MECHANICS REVISION

CIRCULAR MOTION
GRAVITATION
OSCILLATIONS

Errata

NJC P3/Q2

(d) Some of the data for this section is given in earlier sections. Calculate the gravitational potential at the point X referred to in (b).

SAJC P3/Q1

(c) The mass of the Earth is 5.98×10^{24} kg and the moon takes 27.4 days to orbit the earth.

AJC P2/Q4

(b) A body undergoes harmonic motion. Fig. 4.1 shows how the velocity *v* varies with its displacement *x* and Fig. 4.2 shows how its kinetic energy *KE* varies with its displacement.

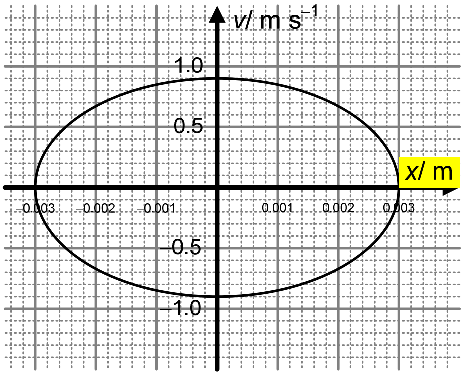


Fig. 4.1

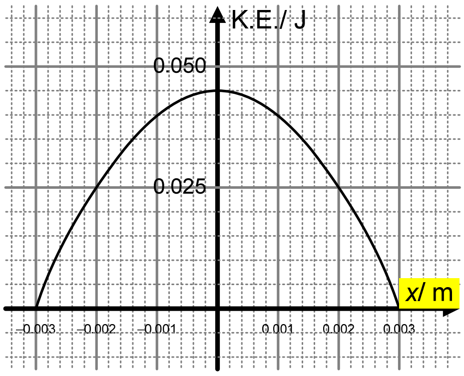


Fig. 4.2

SAJC P2/Q3

(b) Fig. 3.1 shows an apparatus for investigating forced vibrations and resonance of a mass-spring system. Fig. 3.2 shows the displacement-time graph when the system is resonating.

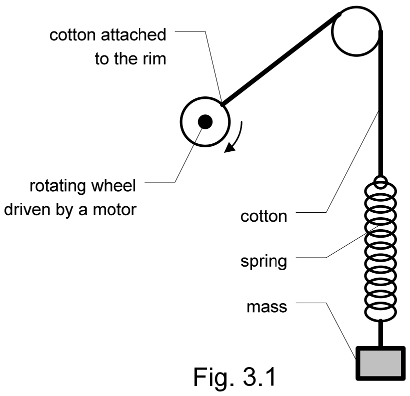


Fig. 3.1

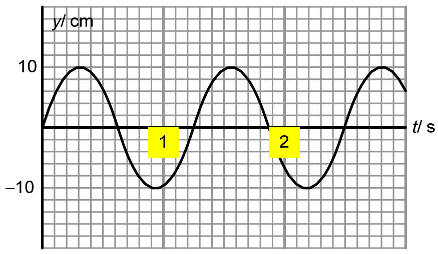


Fig. 3.2

***RJC P2/Q2**

(b) The variation of the gravitational potential near a certain planet of radius 10 000 km is shown by the graph in Fig. 2 below.

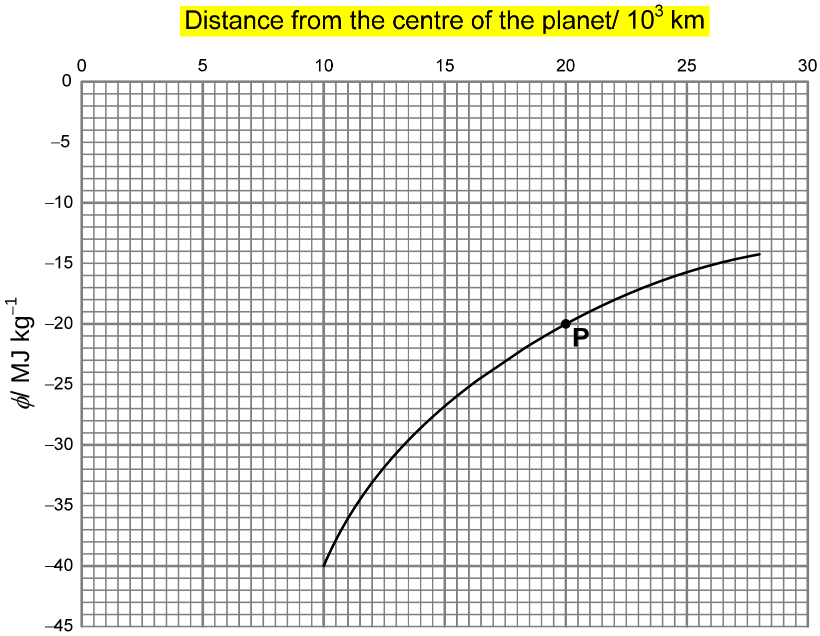


Fig. 2

AJC P2/Q1

(a) An object, when traveling at a constant speed in a circular path, is said to have a centripetal acceleration. Explain why there is acceleration although the speed is constant. [3]

Even though the speed of the object is constant, its *direction* of motion is *constantly changing* hence it has a *constantly changing velocity*. Therefore the object must experience a *centripetal* acceleration.

(b) Fig. 1.1 shows a coin resting 10 cm from the centre of a turntable of radius 20 cm. The turntable was then rotated at a constant rate of 1.5 revolutions per second.

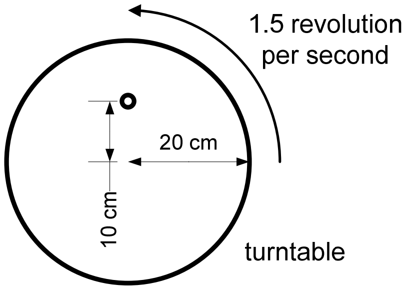


Fig. 1.1

(i) Identify the force acting on the coin to enable it to rotate with the turntable without slipping. [1]

Friction

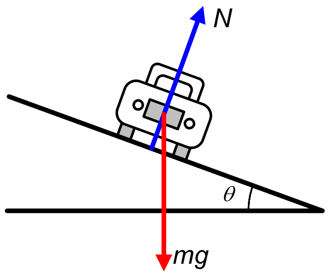
(ii) Calculate the acceleration of the coin towards the centre of the turntable. [2]

$$a = r\omega^2 = 0.10 \times (2\pi \times 1.5)^2 = 8.9 \text{ m s}^{-2}$$

(iii) The speed of the turntable was then gradually increased. When the speed of the turntable was slowly increased the coin remained fixed on the turntable until a certain speed was reached by the turntable, at which the coin slid off. Explain why this was so. [2]

The friction is unable to provide the centripetal force (directly proportional to ω^2) required to keep the coin in the circular motion.

