ERRATA

Book by V.I. Fabrikant *Applications of Potential Theory in Mechanics*, Kluwer Academic, 1989.

The errors and misprints noticed so far by the author are presented below. The formulae and text where omissions took place are reprinted in corrected form. The exact nature of omissions is not specified. They can be exposed by comparing the formulae printed here with similar formulae in the book.

Page 15:

$$\frac{1}{R}=\frac{2}{\pi}\int_{max(\rho_0,\rho)}^{\infty}\frac{\lambda(\frac{\rho\rho_0}{x^2},\phi-\phi_0)\,dx}{(x^2-\rho^2)^{1/2}(x^2-\rho_0^2)^{1/2}}. \tag{1.1.28}$$

Page 37:

$$\sigma(\rho,\phi)=\frac{1}{2\pi^3}\left\{-\Delta\int_0^{2\pi}\int_0^a\frac{1}{R}\tan^{-1}(\frac{\eta}{R})\,\omega(\rho_0,\phi_0)\,\rho_0 d\rho_0 d\phi_0\right.$$

$$\left.-\frac{a}{(a^2-\rho^2)^{3/2}}\int_0^{2\pi}\int_0^a\frac{1-t\bar{t}}{(1-t)(1-\bar{t})}\frac{\omega(\rho_0,\phi_0)}{(a^2-\rho_0^2)^{1/2}}\rho_0 d\rho_0 d\phi_0\right\}$$

$$=\frac{1}{\pi^2}\left\{-\Delta\int_\rho^a\frac{dx}{(x^2-\rho^2)^{1/2}}\int_0^x\frac{\rho_0 d\rho_0}{(x^2-\rho_0^2)^{1/2}}\mathcal{L}\left(\frac{\rho\rho_0}{x^2}\right)\omega(\rho_0,\phi)\right.$$

$$\left.-\frac{a}{(a^2-\rho^2)^{3/2}}\int_0^a\frac{\rho_0 d\rho_0}{(a^2-\rho_0^2)^{1/2}}\mathcal{L}\left(\frac{\rho\rho_0}{a^2}\right)\omega(\rho_0,\phi_0)\right\}, \tag{1.4.40}$$

Page 38:

3. Solve problems 1 and 2 for $v=v_1\rho\cos\phi$, $v_1=\text{const}$.

Answer: $V(\rho,\phi,z)=\frac{2}{\pi}v_1\rho\cos\phi\left[\sin^{-1}(\frac{a}{l_2})-\frac{a}{l_2}\sqrt{1-(a/l_2)^2}\right]$,

Page 42:

$$\sigma(\rho,\phi)=-\frac{1}{\pi^2}\left\{\frac{\chi(a,\rho,\phi)}{(\rho^2-a^2)^{1/2}}\right.$$

$$\left.+\frac{1}{2\pi}\int_0^{2\pi}\int_a^{\infty}\frac{\Delta v(\rho_0,\phi_0)\,\rho_0 d\rho_0 d\phi_0}{[\rho^2+\rho_0^2-2\rho\rho_0\cos(\phi-\phi_0)]^{1/2}}\tan^{-1}\frac{(\rho^2-a^2)^{1/2}(\rho_0^2-a^2)^{1/2}}{a[\rho^2+\rho_0^2-2\rho\rho_0\cos(\phi-\phi_0)]^{1/2}}\right\}. \tag{1.5.10}$$

Page 43:

$$V(\rho,\phi,z)=\frac{1}{\pi^2}\int_0^{2\pi}\int_a^{\infty}\frac{z}{R_0^3}\left[\frac{R_0}{j}+\tan^{-1}\left(\frac{j}{R_0}\right)\right]v(\rho_0,\phi_0)\rho_0 d\rho_0 d\phi_0. \tag{1.5.17}$$

$$\tau_z=\frac{T}{8\pi}\left[\frac{z_1}{(m_1-1)R_1^3}+\frac{z_2}{(m_2-1)R_2^3}-\frac{z_3}{R_3^3}\right]$$

$$-\frac{\overline{T}q^2}{8\pi}\left[\frac{2R_1+z_1}{(m_1-1)R_1^3(R_1+z_1)^2}+\frac{2R_2+z_2}{(m_2-1)R_2^3(R_2+z_2)^2}+\frac{(2R_3+z_3)}{R_3^3(R_3+z_3)^2}\right]$$

$$-\frac{Pq}{4\pi}\left[\frac{m_1}{\gamma_1(m_1-1)R_1^3}+\frac{m_2}{\gamma_2(m_2-1)R_2^3}\right]. \tag{2.2.7}$$

