Block-Krylov Component Synthesis and Minimum Rank Perturbation Theory for Damage Detection in Complex Structures

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Abstract *This paper explores the use of block-Krylov component synthesis (BKCS) and minimum rank perturbation theory (MRPT) as a structural damage detection tool. BKCS is a simple and accurate method for generating a set of component Ritz vectors to use in component synthesis. Ritz vectors are obtained from derived recurrence relations, which represent a suitable basis vectors as an alternative to component normal vibration modes. The MRPT is a model-based damage detection method, which utilizes the fact that discrete damage is manifested in a structural finite element model as a low rank perturbation to the structural property matrices. In this paper, the BKCS method is used along with an analytical finite element beam model subdivided in components. With the coupled healthy model and the measured modal test modes, the MRPT is used to detect the location and extent of the damage or at least the component that contains the damage. Effects of various substructure modeling assumptions on damage detectability are explored using numerical studies.*

Keywords: *Ritz Vectors, block-Krylov , Damage Detection ,Modal Synthesis, Complex Structures.*

Introduction

Most complex structural systems as aircraft, spacecraft, bridges, and offshore platforms are exposed to damage in their service environment. In order to assure safety, it is desirable that this structures be monitored to detect the damage occurrence, its location, and extent. The response of a complex structure to dynamic excitation is usually obtained by finite element model (FEM), which can be modeled with thousands of degree of freedom (DOF), and may have many components (substructures) that are often designed and produced by different organizations. Thus, model complexity demands a component synthesis method to obtain a reduced system representation.

Component mode synthesis (CMS) consist in to modeling individual components of a structure separately, and then to couple them to form an assembled system. The CMS methods can be categorized according to the treatment of interface connection (Hurty, 1965). Each component is modeled by a set of vectors containing some types of component normal modes as subset, augmented with other vectors such as *constrained modes* in the fixed-interface methodology (Craig and Bampton, 1968) or *attachment modes* in the free-interface methodology (Craig and Chang, 1977). However, the CMS can be generalized to permit the use of other shape vectors rather than normal modes. Developed by Craig and Hale, 1988, Block-Krylov Component Synthesis (BKCS) is a method that uses this concept, which is applied in this work with the fixed interface methodology. The method is based on a block-Krylov subspace, which is obtained by simple recurrence relations representing a suitable basis vector as an alternative to component normal modes. Repeated application of the recurrence relations generates a set of component vectors. The subspace spanned by the Ritz vectors is a block-Krylov subspace. Load dependent Ritz vectors have been used extensively in the analytical and numerical areas of model reduction, eigensystem computation, transient response prediction, including CMS (Nour-Omid and Clough, 1984; Craig *et al*, 1988). In recent work, Zimmerman, 1999, have shown some advantages in use Ritz vectors instead mode shapes along with Minimum Rank Perturbation Theory (MRPT) to detect damage in structural systems. The MRPT (Zimmerman and Kaouk, 1994) is an optimal matrix update method based on the modal force vector criteria (Ojavo and Pilon, 1988), which provide an enhancement to this criteria by reducing the effect of measurement errors.

The propose of this work is to develop an algorithm to solve the damage detection problem based on the original n DOFs and m_a block-Krylov component vectors obtained from derived recurrence relations, and the m_e measured modal test modes from the damaged structure. The block-Krylov component vectors from each substructure are used to form an assembled system model. With this model and the measured damaged modes, damage detection algorithm uses the MRPT technique to provide a precise indication and extent of damage, or, at least the component that contain the damage.

Block-Krylov Component Synthesis Method

The equation of motion of a typical undamped component a can be written as

$$\mathbf{M}_a \ddot{\mathbf{u}}_a + \mathbf{K}_a \mathbf{u}_a = \mathbf{f}_a \tag{1}$$

where $\mathbf{M}_a$ and $\mathbf{K}_a$ are the physical mass and stiffness matrices, respectively, $\mathbf{u}_a$ is the displacement vector, and $\mathbf{f}_a$ is the force vector. We can also write the equation (1) in a partitioned form as

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ij} \\ \mathbf{M}_{ji} & \mathbf{M}_{jj} \end{bmatrix}_a \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_j \end{Bmatrix}_a + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ij} \\ \mathbf{K}_{ji} & \mathbf{K}_{jj} \end{bmatrix}_a \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{Bmatrix}_a = \begin{Bmatrix} \mathbf{f}_i \\ \mathbf{f}_j \end{Bmatrix}_a \tag{2}$$

where the subscripts i and j denotes partitions corresponding to N_i and N_j substructure DOFs at the interior and junction, respectively.