(c) The figure below shows a car of mass 1500 kg travelling in a horizontal circle of radius 50.0 m along a banked road with a speed of 15 m s^{-1}. Assuming that the frictional force acting on the car is negligible, calculated the angle, θ, at which the road is banked. [2]



Resolving the forces,

horizontally: $N\sin\theta = \dfrac{mv^2}{r}$;

vertically: $N\cos\theta = mg$

Therefore,

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{15^2}{50.0\times 9.81} = 24.6°$$

NJC P3/Q2

(a) (i) Define gravitational field strength. [1]

The gravitational field strength at a point in a gravitational field is the *gravitational force per unit mass* acting on a small mass placed at that point.

(ii) Use the value of g at the Earth's surface and the ratios below to calculate the gravitational field strength on the surface of the Moon. [2]

$$\frac{\text{radius of the Moon}}{\text{radius of the Earth}} = 0.27$$

$$\frac{\text{mass of the Moon}}{\text{mass of the Earth}} = 0.012$$

Using $g = \frac{GM}{r^2}$,

$$g_M = \frac{M_M}{M_E} \times \frac{r_E^2}{r_M^2} \times g_E = 0.012 \times \frac{1}{(0.27)^2} \times 9.81 = 1.61 \text{ N kg}^{-1}$$

(b) (i) Explain, why, on a line joining E, the centre of the Earth, to M, the centre of the Moon, there is a point X where the resultant gravitational field strength is zero.

Gravitational field strength is a vector quantity. At point X, the gravitational field strength due to Earth has the same magnitude but in the opposite direction to that due to Moon. Hence resultant gravitational field strength is zero.

(ii) If the distance EX is 3.4×10^8 m, calculate the distance MX and hence EM.

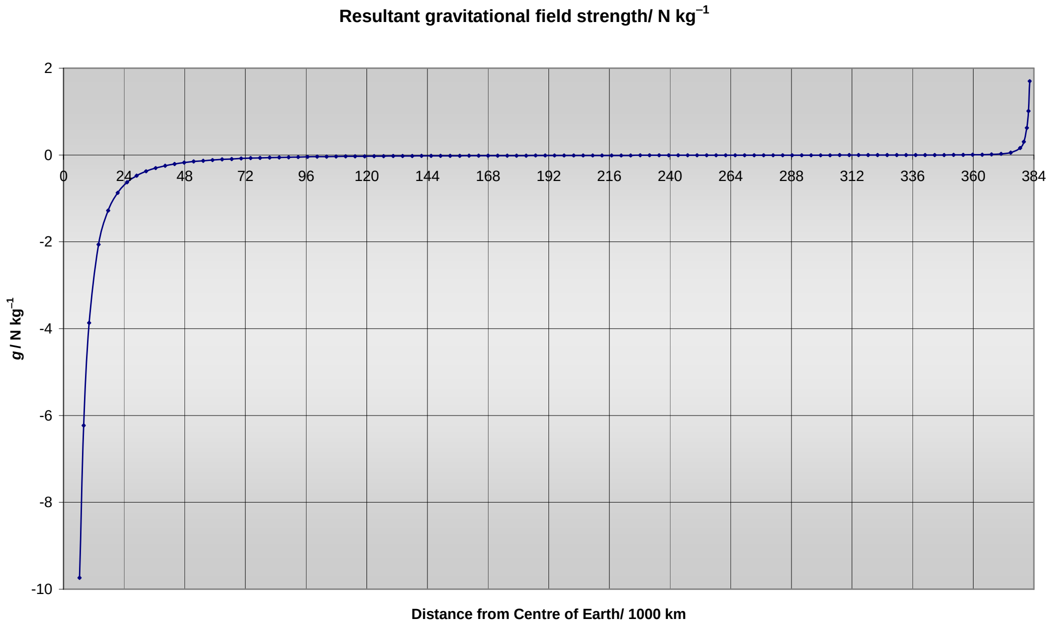
Using $g = \frac{GM}{r^2}$,

$$\frac{GM_E}{EX^2} = \frac{GM_M}{MX^2}$$

$$MX = \sqrt{\frac{M_M}{M_E}} \times EX = \sqrt{0.012} \times 3.4 \times 10^8 = 3.7 \times 10^7 \text{ m}$$

$$EM = EX + MX = 3.4 \times 10^8 + 3.7 \times 10^7 = 3.8 \times 10^8 \text{ m}$$

(iii) Sketch a graph showing qualitatively the variation of gravitational field strength with distance from the surface of the Earth to the surface of the Moon along the line EM. [6]



(c) Given that the mass of Earth is 6.0×10^{24} kg, determine the period of the Moon round the Earth. [2]

The resultant force acting on Moon is the gravitational force of attraction due to Earth.

$$\frac{GM_E M_M}{EM^2} = M_M \times EM \times \omega^2 = M_M \times EM \times \left(\frac{2\pi}{T}\right)^2$$

$$T = 2\pi\sqrt{\frac{EM^3}{GM_E}} = 2\pi\sqrt{\frac{(3.77 \times 10^8)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}} = 2.3 \times 10^6 \text{ s} = 27 \text{ days}$$

(d) Some of the data for this section is given in earlier sections. Calculate the gravitational potential at the point X referred to in (b).