Page 99, Exercise 2.4.2

$$K_1(\rho)=\frac{\sqrt{2}}{\pi\sqrt{\rho}}\int_0^{\rho}\frac{\sigma(x)x\,dx}{(\rho^2-x^2)^{1/2}}.$$

Page 106:

$$\tau_1(\rho) = -\frac{2}{\pi^2 G_1^2 - G_2^2}\frac{\mathrm{d}}{\mathrm{d}\rho}\int_\rho^a \frac{\mathrm{d}x}{(x^2-\rho^2)^{1/2}}\frac{\mathrm{d}}{\mathrm{d}x}\left[x\int_0^x \frac{G_1\chi_1(\rho_0) + G_2\bar{\chi}_1(\rho_0)}{(x^2-\rho_0^2)^{1/2}}\,\mathrm{d}\rho_0\right]. \qquad (2.6.8)$$

The general solution of the system (2.6.5) and (2.6.6) can be presented in the form

$$\tau_{-n+1}(\rho) = \rho^{n-1}\int_\rho^a \frac{f_{-n+1}(t)\,\mathrm{d}t}{(t^2-\rho^2)^{1/2}} + \left(\frac{G_2}{G_1}\bar{C}_n + D_n\right)\frac{\rho^{n-1}}{(a^2-\rho^2)^{1/2}},$$

Page 107:

$$\frac{\pi G_1}{\rho^{n-1}}\left[\int_0^\rho \frac{x^{2n-2}\mathrm{d}x}{(\rho^2-x^2)^{1/2}}\int_x^a f_{-n+1}(t)\,\mathrm{d}t + D_n\frac{\Gamma(\frac{1}{2})\,\Gamma(n-\frac{1}{2})}{2\Gamma(n)}\,\rho^{2n-2}\right]$$

$$-\frac{\pi G_2}{\rho^{n-1}}\int_0^\rho \frac{x^{2n-2}\mathrm{d}x}{(\rho^2-x^2)^{1/2}}\int_x^a \bar{f}_{n+1}(t)\,\mathrm{d}t = \chi_{-n+1}(\rho). \qquad (2.6.11)$$

Pages 109-110:

Here $\delta(\cdot)$ is the Dirac delta-function. The governing integral equation corresponds to (2.6.2), with the right hand side χ, defined by (2.6.3), i.e.

$$\chi(\rho,\phi) = \frac{PH\alpha}{\rho e^{-i\phi} - \rho_0 e^{-i\phi_0}} = -\frac{PH\alpha}{\rho_0 e^{-i\phi_0}}\sum_{n=0}^{\infty}(e^{-i(\phi-\phi_0)}\frac{\rho}{\rho_0})^n.$$

The general solution, presented above, yields the following results

$$\chi_{-n+1}(\rho) = -\frac{PH\alpha e^{i\phi_0}}{\rho_0}(e^{i\phi_0}\frac{\rho}{\rho_0})^{n-1}, \quad \chi_{n+1}(\rho) = 0\,,$$

$$f_{-n+1} = f_{n+1} = C_n = 0, \quad D_n = -\frac{2PH\alpha\Gamma(n)}{\pi^{3/2}G_1\Gamma(n-\frac{1}{2})(\rho_0 e^{-i\phi_0})^n} \qquad (2.6.19)$$

Substitution of (2.6.19) in (2.6.17) and (2.6.9) leads to the solution

$$\tau(\rho,\phi) = -\frac{2PH\alpha}{\pi^{3/2}G_1}\sum_{n=0}^{\infty}\frac{\Gamma(n+1)}{(\rho_0 e^{-i\phi_0})^{n+1}\Gamma(n+\frac{1}{2})}\frac{\rho^n e^{-in\phi}}{(a^2-\rho^2)^{1/2}}.$$

The summation can be performed, according to the scheme

$$\sum_{n=0}^{\infty}\frac{\Gamma(n+1)}{\Gamma(n+\frac{1}{2})}\zeta^n = \pi^{-1/2}F(1,1;\tfrac{1}{2};\zeta) = \frac{\pi^{-1/2}}{1-\zeta}\left[1+\left(\frac{\zeta}{1-\zeta}\right)^{1/2}\sin^{-1}\sqrt{\zeta}\right].$$

