4 bottles) and B (2 boxes, 3 shelves)

A x B = (5 rabbits, 4 bottles) x (2 boxes, 3 shelves) = AB (15 shelf-rabbit, 12 shelf-bottle, 10 box-rabbit, 8 box-bottle)

That means that there will be:

- 15 Integrated factors (o subsets), each one of them formed by a rabbit with shelf.
- 12 Integrated factors (o subsets), each one of them formed by a shelf with bottle.
- 10 Integrated factors (o subsets), each one of them formed by a rabbit in its box.
- 8 Integrated factors (o subsets), each one of them formed by a box with bottle.

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Coefficiente = Number of integrated factors

$$\begin{array}{r}
 2 \square \\
 \times 3 \diamond \\
 \hline
 6 \diamond \square
 \end{array}
 =
 \begin{array}{r}
 \square, \square \\
 \diamond, \diamond, \diamond \\
 \hline
 \diamond \square, \diamond \square, \diamond \square, \diamond \square, \diamond \square, \diamond \square
 \end{array}$$

Six integrated factors

Quantities and coefficients

In the algebraic product of sets, numeric elements that represent different concepts can appear to which we will consider as Quantities or as Coefficients:

Quantities

They will be numeric values of free and pure mathematics, say, they are not subject to any physical element to multiply or to index.

This way these mathematical quantities have value for itself and they don't have to depend on any physical element for their existence.

A (23, 12, 4)

Coefficients

The coefficients are the quantities of elements that a set or factor contains and therefore they are subject to the existence of these elements.

If the elements weren't exist, then the coefficients neither could exist.

In the example:

A (2x, 2y, 3H)

If x, y or H take zero value, then the coefficients 2, 2 or 3 also take zero value and they stop to exist.

Quantity of resultant subsets.

As we can see in the examples, the quantity of subsets or resulting integrated factors of any multiplication among sets of elements, is the product among the total number of elements of each multiplier sets.

In the example:

$$A \times B = (5 \text{ rabbits, } 4 \text{ bottles}) \times (2 \text{ boxes, } 3 \text{ shelves})$$

$$A \times B = (5 + 4 = 9) \times (2 + 3 = 5)$$

$$A \times B = 5 \times 9 = 45 \text{ subsets of integrated factors.}$$

Multiplication of sets of elements

INTEGRATION of factors and Number **N** of resultant factors

$$\begin{array}{r}
 a + b \\
 \times c + g \\
 \hline
 ca + cb + ga + gb \\
 \times \quad \quad l + m \\
 \hline
 lca + lcb + lga + lgb + mca + mcb + mga + mgb
 \end{array}$$

$n = 2$
 $A \{ a, b \}$
 $B \{ c, g \}$
 $C \{ l, m \}$
 + \longleftrightarrow ,
 Equivalent

Integrated factor

$$\begin{aligned}
 N &= n \times n \times n \dots \\
 N &= 2 \times 2 \times 2 = 8
 \end{aligned}$$

Multiple element: (x e)

Sometime we could operate with sets of elements as if they were a single one for the goal of obtaining results more appropriate for us.

For example, if we want to nail five nails in a wood, we cannot multiply the wood for the five nails as separate elements, due to this case the result give us five wood with a nail each one.

$$A (\text{wood}) \times B (5 \text{ nails}) = \text{Wood} \times 5 \text{ nails} = 5 \text{ wood-nail}$$

For this, we convert the separate 5 nails into a single element with 5 nails.

$$B ((5 \text{ nails}))$$

And this way we can multiply the element wood by the element (5 nails)

$$A (\text{wood}) \times B ((5 \text{ nails})) = AB (\text{wood-5 nails})$$

Let us the drawings.

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Multiple element (xe)

$$\begin{array}{r} \square, \triangle \\ \times (3 \diamond) \\ \hline (3 \diamond) \square, (3 \diamond) \triangle \end{array} \quad \text{--- It's not ---} \rightarrow \begin{array}{r} \square, \triangle \\ \times \diamond, \diamond, \diamond \\ \hline \diamond \square, \diamond \triangle, \diamond \square, \diamond \triangle, \diamond \square, \diamond \triangle \end{array}$$

$$\begin{array}{r} (3 \square), \diamond \\ \times 2 \triangle \\ \hline 2 \triangle (3 \square), 2 \triangle \diamond = \triangle (3 \square), \triangle (3 \square), \triangle \diamond, \triangle \diamond \end{array}$$

With the second drawing we can see as the product of sets can give us as result to sets or subsets of different Type, being sometimes Scrupply sets in which their elements don't keep any relationship among them, or as in this previous drawing, in which the resulting set can be a In Relation set, say, their elements have convergence among them.