(i) due to the Earth alone,

$$\phi_E = -\frac{GM_E}{r_{EX}} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.4 \times 10^8} = -1.18 \times 10^6 \text{ J kg}^{-1}$$

(ii) due to the Moon alone,

Since $\phi = -\dfrac{GM}{r}$,

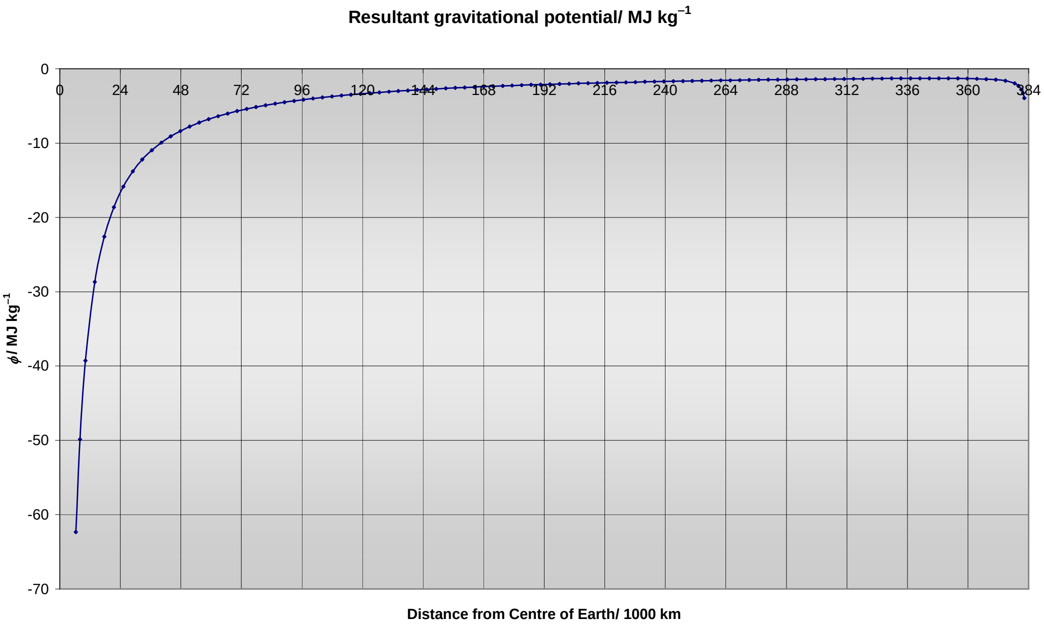
$$\frac{\phi_M}{\phi_E} = \frac{M_M}{M_E} \times \frac{r_{EX}}{r_{MX}}$$

$$\phi_M = \frac{M_M}{M_E} \times \frac{r_{EX}}{r_{MX}} \times \phi_E = 0.012 \times \frac{3.4 \times 10^8}{3.7 \times 10^7} \times 1.18 \times 10^6 = -1.30 \times 10^5 \text{ J kg}^{-1}$$

(iii) due to the Earth and the Moon. [3]

$\phi = \phi_E + \phi_M = -1.31 \times 10^6 \text{ J kg}^{-1}$

(e) **Without further calculation**, sketch a graph showing the variation of gravitational potential with distance from the distance from the surface of the Earth to the surface of the Moon along the line EM as defined in **(b)(iii)**. [2]



(f) A space craft of mass 2.5×10^4 kg is to be sent from the Earth to the Moon.

(i) Explain the significance of point X in determining the minimum energy required.

The space craft needs a minimum energy to overcome the p.d. between the surface of earth and X. Beyond X, the space craft would accelerate towards the moon.

(ii) The gravitational potential on the Earth's surface is -62.5 MJ kg^{-1}. Using your answer from part (d), calculate the minimum energy required to send the space craft from the Earth to the Moon. [4]

The increase in potential from the surface of earth to X is $61.2 \times 10^6 \text{ J kg}^{-1}$. When the space craft is projected when the minium energy, it will reach X when zero K.E.. By the principle of conservation of energy,

$$\text{K.E.}_{min} = m\Delta\phi = 2.5 \times 10^4 \times \left[-1.31 - (-62.5)\right] \times 10^6 = 1.53 \times 10^{12} \text{ J}$$

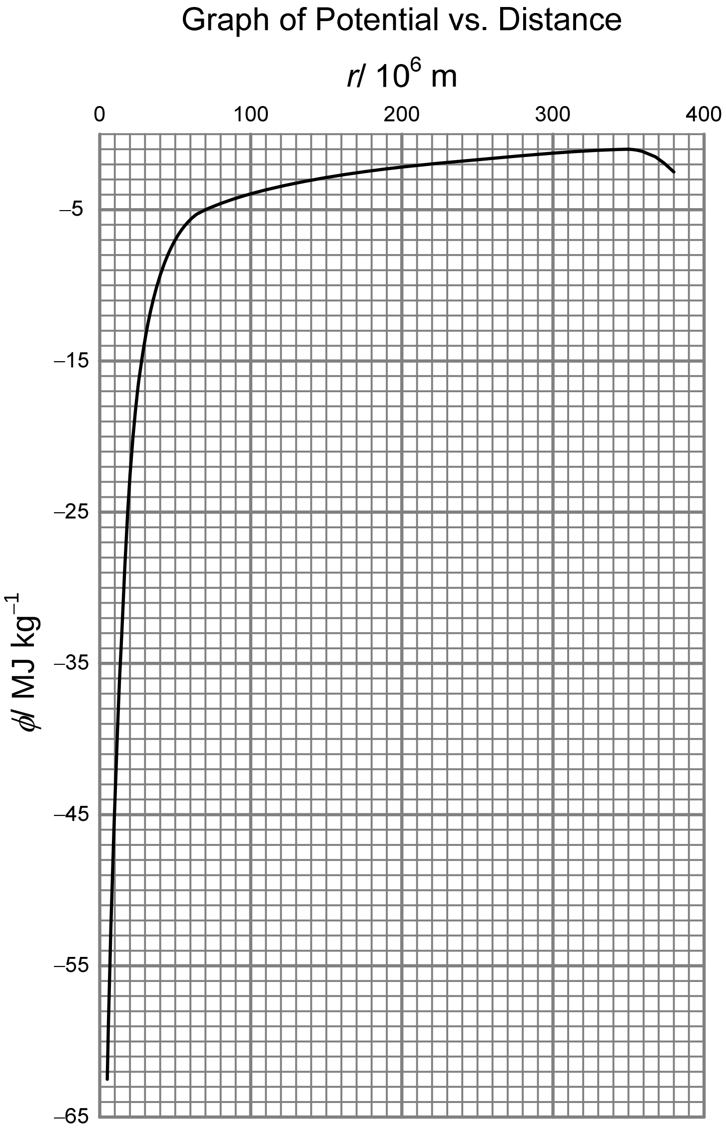
SAJC P3/Q1

(a) Define *gravitational field strength* and *gravitational potential*. [2]

The gravitational field strength at a point in a gravitational field is the *gravitational force per unit mass* acting on a small mass placed at that point.

The gravitational potential at a point in a gravitational field is the *external work done per unit mass* in bringing a mass from infinity to that point.

The graph below shows the variation of the gravitational potential ϕ with distance *r* along a line joining the centres of the moon and the Earth.



(i) Explain why all values of ϕ in the graph are negative. [1]

Positive work done per unit mass is required to bring a mass from any point in a gravitational field to infinity (zero potential).