Here we have used the well known property of the hypergeometric functions (Bateman and Erdélyi, 1955). Now the final result will take the form

$$\tau(\rho,\phi) = -\frac{2PH\alpha}{\pi^2 G_1\rho_0 e^{-i\phi_0}(a^2-\rho^2)^{1/2}}\frac{1}{1-b}\left[1+\left(\frac{b}{1-b}\right)^{1/2}\sin^{-1}\sqrt{b}\right],$$

where $b=(\rho/\rho_0)e^{-i(\phi-\phi_0)}$. In the case of isotropy, the last formula differs in sign with the result of Ufliand (1967).

Page 111:

Answer: $w = \Re(u_0 e^{-i\phi})\dfrac{4H\alpha a}{\pi G_1\rho}$, for $\rho > a$;

$$w = \Re(u_0 e^{-i\phi})\frac{4H\alpha[a-(a^2-\rho^2)^{1/2}]}{\pi G_1\rho}, \quad \text{for } \rho \le a;$$

3. Subject to the conditions of the first problem, find the tangential displacement outside the circle $\rho = a$.

Answer: $u = \dfrac{2}{\pi}\left[u_0\sin^{-1}(\dfrac{a}{\rho}) + \bar{u}_0\dfrac{G_2}{G_1}\dfrac{a(\rho^2-a^2)^{1/2}}{\rho^2}e^{2i\phi}\right].$

Page 130:

2. In the example above find the shear tractions in the plane z=0 outside the crack.

Answer: $$\tau(\rho,\phi)=\frac{2}{\pi}\left\{\left[\sin^{-1}\left(\frac{a}{\rho}\right)-\frac{a}{(\rho^2-a^2)^{1/2}}\right]\tau-\frac{G_2a^3e^{2i\phi}}{G_1\rho^2(\rho^2-a^2)^{1/2}}\bar{\tau}_0\right\}.$$

Page 133:

$$\sigma(\rho,\phi)=-\frac{\mathcal{L}(1/\rho)}{\pi^2H\rho}\frac{d}{d\rho}\int_0^a\frac{x\,dx}{(\rho^2-x^2)^{1/2}}\mathcal{L}(x^2)\frac{d}{dx}\int_x^a\frac{\rho_0 d\rho_0}{(\rho_0^2-x^2)^{1/2}}\mathcal{L}\left(\frac{1}{\rho_0}\right)w(\rho_0,\phi). \qquad (2.8.7)$$

Page 164:

One can notice that $q_1=\Re f$, $q_2=\Im F$, $Q_1=\Re F$, and $Q_2=\Im f$,

Page 177:

$$\tau(\rho)=\frac{d}{d\rho}\left[C_1\int_a^\rho\frac{f_1(t)dt}{(\rho^2-t^2)^{1/2}}+C_2\int_\rho^\infty\frac{f_2(t)dt}{(t^2-\rho^2)^{1/2}}\right]+\frac{C_1Da}{\rho(\rho^2-a^2)^{1/2}}. \qquad (3.3.6)$$

Page 178 (error in font and location):

$$\frac{\alpha^2}{\gamma_1\gamma_2}\int_a^\infty\frac{f_1(t)dt}{t^2-r^2}-\int_a^\infty\frac{(r^2-a^2)^{1/2}f_1(t)t\,dt}{r(t^2-a^2)^{1/2}(t^2-r^2)}=\psi_1(r). \qquad (3.3.9)$$

Page 185 (error in location):

$$-\frac{\alpha}{\pi\sqrt{\gamma_1\gamma_2}}\int_0^a F_1(t)\ln\frac{t^2}{|r^2-t^2|}dt-\int_0^r F_2(t)dt=\frac{1}{\pi^2H\sqrt{\gamma_1\gamma_2}}r\int_0^r\frac{\omega_2(\rho)d\rho}{(r^2-\rho^2)^{1/2}}. \qquad (3.4.15)$$

Page 189:

The structure of equations (3.5.2) is such that we may assume that $\sigma_1 = \bar{\sigma}_{-1}$. The solution may be represented in the form:

$$\sigma_1(\rho) = \bar{\sigma}_{-1}(\rho) = \frac{d}{d\rho}\int_0^{\rho} \frac{\bar{f}(t)dt}{(\rho^2 - t^2)^{1/2}},$$

$$\tau_0(\rho) = -\frac{C}{\rho}\frac{d}{d\rho}\int_{\rho}^{a} \frac{f(t)tdt}{(t^2 - \rho^2)^{1/2}} + \frac{D}{(a^2 - \rho^2)^{1/2}},$$

$$\tau_2(\rho) = -C\rho\frac{d}{d\rho}\left\{\frac{1}{\rho^2}\int_{\rho}^{a} \frac{\bar{f}(t)tdt}{(t^2 - \rho^2)^{1/2}}\right\} - \bar{D}\frac{2a^2 - \rho^2}{\rho^2(a^2 - \rho^2)^{1/2}}. \qquad (3.5.3)$$

Page 193:

$$u_0(\rho) = \pi(G_1 - G_2)\frac{\alpha}{\gamma_1\gamma_2}\int_0^{a} \frac{f(x)dx}{(\rho^2 - x^2)^{1/2}} + \pi(G_1 + G_2)D\sin^{-1}(\frac{a}{\rho}),$$

$$u_2(\rho) = \frac{1}{\rho^2}\left\{2\pi H\alpha\int_0^{a} f(x)d\{x[(a^2 - x^2)^{1/2} - (\rho^2 - x^2)^{1/2}]\}\right.$$

$$\left. + \pi(G_1 + G_2)Da(\rho^2 - a^2)^{1/2}\right\}.$$

Page 195:

$$\sigma_n(\rho) = \bar{\sigma}_{-n}(\rho) = \frac{1}{\rho^n}\int_0^{\rho} \frac{t^{2n-1}df_n(t)}{(\rho^2 - t^2)^{1/2}}$$

$$= -\frac{1}{\rho^{n+1}}\frac{d}{d\rho}\int_0^{\rho} \frac{t^{2n-2}[(2n-1)\rho^2 - 2nt^2]}{(\rho^2 - t^2)^{1/2}} f_n(t)dt;$$

Page 205:

$$u_{n+1}(\rho)=\frac{2\pi H\alpha}{\rho^{n+1}}\int_0^a f_n(x)\mathrm{d}\{x^{2n-1}[(a^2-x^2)^{1/2}-(\rho^2-x^2)^{1/2}]\}$$

$$+\pi(G_1+G_2D_n a^{2n-1}\frac{(\rho^2-a^2)^{1/2}}{\rho^{n+1}}.$$

Page 207:

$$2\pi H\alpha\,\Re\left\{\frac{e^{-in\phi}}{\rho^n}\int_a^\rho \tau_{-n+1}(\rho_0)\rho_0^n\mathrm{d}\rho_0-\rho^n e^{in\phi}\int_\rho^\infty\frac{\tau_{n+1}(\rho_0)\mathrm{d}\rho_0}{\rho_0^n}\right\}$$

$$+4H\rho^n\int_\rho^\infty\frac{\mathrm{d}x}{x^{2n}(x^2-\rho^2)^{1/2}}\int_a^x\frac{\sigma_{-n}(\rho_0)e^{-in\phi}+\sigma_n(\rho_0)e^{in\phi}}{(x^2-\rho_0^2)^{1/2}}\rho_0^{n+1}\mathrm{d}\rho_0=\Re\{e^{in\phi}\Phi_n(\rho)\},$$

$$\text{for } n\geq 0. \qquad (3.7.1)$$

Page 209:

$$\int_a^x\frac{\rho_0^{2n+1}\mathrm{d}\rho_0}{(x^2-\rho_0^2)^{1/2}}\int_{\rho_0}^\infty\frac{\mathrm{d}f_n(t)}{t^{2n}(t^2-\rho_0^2)^{1/2}}=\int_a^\infty\left[(x^2-a^2)^{1/2}(t^2-a^2)^{1/2}Q_n(x,t)\right.$$

$$\left.+\psi_n(x,t)\ln\frac{|(x^2-a^2)^{1/2}+(t^2-a^2)^{1/2}|}{|t^2-x^2|^{1/2}}\right]\frac{\mathrm{d}f_n(t)}{t^{2n}}.$$

Page 210:

$$-\frac{(r^2-a^2)^{1/2}}{r}\int_a^\infty\frac{f_n(t)t\mathrm{d}t}{(t^2-r^2)(t^2-a^2)^{1/2}}+\frac{\alpha^2}{\gamma_1\gamma_2}\int_a^\infty\frac{f_n(t)\mathrm{d}t}{t^2-\rho^2}=\chi_n(r).$$