In component modal synthesis, the component displacement coordinates $\mathbf{u}_a$ are related to a reduced set of generalized coordinates $\mathbf{p}_a$ by the following Ritz transformation

$$\mathbf{u}_a = \mathbf{Ø}_a \mathbf{p}_a \tag{3}$$

where $\mathbf{Y}_a$ is a matrix whose columns are Ritz vectors, or so-called component modes. Generally, the types of modes that make up $\mathbf{Y}_a$ are: constraint modes, attachment modes and normal modes.

The BKCS with the fixed interface methodology uses the constraint modes, and the component normal modes are replaced by a set of Krylov vectors. A constraint mode is defined as the static deflection of a structure when a unit displacement is applied to one coordinate of a specific set of coordinates, while the remaining coordinates of that set are restrained and the remaining DOFs of the structure are force free. These constraint modes $\mathbf{Y}_c$ are obtained by solving the equation

$$\begin{bmatrix}\mathbf{K}_{ii} & \mathbf{K}_{ij}\\ \mathbf{K}_{ji} & \mathbf{K}_{jj}\end{bmatrix}\begin{Bmatrix}\mathbf{Ø}_{ic}\\ \mathbf{I}_{cc}\end{Bmatrix} = \begin{Bmatrix}\mathbf{0}_{ic}\\ \mathbf{R}_{cc}\end{Bmatrix} \tag{4}$$

yielding

$$\mathbf{Ø}_c = \begin{bmatrix}\mathbf{Ø}_{ic}\\ \mathbf{I}_{cc}\end{bmatrix} = \begin{Bmatrix}-\mathbf{K}_{ii}^{-1}\ \mathbf{K}_{ij}\\ \mathbf{I}_{cc}\end{Bmatrix} \tag{5}$$

The BKCS is suggested by recurrence relations, which can be derived from the component dynamic equation

$$\left(\mathbf{K}_a - w^2\mathbf{M}_a\right)\mathbf{ö}_a = \mathbf{g}_a \tag{6}$$

where, w^2 is an eigenvalue of the assembled system, $\mathbf{j}_a$ is the portion of the system eigenvector within the component, and $\mathbf{g}_a$ constitutes boundary forces imposed on the component by adjacent components.

Let the equation (6) be written in partitioned form

$$\begin{bmatrix}\mathbf{K}_{ii} & \mathbf{K}_{ij}\\ \mathbf{K}_{ji} & \mathbf{K}_{jj}\end{bmatrix}\begin{Bmatrix}\mathbf{ö}_i\\ \mathbf{ö}_j\end{Bmatrix} = w^2\begin{bmatrix}\mathbf{M}_{ii} & \mathbf{M}_{ij}\\ \mathbf{M}_{ji} & \mathbf{M}_{jj}\end{bmatrix}\begin{Bmatrix}\mathbf{ö}_i\\ \mathbf{ö}_j\end{Bmatrix} + \begin{Bmatrix}\mathbf{0}\\ \mathbf{g}_i\end{Bmatrix} \tag{7}$$