(ii) Explain whether *r* is measured from the moon or the Earth. [1]

r is measured from centre of Earth since Earth has a greater mass and hence the potential at points nearer the Earth is more negative.

(iii) Estimate from the graph, the value of *r* at which the gravitational field strength is zero. Explain your answer. [2]

3.50×10^8 m . Gravitational field strength is the potential gradient which is zero.

(iv) Determine the minimum energy a 1500 kg meteorite requires to travel from the moon to the Earth. [1]

To travel from moon to earth, the meteorite has to overcome a potential increase of 1.5×10^6 J kg^{-1}.

$$\text{K.E.}_{\text{min}} = m\Delta\phi = 1500 \times 1.5 \times 10^6 = 2.25 \times 10^9 \text{ J}$$

(v) Determine the speed at which the meteorite would hit the Earth. (Neglect atmospheric resistance.) [2]

The decrease in potential from $r = 350 \times 10^6$ m to the surface of earth is 61.5×10^6 J kg^{-1}. By the principle of conservation of energy,

$$\text{gain in K.E.} = \text{loss in G.P.E}$$

$$\text{K.E.} = \tfrac{1}{2}mv^2 = m\Delta\phi$$

$$v = \sqrt{2\Delta\phi} = \sqrt{2 \times 61.5 \times 10^6} = 1.11 \times 10^4 \text{ m s}^{-1}$$

(vi) Suggest why the region where $r < 60 \times 10^6$ m may be referred to as a *potential well*? [1]

Large positive w.d. per unit mass is needed to move a mass beyond $r > 60 \times 10^6$ m.

(c) The mass of the Earth is 5.98×10^{24} kg and the moon takes 27.4 days to orbit the earth.

(i) Show that the distance between the centre of the Earth and the moon is about 384000 km. [2]

The resultant force acting on Moon is the gravitational force of attraction due to Earth.

$$\frac{GM_E M_M}{r^2} = M_M \times r \times \omega^2 = M_M \times r \times \left(\frac{2\pi}{T}\right)^2$$

$$r = \sqrt{\frac{GM_E T^2}{4\pi^2}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (27.4 \times 24 \times 60 \times 60)^2}{4\pi^2}} = 3.84 \times 10^8 \text{ m}$$

(ii) Hence, find the linear speed of the moon. [2]

$$v = r\omega = 3.84 \times 10^8 \times \left(\frac{2\pi}{27.4 \times 24 \times 60 \times 60}\right) = 1020 \text{ m s}^{-1}$$

(d) Using your answer in (c)(i), and given also that the mass of the moon is 7.36×10^{22} kg, verify your answer in (b)(iii). [2]

Let X be the point where resultant gravitational field strength is zero.

$$\frac{GM_E}{r_{EX}^2}=\frac{GM_M}{(r_{EM}-r_{EX})^2}$$

$$\frac{r_{EM}-r_{EX}}{r_{EX}}=\sqrt{\frac{M_M}{M_E}}$$

$$\frac{r_{EM}}{r_{EX}}=1+\sqrt{\frac{M_M}{M_E}}$$

$$r_{EX}=\left(1+\sqrt{\frac{M_M}{M_E}}\right)^{-1}\times r_{EM}=\left(1+\sqrt{\frac{7.36\times10^{22}}{5.98\times10^{24}}}\right)^{-1}\times3.84\times10^{8}=3.46\times10^{8}\text{ m}\simeq3.5\times10^{8}\text{ m}$$

(e) The orbit of the moon is actually elliptical and its distance from the Earth's centre varies from 356000 km to 407000 km. Find the maximum increase in kinetic energy of the moon as it comes closer to the Earth. [2]

By the principle of conservation of energy,

$$\begin{aligned}\text{gain in K.E.}&=\text{loss in G.P.E}\\&=m\Delta\phi\\&=7.36\times10^{22}\times\left[-\frac{6.67\times10^{-11}\times5.98\times10^{24}}{4.07\times10^{8}}-\left(-\frac{6.67\times10^{-11}\times5.98\times10^{24}}{3.56\times10^{8}}\right)\right]\\&=1.03\times10^{28}\text{ J}\end{aligned}$$

(f) Explain why the centre of the Earth is expected to lie on the plane of the moon's orbit. [2]

The resultant force acting on the moon is the gravitational force of attraction due to earth and it is directed towards the centre of earth. Hence the centre of earth must lie on the plane of the moon's orbit since the orbital plane must be parallel to the resultant force.

AJC P2/Q4

(a) Define *simple harmonic motion*. [2]

The acceleration of the a body undergoing S.H.M. is *directly proportional* to its *displacement* from a *fixed point* and is *always directed towards that fixed point*.

(b) A body undergoes harmonic motion. Fig. 4.1 shows how the velocity *v* varies with its displacement *x* and Fig. 4.2 shows how its kinetic energy *KE* varies with its displacement.

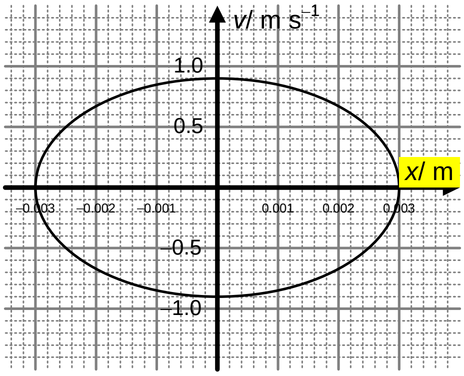


Fig. 4.1

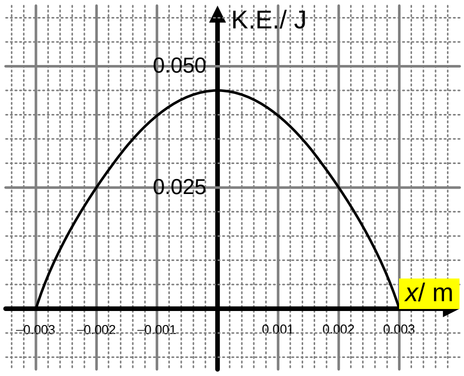


Fig. 4.2

Deduce from the numerical values given on the graph,

(i) the amplitude of the oscillation, [1]