(3.7.8)

Here

$$\chi_n(r)=\frac{1}{4\pi H}\int_r^\infty\frac{\rho^{2n}\mathrm{d}\rho}{(\rho^2-r^2)^{1/2}}\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\frac{\Phi_n(\rho)}{\rho^n}\right]$$

$$-\frac{2}{\pi}\int_r^\infty \frac{\rho^{2n}d\rho}{(\rho^2-r^2)^{1/2}}\frac{d}{d\rho}\int_\rho^\infty \frac{(x^2-a^2)^{1/2}dx}{x^{2n}(x^2-\rho^2)^{1/2}}\int_a^\infty (t^2-a^2)^{1/2}Q_n(x,t)\,\frac{df_n(t)}{t^{2n}}$$

$$+\frac{2}{\pi^{3/2}}\int_r^\infty \frac{\rho^{2n-1}d\rho}{(\rho^2-r^2)^{1/2}}\int_\rho^\infty \frac{dx}{(x^2-\rho^2)^{1/2}(x^2-a^2)^{1/2}}\int_a^\infty q_n(\rho,x,t)(t^2-a^2)^{1/2}\frac{df_n(t)}{t^{2n-2}}, \tag{3.7.9}$$

Page 212:

$$\sigma_1(\rho)=\overline{\sigma}_{-1}(\rho)=\rho\int_\rho^\infty \frac{df_1(t)}{t^2(t^2-\rho^2)^{1/2}},$$

Page 219:

$$\int_a^\infty \frac{Y_c(x)dx}{x^3(x^2-a^2)^{1/2}}=\frac{\pi[(1/4)-\theta^2]}{a^3\cosh(\pi\theta)},$$

Page 220:

$$\int_a^\infty \frac{Y_s(x)dx}{(x^2-a^2)^{1/2}(x^2-r^2)}=-\frac{\pi\tanh(\pi\theta)}{2r(r^2-a^2)^{1/2}}\,Y_c(r),$$

Page 220:

$$\int_a^\infty \frac{Y_s(x)dx}{x^2(x^2-a^2)^{1/2}(x^2-r^2)}=-\frac{\pi}{r^2}\left[\frac{\tanh(\pi\theta)}{2r(r^2-a^2)^{1/2}}\,Y_c(r)+\frac{\theta}{a^2\cosh(\pi\theta)}\right],$$

Page 222:

Simplification of two consecutive integrals:

$$\int_\rho^a \frac{x^2\mathrm{d}x}{(x^2-\rho^2)^{1/2}}\frac{\mathrm{d}}{\mathrm{d}x}\int_x^a \frac{f(y)\mathrm{d}y}{(y^2-x^2)^{1/2}} = -\frac{\pi}{2}\left[\rho f(\rho) + \int_\rho^a f(y)\mathrm{d}y\right]$$

$$-\frac{a}{(a^2-\rho^2)^{1/2}}\lim_{r\to a}(f(r)(a^2-r^2)^{1/2});$$

Page 223:

$$\int_0^\rho \frac{\mathrm{d}x}{(\rho^2-x^2)^{(1+\kappa)/2}}\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x \frac{f(r)r\mathrm{d}r}{(x^2-r^2)^{(1-\kappa)/2}} = \frac{\pi}{2\cos(\pi\kappa/2)}\left[f(\rho)-\frac{1}{\rho}\lim_{r\to 0}[rf(r)]\right]$$

Page 225: x^{2n-2}

$$\int_x^a \frac{\mathrm{d}\rho}{\rho^{2n-1}(\rho^2-x^2)^{1/2}(\rho^2-t^2)^{1/2}} = \frac{(a^2-x^2)^{1/2}(a^2-t^2)^{1/2}}{a^2x^{2n-2}(x^2-t^2)}\sum_{m=0}^{n-1}\left(\frac{a^2-x^2}{a^2}\right)^m$$

$$\times\frac{\Gamma(n)}{\Gamma(n-m)(m!)^2}\frac{\mathrm{d}^m}{\mathrm{d}\zeta^m}\left[(1-\zeta)^{m-1/2}\frac{\sin^{-1}\sqrt{\zeta}}{\sqrt{\zeta}}\right],\text{ with } \zeta = -\frac{t^2(a^2-x^2)}{a^2(x^2-t^2)}.$$

