Time-Shift-Keyed Equicorrelated Signal Sets for Impulse Radio M-ary Modulation

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ABSTRACT— **In this paper we study M-ary time-shift-keyed signal sets constructed using impulse technology. For the equicorrelated signals case the construction method is given, the correlation properties are discussed, the performance in additive white Gaussian noise is analyzed, and receiver simplification is illustrated.**

I. Introduction

THE technology for receiving and generating subnanosecond impulses controlling their relative position in the time axis with great accuracy is now available [1]. Impulse radio modulation is a spread spectrum technique that uses impulse technology to generate ultra-wideband (UWB) communication signals[1] that consists of trains of time-shifted impulses [2]. The UWB nature of impulse radio makes it convenient for communications over the indoor wireless channel. With UWB signals the dense multipath (produced by signals arriving at the receiver with different time delays that can be as small as fractions of nanoseconds [3]) can be resolved, allowing the use of a Rake receiver [4] for signal demodulation. With UWB signals the impulse radio link can be operated with reduced fading margin and the signals can be received by correlation detection literally at the antenna terminals, making a relatively simple and low-cost, low-power transceiver viable [2].

The use of M-ary (block-waveform) TSK signals is attractive. In one-user link it allows to increase the data transmission rate, making efficient use of the available signal-to-noise-ratio. In a multiple-access environment, the use of M-ary signals allows to increase the number of users supported by the system for a given multiple access performance and bit transmission rate.

In this paper we study M-ary TSK equicorrelated signal sets. In the next sections the construction method is given, the correlation properties are discussed, the performance in additive white Gaussian noise (AWGN) is analyzed, and receiver simplification is illustrated.

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[1] The range of frequencies occupied by the UWB signals goes from a few hundreds of Kilohertz up to a few Gigahertz.

II. M-ary TSK signal description

The signal $p(t)$ is the basic subnanosecond impulse used to convey information. It has duration T_p and energy $E_p = \int_{-\infty}^{\infty} [p(t)]^2 \, dt$. The normalized signal correlation function of $p(t)$ is

$$\gamma_p(\tau) \triangleq \frac{1}{E_p} \int_{-\infty}^{\infty} p(t)p(t-\tau)dt > -1 \, \forall \tau.$$

The M-ary TSK signals studied in this paper consists of N_s time-shifted impulses

$$\Psi_j(t) = \sum_{k=0}^{N_s-1} p(t - kT_f - \rho_j^k), \quad j = 1, 2, \ldots, M.$$

Here $\Psi_j(t)$ represents the j^{th} signal in an ensemble of M signals, each signal completely identified by the sequence of time shifts $\{\rho_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$. The complete ensemble of signals $\{\Psi_j(t)\}$ can be represented by the $M \times N_s$ matrix

$$\Delta = \begin{bmatrix} \rho_1^1 & \rho_1^2 & \ldots & \rho_1^k & \ldots & \rho_1^{N_s} \\ \rho_2^1 & \rho_2^2 & \ldots & \rho_2^k & \ldots & \rho_2^{N_s} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \rho_j^1 & \rho_j^2 & \ldots & \rho_j^m & \ldots & \rho_j^{N_s} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_M^1 & \rho_M^2 & \ldots & \rho_M^k & \ldots & \rho_M^{N_s} \end{bmatrix},$$

where each row corresponds to the time shifts $\{\rho_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$ defining the j^{th} signal. Each signal $\Psi_j(t)$ has duration $T_\Psi = N_s T_f$ and energy $E_\Psi = \int_{-\infty}^{\infty} [\Psi_j(t)]^2 \, dt$. In impulse radio, the impulse duration satisfies $T_p << T_f$, where T_f is the time shift value corresponding to the frame period; and the time shift corresponding to the data modulation is $\rho_j^k \in \{\tau_1 < \tau_2 < \ldots < \tau_{\text{N}}\}$, with $\tau_{\text{N}} < T_f - T_p$ to avoid shifted impulses to overlap between different frames. The complete set of normalized correlation values

$$\begin{aligned} \beta_{ij} &\triangleq \frac{1}{E_\Psi} \int_{-\infty}^{\infty} \Psi_i(t) \Psi_j(t) dt \\ &= \frac{1}{N_s} \sum_{k=0}^{N_s-1} \gamma_p(\rho_i^k - \rho_j^k), \end{aligned}$$

is given by the $M \times M$ symmetric non negative definite correlation matrix

$$, \triangleq \begin{bmatrix} 1 & \beta_{21} & \ldots & \beta_{M1} \\ \beta_{21} & 1 & \ldots & \beta_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{M1} & \beta_{M2} & \ldots & 1 \end{bmatrix}.$$