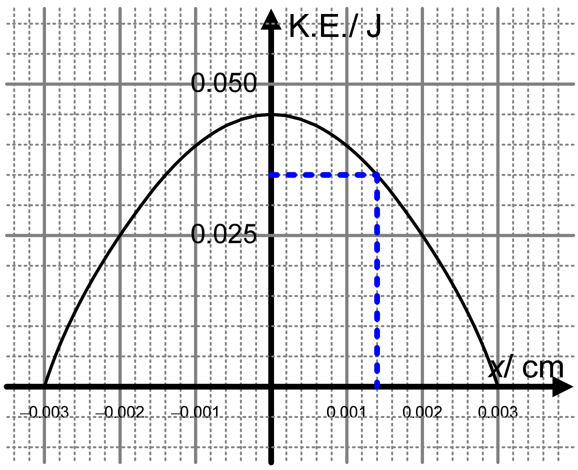
0.0030 m

(ii) the mass of the body, [2]

Since $K.E. = \frac{1}{2}mv^2$,

$$m = \frac{2K.E.}{v^2} = \frac{2 \times 0.045}{0.90^2} = 0.11\text{ kg}$$

(iii) the position *x* where the ratio of the body's potential energy to the kinetic energy is 10/35. [3]

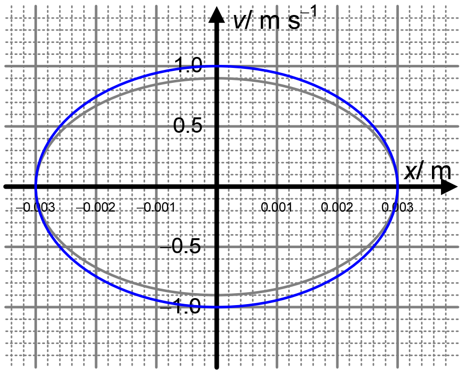


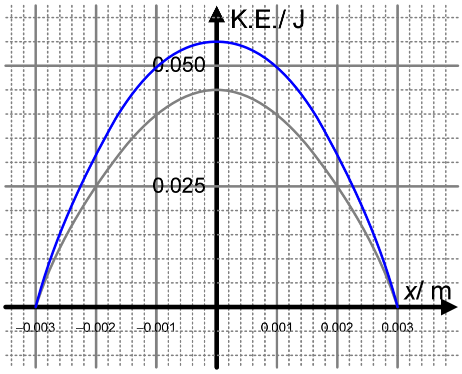
$x = 0.0012$ cm

(c) The frequency of the simple harmonic motion is increased while the amplitude is kept constant. Sketch another graph in Fig. 4.1 and 4.2 to show how this affects

(i) the velocity and [1]

(ii) kinetic energy respectively [1]





ACJC P2/Q2

(a) Define *simple harmonic motion* of a body. [1]

The acceleration of the a body undergoing S.H.M. is *directly proportional* to its *displacement* from a *fixed point* and is *always directed towards that fixed point*.

(b) A light spring is loaded with a mass m, of 200 g and made to execute vertical oscillations.

Fig. 2.1 shows the graph of restoring force, F against extension, x for the spring.

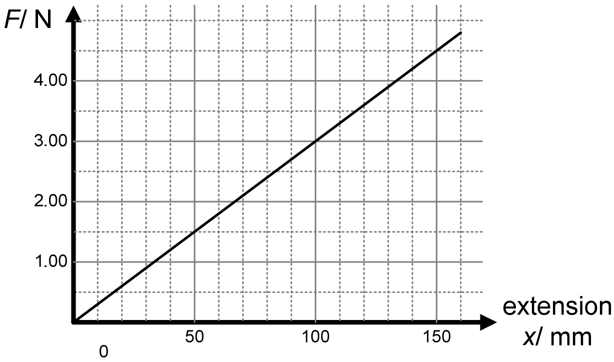


Fig. 2.1

(i) Explain why the oscillations are likely to be harmonic. [3]

The resultant force acting on the mass is directly proportional to its displacement from the equilibrium position (extension of approximately 65 mm). Therefore the acceleration of the mass is directly proportional to its displacement from the equilibrium position and hence the oscillations are S.H.M.

(ii) Find the spring constant k of the spring in N m^{-1} from the graph in Fig. 2.1. Hence calculate the angular frequency of the oscillation by the 200 g mass given that the period is given by the formula $T = 2\pi\sqrt{\frac{m}{k}}$. [3]

Using $F = kx$

$$k = \frac{F}{x} = \frac{4.50}{0.150} = 30.0 \text{ N m}^{-1}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} = \sqrt{\frac{30.0}{0.200}} = 12.2 \text{ rad s}^{-1}$$

(c) A particular ice-skating ring has surrounding vertical walls with special features. One such feature is shown in Fig. 2.2. In this section of the wall, 64 springs, each of spring constant equal to that in (b)(ii), are arranged in parallel to each other and each is attached to the vertical wall and a small light board. The effective spring constant of this arrangement is 64 times that of a single spring.

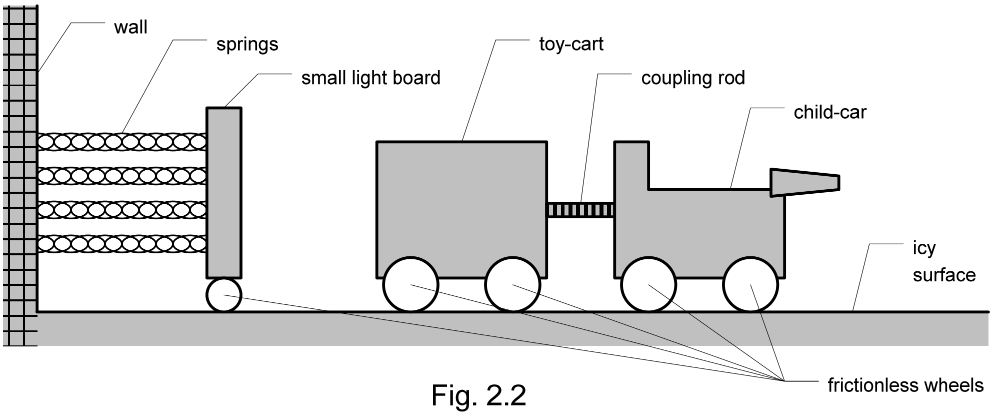


Fig. 2.2

Fig. 2.2 shows a girl in her child-car moving backwards towards the light board. The toy-cart and child-car system are momentarily brought to rest by the small light board when the springs are compressed by 45.0 cm from their natural length. Thereafter, the cart-car system begins to move forward towards the right. The total mass of the toy-cart is 9.25 kg and the total mass of the child-car and its contents is 42.15 kg.

(i) Explain when the car would lose contact with the light board and at what speed does this occur. State any assumptions made. [4]

The toy-cart loses contact with light board when the springs are at their natural length and both the toy-cart and light board are moving with maximum speed. Thereafter, the toy-cart moves with constant velocity while the light board experience an acceleration in the opposite direction.