Addition and cloning of elements in the algebraic multiplication

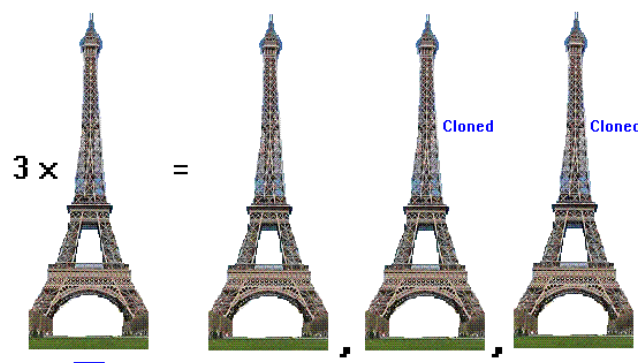
In the sum, subtraction and division, the resulting elements of these operations were already in the primary factors of these operations, but in the algebraic multiplication an addition of elements that were not in these primary operative factors takes place.

If they are common elements, (for example pencils, rabbits, squares, roses, etc.) it is enough to look for new elements, introducing them to complete the resulting product.

Nevertheless, if what we want to multiply a concrete element, (for example, 3 x Einstein = 3 Einstein) then to the theoretical added elements we can denominate Cloned elements.

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Particular elements



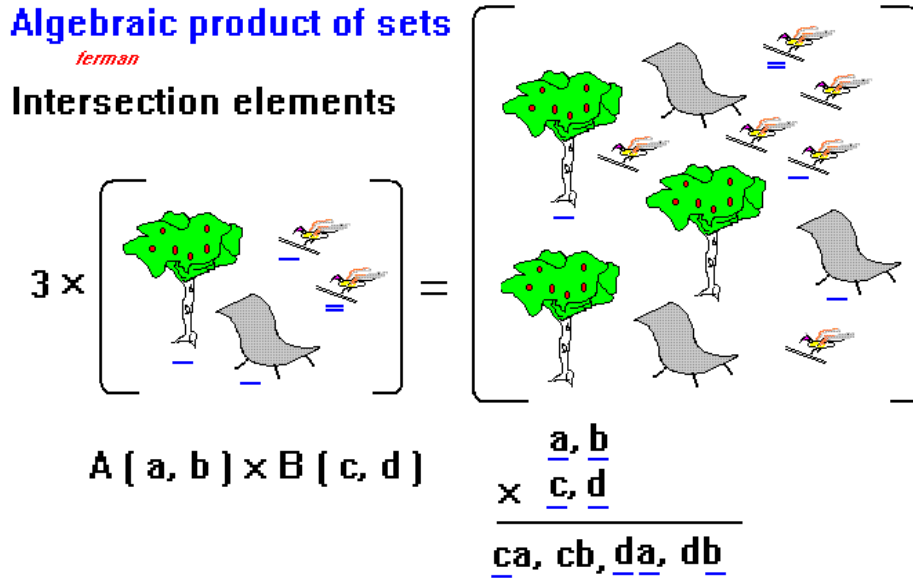
Common elements



Intersection of elements among factors and product

In the algebraic product of sets, when having to introduce new elements from the exterior to be able to complete the product of the multiplication, some time we could need to know which are the elements that were in the factors to multiply and which are those that we have introduced into the product.

For it, we can use the intersection method of this theory, that is to say, to underline the elements that are at the same time in the factors and in the product, such as we can see in the drawing.



Let us remember the intersection of elements in this theory:

Given two sets A (a, b, c,) and B (c, d, e) we call intersection elements to those that belong to the two sets.

The intersection elements are expressed underlining them:

Given two sets A (a, b, c) and B (c, d, e) where c is the intersection elements that belongs to both sets.

Commutative property in the algebraic product of sets.

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Commutative property

$$A(\square, \triangle) = A'(\triangle, \square)$$

$$\square \diamond = \diamond \square$$

Integrated factor F = Integrated factor F'

$$\begin{array}{c}
 \square, \triangle \\
 \times \diamond, \circ \\
 \hline
 \diamond \square, \diamond \triangle, \circ \square, \circ \triangle \\
 1 \quad 2 \quad 3 \quad 4
 \end{array}
 =
 \begin{array}{c}
 \diamond, \circ \\
 \times \square, \triangle \\
 \hline
 \square \diamond, \square \circ, \triangle \diamond, \triangle \circ \\
 1 \quad 3 \quad 2 \quad 4
 \end{array}$$

A × B = B × A

By the moment we will consider that the algebraic product of set has the commutative property, and so that, as much the situation of the elements inside the set as the order of the elements inside the integrated factors follow this property.

$$A(a,b) = A(b,a) ; ab = ba$$

Ubiquity Principle

As I have exposed several times, this theory on the physical elements tries to take mathematics of set to the reality of the physical elements and therefore the mathematical operations must to be subject the characteristics of these elements.