The top portion can be solved for $\mathbf{j}_i$ and the result combined with an identity for $\mathbf{j}_j$ to give

$$\mathbf{ö} \equiv \begin{Bmatrix}\mathbf{ö}_i\\ \mathbf{ö}_j\end{Bmatrix} = \begin{bmatrix}-\mathbf{K}_{ii}^{-1}\mathbf{K}_{ij}\\ \mathbf{I}\end{bmatrix}\left\{\mathbf{ö}_j\right\} + w^2\begin{bmatrix}\mathbf{K}_{ii}^{-1}\mathbf{M}_{ii} & \mathbf{K}_{ii}^{-1}\mathbf{M}_{ij}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\mathbf{ö}_i\\ \mathbf{ö}_j\end{Bmatrix} \tag{8}$$

or in abbreviated form the following *fixed interface recurrence formula* may be defined

$$\mathbf{ö}^{(k)} = \mathbf{Ø}_c\mathbf{ö}_j^{(k)} + w^2\mathbf{G}_c\mathbf{ö}^{(k-1)} \qquad k = 1,2,\ldots \tag{9}$$

where

$$\mathbf{G}_c = \begin{bmatrix}\mathbf{K}_{ii}^{-1}\mathbf{M}_{ii} & \mathbf{K}_{ii}^{-1}\mathbf{M}_{ij}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} \tag{10}$$

Starting with $\mathbf{j}^{(0)} = 0$, apply equation (9) repeatedly to obtain

$$\mathbf{ö}^{(k)} = \mathbf{Ø}_c\mathbf{ö}_j^{(k)} + \sum_{n=1}^{k-1} w^{2n}\mathbf{G}_c^{n}\mathbf{Ø}_c\mathbf{ö}^{(k-1)} \tag{11}$$

Equation (11) shows that $\mathbf{j}$ is a combination of the vectors in $\mathbf{Y}_c$, $\mathbf{Y}_c\mathbf{G}_c$, $\mathbf{Y}_c\mathbf{G}_c^2$, etc. Thus, a component *fixed interface block-Krylov subspace* will be defined by

$$\mathbf{Ø}_c^k \equiv \left[\mathbf{Ø}_c, \mathbf{Ø}_c^{(1)}, \mathbf{Ø}_c^{(2)}, \ldots, \mathbf{Ø}_c^{(k-1)}\right] \tag{12}$$

where $\mathbf{Y}_c$ is given by equation (5), and where

$$\mathbf{Ø}_c^{(r)} = \begin{bmatrix}\mathbf{Ø}_{ic}^{(r)}\\ \mathbf{0}\end{bmatrix} = \mathbf{G}_c^{(r)}\mathbf{Ø}_c \tag{13}$$

Then,

$$\mathbf{Ø}_c^{(1)} = \begin{bmatrix}\mathbf{Ø}_{ic}^{(1)}\\ \mathbf{0}\end{bmatrix} = \mathbf{G}_c\mathbf{Ø}_c = \begin{bmatrix}\mathbf{K}_{ii}\left(\mathbf{M}_{ii}\mathbf{Ø}_{ic} + \mathbf{M}_{ij}\right)\\ \mathbf{0}\end{bmatrix} \tag{14}$$

and subsequently,

$$\mathbf{Ø}_c^{(r)} = \begin{bmatrix} \mathbf{K}_{ii}^{-1}\mathbf{M}_{ii}\mathbf{Ø}_{ic}^{(r-1)} \\ \mathbf{0} \end{bmatrix} \qquad r = 2,3,\ldots \tag{15}$$

The deformation modes that characterize $\mathbf{Y}_c^{(r)}$ have fully restrained boundary freedoms, and the interior displacement is the static deflection due to inertia loading associated with the preceding deflection shapes of $\mathbf{Y}_c^{(r-1)}$. The vectors are orthogonalized at each step by the Gram-Schmidt orthogonalization and normalized with the mass matrix. A detailed development of this method can be found in Craig and Hale (1988).

Minimum Rank Perturbation Theory and Subspace Selection Algorithm

In this section, a brief overview of minimum rank perturbation for undamped structures is presented in which the mass matrix is assumed correct. A detailed development can be found in Zimmerman and Kaouk (1994). The problem to be solved is to determine a minimum rank perturbation matrix to the stiffness matrix such that the measured and analytical modal properties are in agreement. The eigenproblem for the structure with p measured eigenvalues can be written as

$$\left(\mathbf{K} - w_{di}^2\mathbf{M}\right)f_{di} = \Delta\mathbf{K}_d\mathbf{ö}_{di} \equiv \mathbf{b}_i \qquad i = 1,2,\ldots,p \tag{16}$$