By the principle of conservation of energy,

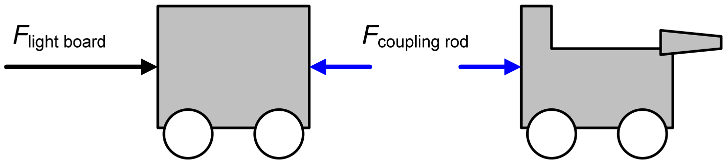
$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2$$

$$v = \sqrt{\frac{k}{m}} \times x = \sqrt{\frac{64 \times 30.0}{9.25 + 42.15}} \times 0.450 = 2.75 \text{ m s}^{-1}$$

(ii) Find the maximum acceleration experienced by the toy-cart and child-car system and the force exerted by the horizontal coupling rod on the toy-cart at that instant. [4]

Maximum acceleration is experienced when the spring has maximum compression. Using $F_{\text{light board}} = kx_0 = ma$,

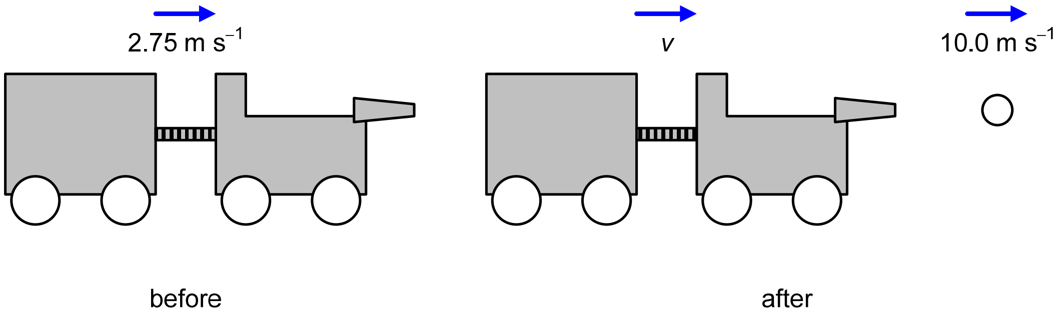
$$a_0 = \frac{kx_0}{m} = \frac{64 \times 30.0 \times 0.450}{9.25 + 42.15} = 16.8 \text{ m s}^{-2}$$



The force acting on toy-cart by the coupling rod is equal and opposite to that acting on the child-car. Both toy-cart and child-car has the same acceleration hence

$$F_{\text{coupling rod}} = \frac{m_{\text{child-car}}}{m_{\text{total}}} \times F_{\text{light board}} = \frac{42.15}{9.25 + 42.15} \times 64 \times 30.0 \times 0.450 = 709 \text{ N}$$

(iii) Shortly after the toy-cart loses contact with the light board, the child fires her "cannon" and an object of mass 0.45 kg leaves the car in the forward direction with a horizontal speed of 10.0 m s^{-1} with respect to the vertical wall. Find the subsequent velocity of the toy-cart and child-car system. [3]



By the principle of conservation of linear momentum,

$$(9.25 + 42.15) \times 2.75 = (9.25 + 42.15 - 0.45)v + 0.45 \times 10.0$$

$$v = 2.69 \text{ m s}^{-1}$$

SAJC P2/Q3

(a) A body is said to be moving in simple harmonic motion when $a = -\omega^2 x$. State clearly what a, ω, x represents. [3]

a is acceleration of the body

ω is the angular velocity of the body

x is the displacement of the body from the equilibrium position

(b) Fig. 3.1 shows an apparatus for investigating forced vibrations and resonance of a mass-spring system. Fig. 3.2 shows the displacement-time graph when the system is resonating.

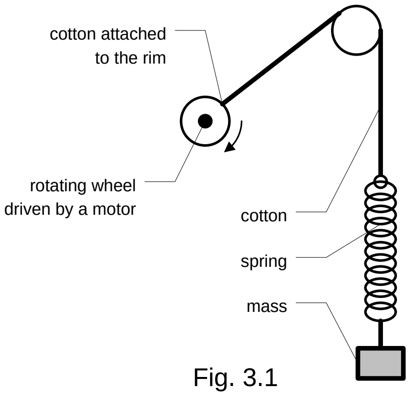


Fig. 3.1

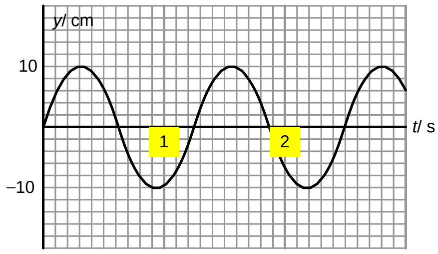


Fig. 3.2

(i) State what is meant by a forced vibration. [1]

An oscillating driving force causing a system to oscillate.

(ii) State the condition for resonance to occur in the system shown in Fig. 3.1. [1]

Driving frequency is equal to natural frequency of mass-spring system.

(iii) The spring constant of the spring used in the experiment was 9.0 N m^{-1}. Using the information from Fig. 3.2, determine the value of the mass suspended from the spring. [3]

$$-kx = ma$$

$$a = -\frac{k}{m}x = -\omega^2 x$$

$$m = \frac{k}{\omega^2} = \frac{kT^2}{4\pi^2} = \frac{9.0 \times \left(\frac{2.5}{2}\right)^2}{4\pi^2} = 0.356 \text{ kg}$$

(iv) When the rotating wheel stops, Fig. 3.3 shows how the amplitude of the oscillations of the mass subsequently varies with time.