Page 225:

$$\int_a^\infty \frac{\mathrm{d}x}{x^{2n+1}(x^2-a^2)^{1/2}(x^2-\rho^2)} = \frac{1}{a^{2n+1}\rho^{2n+2}}\int_0^\rho \frac{x^{2n+2}\,\mathrm{d}x}{(\rho^2-x^2)^{1/2}(a^2-x^2)}$$

$$= \frac{1}{a^{2n+1}\rho^{2n+2}}\left[\frac{\pi a^{2n+1}}{2(a^2-\rho^2)^{1/2}} - \frac{\sqrt{\pi}}{2}\sum_{k=0}^{n}\frac{\Gamma(k+1/2)}{\Gamma(k+1)}a^{2(n-k)}\rho^{2k}\right],$$

Page 226:

Rules of interchanging the order of integration:

$$\int_0^a F(r)\mathrm{d}r\,\frac{\mathrm{d}}{\mathrm{d}r}\int_0^r \frac{f(\rho)\rho\mathrm{d}\rho}{(r^2-\rho^2)^{1/2}} = -\int_0^a f(\rho)\mathrm{d}\rho\,\frac{\mathrm{d}}{\mathrm{d}\rho}\int_\rho^a \frac{F(r)r\mathrm{d}r}{(r^2-\rho^2)^{1/2}} + \lim_{\rho\to 0}\left\{\rho f(\rho)\frac{\mathrm{d}}{\mathrm{d}\rho}\int_\rho^a \frac{F(r)r\mathrm{d}r}{(\mathrm{rr}2-\rho^2)^{1/2}}\right\} +$$

$$\lim_{\rho\to a}\left\{f(\rho)\int_{\rho}^{a}\frac{F(r)r\,\mathrm{d}r}{(\mathrm{rr}2-\rho^2)^{1/2}}\right\},$$

Page 229:

The governing integral equation (4.1.9) can be rewritten in polar coordinates as follows (see section 2.8)

Page 231:

$$\frac{\partial F}{\partial z}=-8\pi H\int_{l_2(0)}^{l_2}\frac{\mathrm{d}x}{(x^2-\rho^2)^{1/2}}\int_0^{g(x)}\frac{\rho_0\mathrm{d}\rho_0}{[g^2(x)-\rho_0^2]^{1/2}}\mathcal{L}\left(\frac{\rho\rho_0}{x^2}\right)p(\rho_0,\phi). \tag{4.1.20}$$

Page 232:

$$\frac{\partial F}{\partial z}=-8\pi H\int_0^a\frac{\mathrm{d}l_2(t)}{[l_2^2(t)-\rho^2]^{1/2}}\int_0^t\frac{\rho_0\mathrm{d}\rho_0}{(t^2-\rho_0^2)^{1/2}}\mathcal{L}\left(\frac{\rho\rho_0}{l_2^2(t)}\right)p(\rho_0,\phi). \tag{4.1.22}$$

Page 232:

$$F(\rho,\phi,z)=-8\pi H\int_\infty^z\mathrm{d}z\int_0^a\frac{\mathrm{d}l_2(t)}{[l_2^2(t)-\rho^2]^{1/2}}\int_0^t\frac{\rho_0\mathrm{d}\rho_0}{(t^2-\rho_0^2)^{1/2}}\mathcal{L}\left(\frac{\rho\rho_0}{l_2^2(t)}\right)p(\rho_0,\phi). \tag{4.1.23}$$

Page 232:

$$F(\rho,\phi,z)=8\pi H\int_0^a\mathrm{d}t\int_0^t\frac{\rho_0\mathrm{d}\rho_0}{(t^2-\rho_0^2)^{1/2}}\int_0^{l_1(t)}\frac{\mathrm{d}y}{(\rho^2-y^2)^{1/2}}\mathcal{L}\left(\frac{y^2\rho_0}{t^2\rho}\right)p(\rho_0,\phi). \tag{4.1.24}$$

Page 232:

$$F(\rho,\phi,z) = 8\pi H\int_0^a t\mathrm{d}t \int_0^t \frac{\rho_0\mathrm{d}\rho_0}{(t^2-\rho_0^2)^{1/2}}\int_{l_2(t)}^{\infty} \frac{\mathrm{d}x}{x(x^2-t^2)^{1/2}}\,\mathcal{L}\left(\frac{\rho\rho_0}{x^2}\right)p(\rho_0,\phi). \tag{4.1.25}$$