The matrix $\Delta\mathbf{K}_d$ is the stiffness loss to be determined, $\mathbf{j}_{di}$ is the measured mode shape, w_{di}^2 is the corresponding natural frequency, And $\mathbf{b}_i$ is the dynamic residual vector. Equation (16) can be written in matrix form as

$$\mathbf{KÖ}_d - \mathbf{MÖ}_d\mathbf{Ë}_d = \Delta\mathbf{K}_d\mathbf{Ö}_d \equiv \mathbf{B} \tag{17}$$

It should be noted that $\mathbf{B}$ can be calculated from equation (17) in terms of the measured modal properties and the original synthetized FEM matrices. The matrix $\mathbf{B}$ is the dynamic modal residual matrix. By applying the MRPT concept in equation (17) $\Delta\mathbf{K}_d$ can be written as

$$\Delta\mathbf{K}_d = \mathbf{B}\left(\mathbf{B}^{\mathrm{T}}\mathbf{Ö}_d\right)^{-1}\mathbf{B}^{\mathrm{T}} \tag{18}$$

Due to errors present in the measured modal data, a perfect zero/nonzero pattern of the dynamic residual $\mathbf{b}_i$ rarely occurs in practice. In this case the dynamic residual matrix $\mathbf{B}$ may lead to incorrect conclusions about the location of damage. To avoid this problem we can define $\mathbf{Z}_{di} = \mathbf{K} - w_{di}^2\mathbf{M}$ and write equation (16) as

$$b_i^j \equiv \mathbf{z}_{di}^j = \left\|\mathbf{z}_{di}^j\right\|\left\|\mathbf{ö}_{di}\right\|\cos\left(è_i^j\right) \tag{19}$$

where b_i^j is the jth DOF of the ith damage vector, $\mathbf{z}_{di}^j$ is the jth row of the matrix $\mathbf{Z}_{di}$ and $è_i^j$ is the angle between the vectors $\mathbf{z}_{di}^j$ and $\mathbf{ö}_{di}^j$. A better indication of damage can be done through the deviation of the angle from ninety degrees,

$$á_i^j = è_i^j\left(\frac{180^o}{p}\right) - 90^o \tag{20}$$

where $è_i^j$ is obtained from equation (19), $á_i^j$ is the jth component of angle residual vector $\mathbf{á}_i$, and the damaged DOFs are the components substantially different than zero degrees. In analogous form to matrix $\mathbf{B}$, it is possible to assembly a matrix $\mathbf{A}$ (*angle residual matrix*) of the p measured eigenparameters.

Consider the MRPT perturbation matrix constraint equation The subspace selection algorithm (Zimmerman and Simmermacher, 1994) can be defined as a search for a matrix $\mathbf{X} \in R^{pxt}$ such that,

$$\Delta\mathbf{KÖ}_d\mathbf{X} = \mathbf{BX} \tag{21}$$

is well conditioned. The unknowns are t, the numerical rank of $\mathbf{B}$, and $\mathbf{X}$. Consider the singular value decomposition (SVD) of $\mathbf{B}$,

$$\mathbf{B} = \left[\mathbf{U}_1 \quad \mathbf{U}_2\right]\begin{bmatrix} \mathbf{Ó} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\left[\mathbf{V}_1 \quad \mathbf{V}_2\right]^{\mathrm{T}} \tag{22}$$

where $\Sigma \in R^{txt}$ is the matrix of non-zero (or grater than some prescribed tolerance) singular values, and $\mathbf{U}$ and $\mathbf{V}$ are the left and right singular vectors partitioned conformable. When $\mathbf{B}$ is rank deficient, its range is spanned by the t columns of $\mathbf{U}_1$. Thus, it is desired to find a $\mathbf{X}$ such that,

$$\mathbf{BX} = \mathbf{U}_1 \tag{23}$$

The matrix $\mathbf{X}$ can be approximated by using the pseudo-inverse of $\mathbf{B}$ as

$$\mathbf{X} = \mathbf{B}^+\mathbf{U}_1 = \mathbf{V}_1\mathbf{Ó}^+\mathbf{U}_1^T\mathbf{U}_1 = \mathbf{V}_1\mathbf{Ó}^+ \tag{24}$$

Then, the solution of equation (21) is given by,

$$\Delta\mathbf{K} = \mathbf{BX}\left(\mathbf{X}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{Ö}_d\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} \tag{25}$$

The columns of $\mathbf{U}_1$ represent the principle components of the dynamic residual matrix **B**, which are termed *principle component damage vectors*. An important improvement of this calculation is the noise reduction due both measurements and substructure modeling errors.