The signals studied in this paper are equicorrelated (EC), i.e., the matrix of correlation values is

$$,_{\text{EC}} = \begin{bmatrix} 1 & \lambda & \ldots & \lambda \\ \lambda & 1 & \ldots & \lambda \\ \vdots & \vdots & \ddots & \vdots \\ \lambda & \lambda & \ldots & 1 \end{bmatrix}_{M \times M}$$

with $-1 < \lambda < 1$. We are interested in M-ary TSK signal sets in which the basic structure of $,_{\text{EC}}$ does not depend on the shape of the impulse waveform $p(t)$.[2] In the next section we describe different constructions that satisfy this requirement.

III. M-ary TSK equicorrelated signal sets

Let H be an $(N_s+1) \times (N_s+1)$ cyclic Hadamard matrix, where the value of N_s satisfies one of the following three conditions [5]

(a) $N_s = 2^m - 1$, $m \geq 1$, or
(b) $N_s = p$, p a prime, or
(c) $N_s = p(p+2)$, p and $(p+2)$ form a twin prime,

We can generate EC signals $\{\Psi_j(t)\}$ for $M \leq N_s$ by deleting the first column and the first row of H, and then use the j^{th} row of this modified matrix $\hat{H}$ and the mapping $(+1) \to \tau_2$ and $(-1) \to \tau_1 = 0$ to produce the time shift pattern $\{\delta_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$ defining the j^{th} signal. For example, if we use $N_s = 2^m - 1$, $m \geq 1$, the j^{th} row of $\hat{H}$ is the j^{th} cyclic shift, $j = 1, 2, \ldots, M$ of an m-sequence of length N_s [5]. Continuing this example, with $M = 8$ and $N_s = 2^m - 1$, $m = 3$, the following modified Hadamard matrix

$$\hat{H} = \begin{bmatrix} -1 & -1 & -1 & +1 & -1 & +1 & +1 \\ +1 & -1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & -1 & +1 & -1 \\ -1 & +1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & -1 & -1 \\ -1 & +1 & -1 & +1 & +1 & -1 & -1 \\ -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix}$$

[2]The impulse shape of $p(t)$ is not standard and depends on the device used to generate the signal.

results in the ensemble of signals represented by

$$
\Delta_{\text{EC}} = \begin{bmatrix} 0 & 0 & 0 & \tau_2 & 0 & \tau_2 & \tau_2 \\ \tau_2 & 0 & 0 & 0 & \tau_2 & 0 & \tau_2 \\ \tau_2 & \tau_2 & 0 & 0 & 0 & \tau_2 & 0 \\ 0 & \tau_2 & \tau_2 & 0 & 0 & 0 & \tau_2 \\ \tau_2 & 0 & \tau_2 & \tau_2 & 0 & 0 & 0 \\ 0 & \tau_2 & 0 & \tau_2 & \tau_2 & 0 & 0 \\ 0 & 0 & \tau_2 & 0 & \tau_2 & \tau_2 & 0 \end{bmatrix}
$$

If in $\hat{H}$ we use the mapping $(+1) \rightarrow (+1)$ and $(-1) \rightarrow (0)$, then the matrix

$$
\begin{aligned}
\tilde{H} &= \begin{bmatrix} b_1^1 & b_1^2 & b_1^3 & b_1^4 & b_1^5 & b_6^1 & b_7^1 \\ b_2^1 & b_2^2 & b_2^3 & b_2^4 & b_2^5 & b_2^6 & b_2^7 \\ b_3^1 & b_3^2 & b_3^3 & b_3^4 & b_3^5 & b_3^6 & b_3^7 \\ b_4^1 & b_4^2 & b_4^3 & b_4^4 & b_4^5 & b_4^6 & b_4^7 \\ b_5^1 & b_5^2 & b_5^3 & b_5^4 & b_5^5 & b_5^6 & b_5^7 \\ b_6^1 & b_6^2 & b_6^3 & b_6^4 & b_6^5 & b_6^6 & b_6^7 \\ b_7^1 & b_7^2 & b_7^3 & b_7^4 & b_7^5 & b_7^6 & b_7^7 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}
\end{aligned}
$$