Now well, one of these characteristics is that a physical element cannot be at the same time in two or more different places.

Therefore we will take this definition for the Ubiquity Principle:

"No physical element can be at the same time in two or more places."

And of this consequence we can consider the following points:

1.-

A physical element can't be repeated inside a single set.

Against, any physical element can belong to two or more different sets.

If any physical element were represented two or more times inside a set, we should consider it as a single element when operating with it.

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Ubiquity Principle of the physical elements

"Every physical element can't be in two or more places at the same time"

$$A(\underline{a}, \underline{b}, \underline{c}, \underline{a})$$

$$A(\underline{a}, \underline{b}, \underline{c}) \times B(\underline{d}, \underline{c}) = A(\underline{a}, \underline{b}, \underline{c}) \times B(\underline{d}, \underline{c}) =$$

$$\underline{(\underline{a}, \underline{b}, \underline{c}) \times (\underline{d})} = (\underline{da}, \underline{db}, \underline{dc})$$

$$A^2(\underline{a}, \underline{b}, \underline{c}) \times A(\underline{a}, \underline{b}, \underline{c}) =$$

$$\underline{A(\underline{a}, \underline{b}, \underline{c}) \times A(\underline{a}, \underline{b}, \underline{c})} = A^N(\underline{abc})$$

With quantities

Quantities are not subject to the Ubiquity principle
when they aren't physical elements

$$\begin{array}{r} 5, 6 \\ \times 5, 6 \\ \hline 25, 30, 30, 36 \end{array}$$

2.- In the algebraic product among different sets where one or more intersection elements exist, these elements won't be able to operate on themselves, and therefore they will be kept inside alone one factor, preferably in the multiplicand.

$$A(\underline{a}, \underline{b}, \underline{c}) \times B(\underline{c}, \underline{d}, \underline{e}) = A(\underline{a}, \underline{b}, \underline{c}) \times B(\underline{d}, \underline{e})$$

This is due to a physical element cannot fuse with itself, and therefore, this physical element cannot multiply for itself neither.

In the case of the square (cube... N) of a set, the resulting product will be the fusion of its elements.

$$A(a,b,c) \times A(a,b,c) = A^2(abc)$$

3.- A. - The Ubiquity Principle doesn't affect to numeric quantities since they are not physical elements.

$$A(2, 5) \times B(3, 5) = A \times B(10, 25, 6, 15)$$

B. - The Ubiquity Principle affects to the coefficients since if we eliminate the elements of a set, we also eliminate the multiplier coefficient.

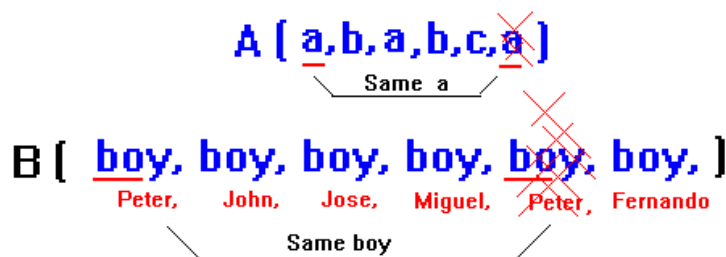
$$A(2U, 3H) \times B(2U, 2K) = A(2U, 3H) \times B(2K)$$

Due to we have eliminated the elements 2U of the multiplier. (see 2.-)

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Ubiquity Principle of the physical elements

"A physical element can't be in two or more times in the same set "



In product

$$3 \times A(\text{rabbit}) = 3A(\text{rabbit, rabbit, rabbit})$$

Same rabbit Addition element Addition element

Internal operations among elements of any set

Algebraic product of sets *ferman* Internal operations

Types of internal product of any set of elements

Integrated product k

$$[6, 2, 4, 3]_k = [144]$$

6x2x4x3

$$[\square, \circ, \triangle, \blacktriangle]_k = [\square \circ \triangle \blacktriangle]$$

Internal product i

$$[6, 2, 4, 3]_i = [12, 24, 18, 8, 6, 12]$$

6x2 6x4 6x3 2x4 2x3 4x3

$$[\square, \circ, \triangle, \blacktriangle]_i = [\square \circ, \square \triangle, \square \blacktriangle, \circ \triangle, \circ \blacktriangle, \triangle \blacktriangle]$$