Simulation Examples

Figure 1 shows the same structure used by Dos Santos and Zimmerman, 1996. It is a 2-dimensional clamped-clamped beam with 18 nodes, 3 DOFs/nodes and was artificially divided into 5 substructures. The damage is characterized by a 1/4 reduction in the original finite element inertia moment.

To simulate the realistic case of noisy measurements, the eigenvectors were corrupted with a random noise of the form

$$\hat{f}_{ij} = f_{ij} + f_{ij}(a/100)rand[-1,1] + rms(f_j)(b/100)rand[-1,1] \tag{26}$$

where, $\hat{f}_{ij}$ is the noise corrupted shape vector element, f_{ij} is the *i*th component of the *j*th shape vector, a and b are the scale and bias noise factors respectively, *rms*(-) is the root mean square operator, and *rand*[-1,1] is a random number generator uniformly distributed between -1 and 1. It is assumed that only the first five modes of vibration are "measured".

Figure 2 shows the exactly damage location (element 10 damage, DOFs 25-30) using angle residual approach without and with corrupted noise eigenvectors with 5% scale and 1% bias noise. It can be observed that the angle residual gives a strong indication to the location of damage in both cases.

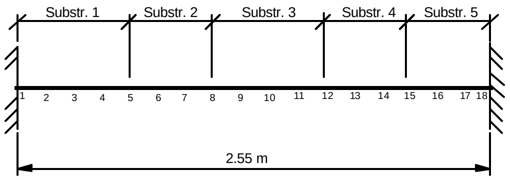

Figure 1 - Simulated example

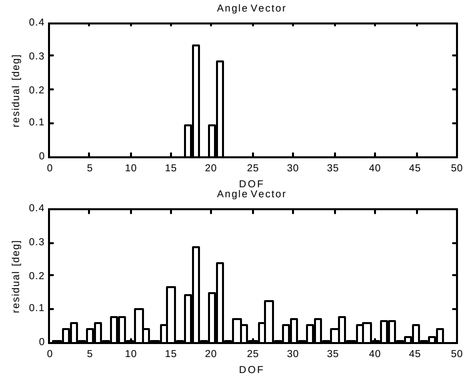

Figure 2 – Damage Localization without and with noise

Six different sets of block-Krylov truncation (A, B, C, D, E and F) were used (figure 3) in the damage location of the element 7 (DOFs 16-21, substructure S2), and all of them using corrupted noise eigenvectors with 5% scale and 1% bias noise. Table 1 shows the quantity of block-Krylov vectors used at each set. It can be observed that the quantity of Ritz vectors in each block-Krylov corresponds to the number of DOFs in each component junctions. As in sets C to F, it is possible to obtain sets of component block-Krylovs with a total number of vectors greater than the FEM component order. Obviously, it is a trade-off solution which depends on the size of each FEM component and computational resources available. Also, there is a maximum number of block-Krylovs that can be generated from which no benefits will be taken (see figure 3, sets D and F), and this number is case dependent. By observing the graphics in figure 3, one can see that all of them give an indication to the component that contains the damaged element. To quantify this indication, we use the *component peak factor* (CPF), which is defined as (Dos Santos and Zimmerman, 1996),

$$CPF = \frac{1}{n_i}\left(\sum_{i=1,\, j=1}^{k,n_i} U_1(i,j)\right) \tag{27}$$

where $U_1(i,j)$ is obtained through the $j = 1,2,...,k$ left singular vectors at the $i = 1,2,...,n_i$ substructure DOFs.