Page 251:

$$k_2 + ik_3 = \frac{(a^2-\rho_0^2)^{1/2}}{\pi^2(2a)^{1/2}}\left[\frac{T\mathrm{e}^{-i\phi}}{\rho_0^2 + a^2 - 2a\rho_0\cos(\phi-\phi_0)}\right.$$

$$\left.+\frac{G_2}{G_1}\frac{\mathrm{e}^{i\phi}(3a-\rho_0\mathrm{e}^{i(\phi-\phi_0)})\overline{T}}{a(a-\rho_0\mathrm{e}^{i(\phi-\phi_0)})^2}\right]. \tag{4.4.58}$$

Page 251:

$$k_2 + ik_3 = \int_0^{2\pi}\int_0^a \frac{(a^2-\rho_0^2)^{1/2}}{\pi^2(2a)^{1/2}}\left[\frac{\mathrm{e}^{-i\phi}\tau(\rho_0,\phi_0)}{\rho_0^2 + a^2 - 2a\rho_0\cos(\phi-\phi_0)}\right.$$

$$\left.+\frac{G_2}{G_1}\frac{(3a-\rho_0\mathrm{e}^{i(\phi-\phi_0)})\,\mathrm{e}^{i\phi}\,\overline{\tau}(\rho_0,\phi_0)}{a(a-\rho_0\mathrm{e}^{i(\phi-\phi_0)})^2}\right]\rho_0\mathrm{d}\rho_0\mathrm{d}\phi_0, \tag{4.4.59}$$

Page 258:

$$w = \frac{G_1^2 - G_2^2}{2G_1}(\overline{\tau}\rho\mathrm{e}^{i\phi} + \tau\rho\mathrm{e}^{-i\phi})\sum_{\mathrm{k}=1}^{2}\frac{m_\mathrm{k}}{m_\mathrm{k}-1}\left[\sin^{-1}\left(\frac{a}{l_{2\mathrm{k}}}\right) - \frac{a(l_{2\mathrm{k}}^2-a^2)^{1/2}}{l_{2\mathrm{k}}^2}\right]\frac{1}{\gamma_\mathrm{k}}, \tag{4.6.3}$$

Page 263:

$$Q_\mathrm{k} = [\cos^2\theta + (1/\gamma_\mathrm{k}^2)\sin^2\theta]^{1/2} \qquad S_\mathrm{k} = [Q_\mathrm{k} - \cos\theta]^{1/2},$$

$$T_\mathrm{k} = [Q_\mathrm{k} + \cos\theta]^{1/2}, \quad \text{for } k = 1,2,3. \tag{4.7.2}$$

Page 330:

Since the integration was indefinite, we might have lost a function of the variables, other than z. This function can be found from the condition that the result of integration should not have a logarithmic singularity at ρ=0 or at q=0. The functions eliminating such a singularity are $\tan^{-1}[(\zeta - 1)^{1/2}]$ and $\tan^{-1}[(\bar{\zeta} - 1)^{1/2}]$. The final result can now be represented in the form

$$\int \frac{z}{R_0^3}\left[\frac{R_0}{h} + \tan^{-1}\left(\frac{h}{R_0}\right)\right]dz$$

$$= -\frac{1}{R_0}\tan^{-1}\left(\frac{h}{R_0}\right) + \frac{1}{(a^2-\rho_0^2)^{1/2}}\left\{\ln[l_2 + (l_2^2-\rho^2)^{1/2}]\right.$$

$$\left. -2\Re\left(\frac{1}{(\zeta-1)^{1/2}}\left[\tan^{-1}\left(\frac{a(\zeta-1)^{1/2}}{(a^2-l_1^2)^{1/2}}\right) - \tan^{-1}(\bar{\zeta}-1)^{1/2}\right]\right)\right\}, \quad \zeta = \frac{\rho}{\rho_0}e^{i(\phi-\phi_0)}$$

The last expression proves the correctness of formula (5.1.13).