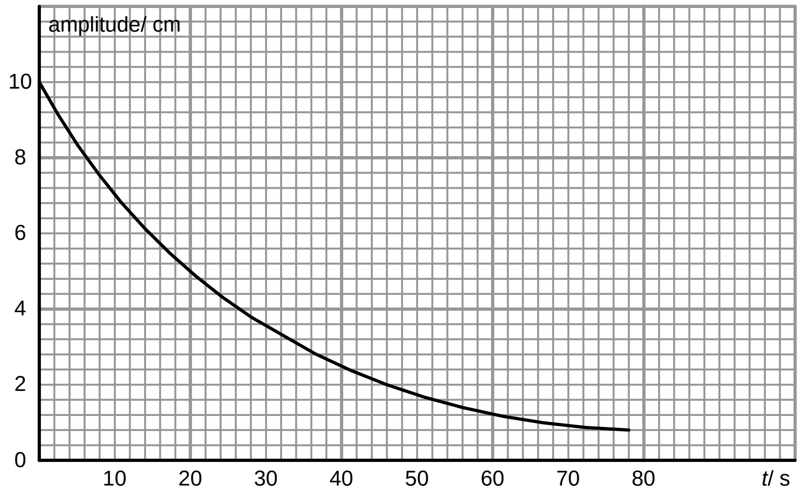


Fig. 3.3

Determine the ratio $\dfrac{\text{energy of the oscillator after 8 oscillations}}{\text{energy of the oscillator at time } t = 0}$. [2]

Since energy of the oscillator is directly proportional to A^2,

$$\frac{\text{energy of the oscillator after 8 oscillations}}{\text{energy of the oscillator at time } t = 0} = \frac{7.0^2}{10.0^2} = 0.49$$

***RJC P2/Q2**

(a) (i) State in words the relationship between *gravitational potential* and *gravitational field strength*. [2]

Magnitude of gravitational field strength is the *potential gradient*.
The direction of gravitational field strength is *towards lower potential*.

(ii) Explain why gravitational potential has a negative value. [2]

Positive work done per unit mass is required to bring a mass from any point in a gravitational field to infinity (zero potential).

(b) The variation of the gravitational potential near a certain planet of radius 10 000 km is shown by the graph in Fig. 2 below.

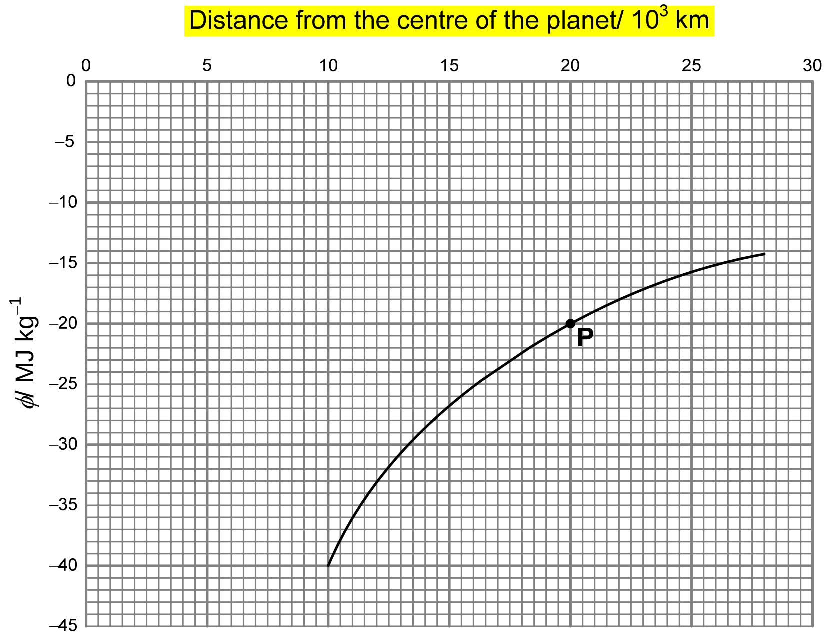


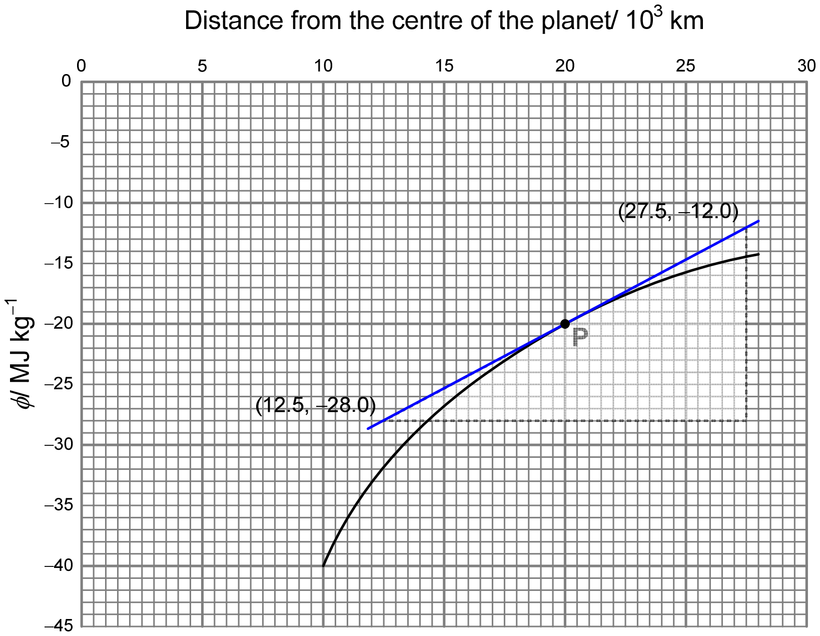
Fig. 2

Use the graph of Fig. 2 to determine

(i) the gravitational potential energy of a 2.0 kg mass at P, a distance of 20000 km from the centre of the planet, [1]

$$U = m\phi = 2.0 \times \left(-20 \times 10^6\right) = -4.0 \times 10^7 \text{ J}$$

(ii) the gravitational field strength at **P**, [2]



$$g = \frac{[-12.0-(-28.0)]\times 10^6}{(27.5-12.5)\times 10^6} = 10.7 \text{ N kg}^{-1}$$

(iii) the difference in gravitational potential between the planet's surface and infinity. Hence, find the escape velocity for an object at the surface of the planet. [3]

$$\Delta\phi = \phi_{\text{infinity}} - \phi_{\text{surface}} = 0 - (-40\times 10^6) = 4.0\times 10^7 \text{ N kg}^{-1}$$

To escape from the planet, the object must have sufficient energy to reach infinity.

$$\tfrac{1}{2}mv^2 = m\Delta\phi$$

$$v = \sqrt{2\Delta\phi} = \sqrt{2\times 4.0\times 10^7} = 8.9\times 10^3 \text{ m s}^{-1}$$

***AJC P3/Q2**

(a) Write down an equation expressing Newton's law of gravitation. Define your symbols. [2]

(b) Use the equation in (a) to derive a value for g, the acceleration due to gravity, [3]

mass of Earth $= 5.98 \times 10^{24}$ kg

mean radius of Earth $= 6.37 \times 10^{6}$ m

(c) Under the heading *Data* there is an entry

acceleration of free fall $g = 9.81$ m s^{-2}

Compare and comment on small differences between this value, the value you obtained in (b), and the value of 9.79 m s^{-2} (which is the value obtained by making accurate measurements near the equator). [5]

(d) Explain what is meant by the term *geostationary orbit*. [1]

(e) The planet Saturn has a number of rings around it. Two such rings are illustrated in Fig. 2.