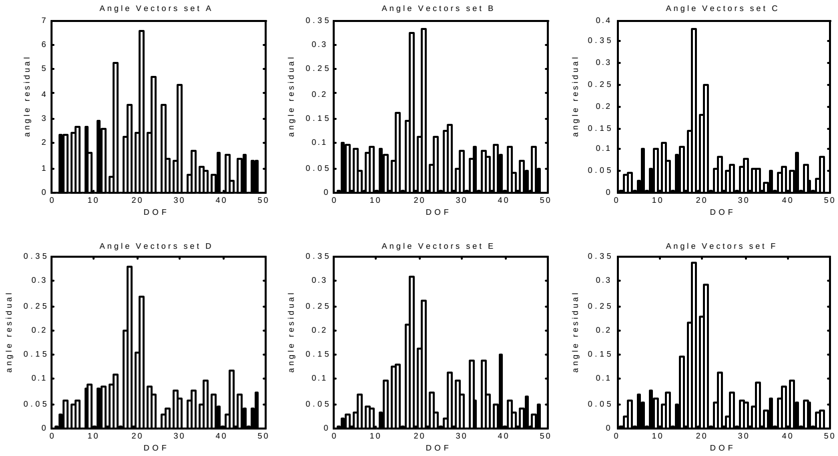

Figure 3 – Mode Set A to F with 5% scale and 0.1 % bias noise factor

Table 2 contain the CPF results for each case in figure 3, where it indicates substructure S2 in all the cases, as would be expected.

Table 1 – Component Modes Sets

Set	Substructure				
	S1	S2	S3	S4	S5
A	1	1	1	1	1
B	2	2	2	2	2
C	3	3	3	3	3
D	4	4	4	4	4
E	3	2	1	4	3
F	10	10	10	10	10

Table 2 – Component Peak Factors

Set	Substructure				
	S1	S2	S3	S4	S5
A	1.6326	2.1831	1.9425	0.7268	0.8417
B	0.0570	0.1093	0.0789	0.0528	0.0427
C	0.0473	0.1114	0.0626	0.0367	0.0393
D	0.0442	0.1111	0.0621	0.0457	0.0421
E	0.0312	0.1113	0.0690	0.0578	0.0315
F	0.0396	0.1165	0.0692	0.0451	0.0373

Figures 4 to 9 show different cases of damage detection (damaged element 2, DOFs 1-6, substructure S1) using different component blocks of vectors truncation sets, different noise factors and with principle component analysis using both the first and second angle residual vector. For each set and noise factor, 100 simulations runs were made. The difference in each run was the random noise generated in each measurement. These results show the mean and standard deviation of each component of the angle residual vector. In the top of these graphs are indicated the DOFs that belong to the interfaces (+++) and the substructures. It can be observed that in all of these simulations, the angle residual algorithm still can localize the damage to the correct substructure even using few block-Krylov vectors and relative high levels of noise.