Page 334:

$$\Lambda^3\ln[R(M,N)+z] = (\rho e^{i\phi} - re^{i\psi})^3\,\frac{8R^2(M,N) + 9R(M,N)z + 3z^2}{R^5(M,N)\,[R(M,N)+z]^3}\,. \tag{A4.4.12}$$

Page 335:

The following indefinite integrals were used here

$$\int \tan^{-1}\left(\frac{\bar{s}}{(l_2^2-a^2)^{1/2}}\right)dz = z\tan^{-1}\left(\frac{\bar{s}}{(l_2^2-a^2)^{1/2}}\right)$$

$$+\bar{s}\left[\ln[l_2 + (l_2^2-\rho^2)^{1/2}] + (\zeta-1)^{1/2}\tan^{-1}\left(\frac{a(\zeta-1)^{1/2}}{(a^2-l_1^2)^{1/2}}\right)\right],$$

Page 340:

$$\mathcal{K}(\rho,\phi,z;\rho_0,\phi_0) = -\frac{1}{R_0}\tan^{-1}\left(\frac{h}{R_0}\right) + \frac{1}{(a^2-\rho_0^2)^{1/2}}\left\{\ln[l_2 + (l_2^2-\rho^2)^{1/2}]\right.$$

$$-2\Re\left(\frac{1}{(\zeta-1)^{1/2}}\left[\tan^{-1}\left(\frac{a(\zeta-1)^{1/2}}{(a^2-l_1^2)^{1/2}}\right)-\tan^{-1}(\zeta-1)^{1/2}\right]\right)\Bigg\},\quad \zeta=\frac{\rho}{\rho_0}e^{i(\phi-\phi_0)} \tag{5.1.13}$$

Page 340:

$$\Lambda\mathcal{K}=\frac{q}{R_0^3}\left[\frac{R_0}{h}+\tan^{-1}\left(\frac{h}{R_0}\right)\right]+\frac{1}{h\bar{q}}\left\{1-\frac{(a^2-l_1^2)^{1/2}}{a}\right.$$
$$\left.-\frac{(a^2-l_1^2)^{1/2}}{a(\bar{\zeta}-1)^{1/2}}\left[\tan^{-1}\left(\frac{a(\bar{\zeta}-1)^{1/2}}{(a^2-l_1^2)^{1/2}}\right)-\tan^{-1}(\bar{\zeta}-1)^{1/2}\right]\right\}, \tag{5.1.15}$$

Pages 340-341:

$$\Lambda^2\mathcal{K}=-\frac{3q^2}{R_0^5}\left[\frac{R_0}{h}+\tan^{-1}\left(\frac{h}{R_0}\right)\right]+\frac{1}{h(R_0^2+h^2)}\left[\frac{q^2}{R_0^2}-\frac{l_2^2-\rho^2}{l_2^2-l_1^2}e^{2i\phi}\right]$$
$$+\frac{3}{\bar{q}^2}\left\{\frac{1}{h}-\frac{1}{(a^2-\rho_0^2)^{1/2}}\left(1+\frac{1}{(\bar{\zeta}-1)^{1/2}}\left[\tan^{-1}\left(\frac{a(\bar{\zeta}-1)^{1/2}}{(a^2-l_1^2)^{1/2}}\right)-\tan^{-1}(\bar{\zeta}-1)^{1/2}\right]\right)\right\}$$
$$+\frac{e^{i\phi}}{\rho\bar{q}(a^2-\rho_0^2)^{1/2}}\left[1-\frac{a^3\bar{\zeta}}{(a^2-l_1^2)^{1/2}(a^2\bar{\zeta}-l_1^2)}\right]. \tag{5.1.18}$$

Page 349:

$$\int_0^\infty \sin ax\, J_1(\rho x)e^{-zx}\frac{dx}{x^2}=\frac{(2a^2-l_1^2)(l_2^2-a^2)^{1/2}-2a^2z}{2a\rho}+\frac{\rho}{2}\sin^{-1}\left(\frac{a}{l_2}\right). \tag{5.3.16}$$

Page 435:

$$u_1(x_1,y_1)=\frac{G_1}{2}\Delta\int\int_{S_1} R_{11}\tau_1(\xi,\eta)\,d\xi d\eta-\frac{G_2}{2}\Lambda^2\int\int_{S_1} R_{11}\bar{\tau}_1(\xi,\eta)\,d\xi d\eta$$
$$+\sum_{n=2}^{N}\left\{\frac{G_1}{2}\Delta\int\int_{S_n} R_{1n}\tau_n(\xi,\eta)\,d\xi d\eta-\frac{G_2}{2}\Lambda^2\int\int_{S_n} R_{1n}\bar{\tau}_n(\xi,\eta)\,d\xi d\eta\right\}.$$

(5.10.6)