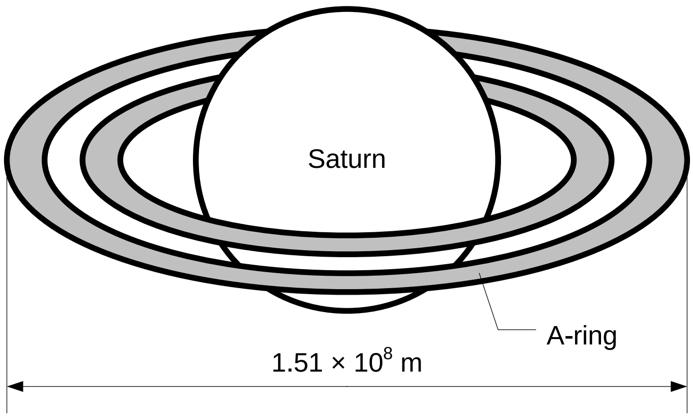


Fig. 2

Each ring consists of many small particles orbiting Saturn. One such ring, the A-ring, has an outer diameter of 1.51×10^{8} m and a particle on the outer circumference of the ring has a speed of 1.59×10^{4} m s^{-1}.

(i) Calculate the angular velocity about Saturn of a particle in the outermost region of the A-ring. [2]

(ii) The rotational period of Saturn is 10 hours 14 minutes. Use your answer in (i) to deduce whether the particle is in a square [3]

(iii) Show that the radius r of the orbit of a particle moving with angular velocity ω around Saturn is given by the expression [2]

$$GM = r^3\omega^2$$

where M is the mass of Saturn.

[Assume that the gravitational field outside Saturn is the same as that of a point mass M at the centre of Saturn.]

(iv) Using your answer to (i), calculate the mass of Saturn. [2]

***NJC P2/Q4**

4 The needle-carrier of a sewing machine is constrained to move in a vertical line by low friction guides as shown in Fig. 4.1. The simple harmonic motion of the needle-carrier is produced by a rotating disc carrying a peg which moves in a circle and engages with a slot attached to the needle-carrier as shown.

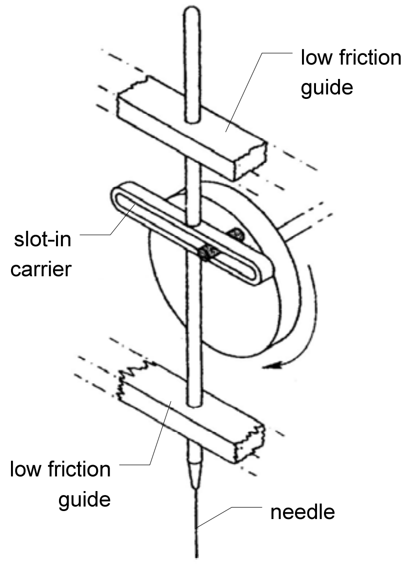


Fig. 4.1

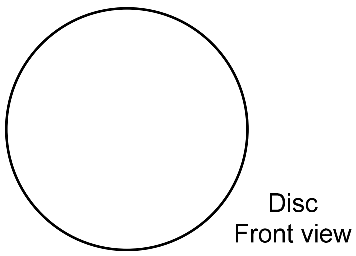
(a) Calculate the angular speed of the peg's circular motion which gives a frequency of oscillation of 12 Hz (which corresponds to the machine's maximum rate of twelve stitches per second). [1]

(b) The carrier and needle together have a mass of 25 g and the needle point moves a distance of 32 mm between the externalities of its movement. Assuming that the fabric being sewn requires negligible force for the needle to penetrate, calculate

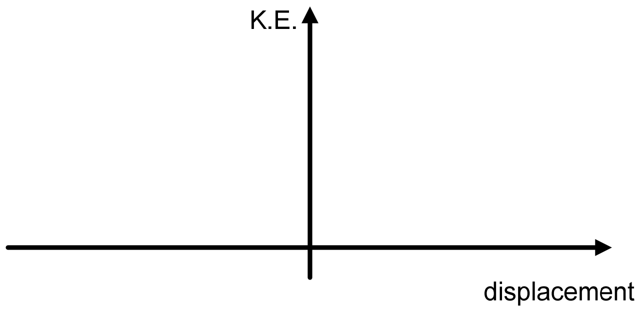
(i) the maximum speed of the needle, and [2]

(ii) the maximum contact force acting on the peg by the slot. [3]

(c) State clearly on the diagram below the position of the peg corresponding to your answers to (b)(i) and (b)(ii). [2]



(d) Sketch on the figure a labeled graph of the kinetic energy of the mass (carrier and needle) against its vertical displacement, indicating all relevant values. [2]



***RJC P2/Q4**

2 A particle executes simple harmonic motion along a vertical line according to the equation $y = 1.2\sin\left(6.3t + \frac{\pi}{4}\right)$, where y is the displacement (measured in metres) from the equilibrium position at time t (measured in seconds). [2]

(a) Define *simple harmonic motion*. [1]

The acceleration of the a body undergoing S.H.M. is *directly proportional* to its *displacement* from a *fixed point* and is *always directed towards that fixed point*.

(b) Determine, for the motion, the following:

(i) *amplitude* [1]

$y_0 = 1.2$ m

(ii) *frequency* [1]

$\omega = 2\pi f = 6.3$
$f = 1.0$ Hz

(iii) maximum *acceleration*. [1]

$a_0 = y_0\omega^2 = 1.2 \times 6.3^2 = 48 \text{ m s}^{-2}$

(c) Explain the significance of the $\frac{\pi}{4}$ term in the equation for y given above. [1]

$\pi/4$ refers to the phase angle of the S.H.M indicating that at $t = 0$, the particle has completed $1/8$ of its cycle from the equilibrium position.

(d) Given that the mass of the particle is 0.20 kg,

(i) write down an expression for the kinetic energy, K, of the particle in terms of t. [3]

$$v = \frac{dy}{dt} = 1.2 \times 6.3\cos\left(6.3t + \frac{\pi}{4}\right) = 7.56\cos\left(6.3t + \frac{\pi}{4}\right)$$
$$\text{K.E.} = \tfrac{1}{2}mv^2 = 5.7\cos^2\left(6.3t + \frac{\pi}{4}\right)$$

(ii) sketch the graph of K against time t over one cycle of the oscillation. [2]