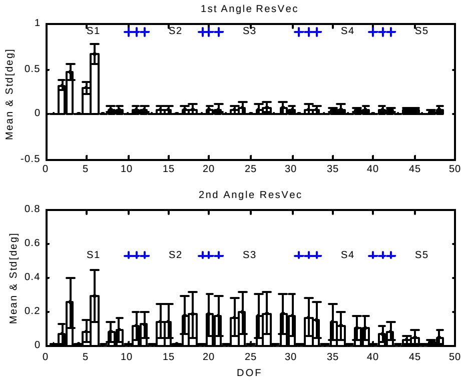

Figure 4 – Mode Set D with 5% scale and 0.1 % bias noise factor

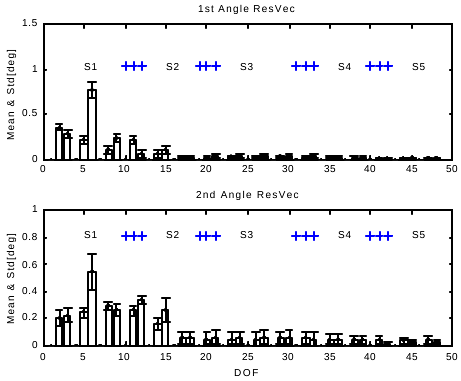

Figure 5 – Mode Set B with 5% scale and 0.1 % bias noise factor

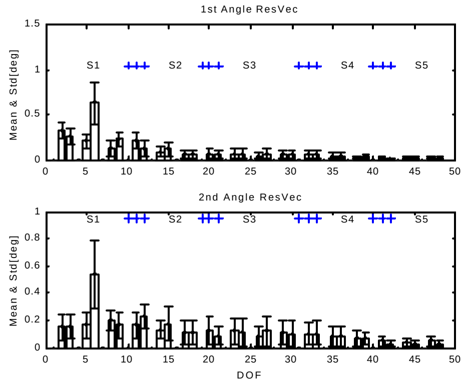

Figure 6 – Mode Set B with 10% scale and 0.1 % bias noise factor

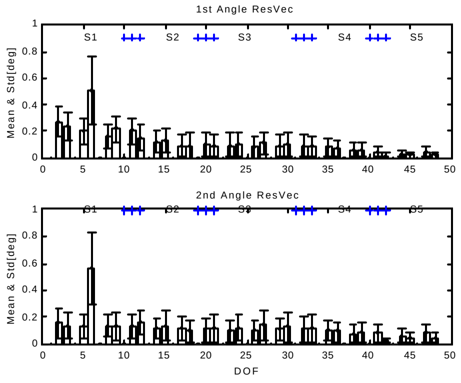

Figure 7 – Mode Set B with 15% scale and 0.1 % bias noise factor

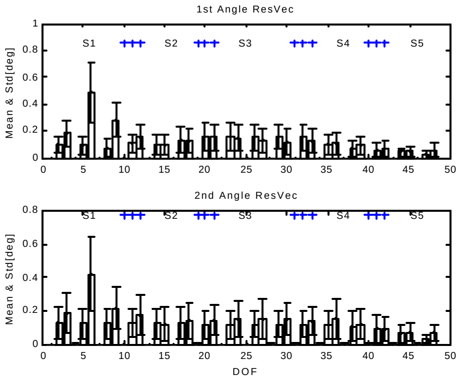

Figure 8 – Mode Set E with 10% scale and 0.1 % bias noise factor

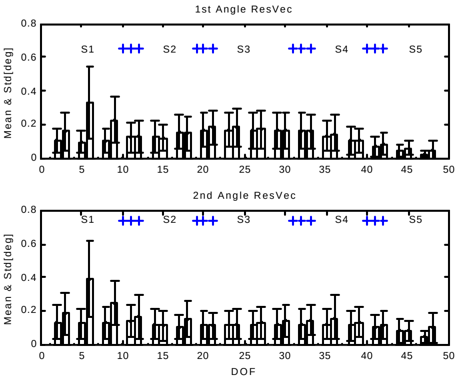

Figure 9 – Mode Set E with 15% scale and 0.1 % bias noise factor

Conclusions

In this work an approach for structural damage detection using a substructure approach and residual modal force vector techniques is presented. The propose procedure was verified using simulated measurements with modal parameters obtained from a FEM of the assembled substructures. A correlated analytical FEM obtained with the BKCS method is used to locate damage regions by using MRPT technique. The results obtained here are similar to that obtained by Dos Santos and Zimmerman, 1996. The main difference is the use of Ritz vectors instead of normal modes, which results in some improvements to the damage detection algorithm:

(i) Ritz vectors are generated from easily derived recurrence relations, which are computationally less expensive to compute then eigenvectors and already includes the static correction term (Zimmerman, 1999);

(ii) Ritz vectors provides a more realistic mathematical basis for the transformation which is used in the reduction of a large structural model to a system with small number of coordinates (Wilson, et al., 1982);

(iii) Numerical examples suggest that only a few blocks of vectors should suffice to give good accuracy, but the question of how many blocks to include still requires further research.

Must be recognized that this investigation represents only a limited test of this concept. Further studies are currently underway to apply the method to a more realistic and complex substructured systems as well as experimental studies. The results presented indicate that the method is promising.

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