provides an alternate way to represent the set of signals Δ_{EC} as follows

$$
\Delta_{EC} = \tau_2 \times \begin{bmatrix} b_1^1 & b_1^2 & b_1^3 & b_1^4 & b_1^5 & b_6^1 & b_7^1 \\ b_2^1 & b_2^2 & b_2^3 & b_2^4 & b_2^5 & b_2^6 & b_2^7 \\ b_3^1 & b_3^2 & b_3^3 & b_3^4 & b_3^5 & b_3^6 & b_3^7 \\ b_4^1 & b_4^2 & b_4^3 & b_4^4 & b_4^5 & b_4^6 & b_4^7 \\ b_5^1 & b_5^2 & b_5^3 & b_5^4 & b_5^5 & b_5^6 & b_5^7 \\ b_6^1 & b_6^2 & b_6^3 & b_6^4 & b_6^5 & b_6^6 & b_6^7 \\ b_7^1 & b_7^2 & b_7^3 & b_7^4 & b_7^5 & b_7^6 & b_7^7 \end{bmatrix},
$$

i.e, the j^{th} row $\{b_j^k, k = 0, 1, 2, \ldots, N_s - 1\}$ of $\tilde{H}$ is an equivalent representation of the time shift pattern $\{\rho_j^k = b_j^k \tau_2; k = 0, 1, 2, \ldots, N_s - 1\}$ defining the j^{th} signal. Hence, the TSK EC signals can be written

$$
\Psi_j(t) = \sum_{k=0}^{N_s-1} p(t - kTf - b_j^k \tau_2), \quad i = 1, 2, \ldots, M. \tag{1}
$$

For the EC PPM signals in (1) the correlation matrix is given by $,_{\text{EC}}$ in with

$$
\lambda = \frac{\frac{N_s-1}{2}\gamma_w(0) + \frac{N_s+1}{2}\gamma_w(\tau_2)}{N_s}. \tag{2}
$$

Notice that for $N_s >> 1$

$$
\lambda \approx \frac{1 + \gamma_w(\tau_2)}{2}. \tag{3}
$$

A. *Selection of τ_2*

The signal design in (1) depends on $p(t)$ and τ_2. From (3) we can see that by using $\tau_2 = \tau_{\min}$ the minimum value of λ is

$$
\begin{aligned}
\lambda_{\min} &\triangleq \frac{\frac{N_s-1}{2}\gamma_w(0) + \frac{N_s+1}{2}\gamma_{\min}}{N_s} \\
&\approx \frac{1 + \gamma_{\min}}{2} \text{ for } N_s >> 1.
\end{aligned} \tag{4}
$$

The actual value of $\gamma_{\min}$ depends on the particular impulse $p(t)$ used in the IR communication link. Figure 1 plots $\lambda_{\min}$ versus N_s for different hypothetical values of $\gamma_{\min}$. Note that $\lambda_{\min} \leq 0$ only for $N_s \leq \frac{1-\gamma_{\min}}{1+\gamma_{\min}}$, and that for $N_s >> 1$ the $\lambda_{\min}$ value is strictly positive.

IV. Performance in AWGN

The symbol error probability for EC signals is [7]

$$
P_e = 1 - \int_{-\infty}^{\infty} \left[1 - Q\left(\xi + \sqrt{\frac{2E_\Psi(1-\lambda)}{N_o}} \right) \right]^{M-1} \frac{\exp(-\xi^2/2)}{\sqrt{2\pi}} \, d\xi.
$$

The bit error probability is simply

$$
P_b = \frac{1}{2}\frac{M}{M-1} P_e.
$$

A. *Example*

In this example we calculate the error probability in AWGN for the equicorrelated signals with $\lambda_{\min}$ given in (4). In IR modulation, the UWB received pulse $p(t)$ can be modeled by

$$
p(t) = \left[1 - 4\pi\left[\tfrac{t}{t_n}\right]^2\right] \exp\left(-2\pi\left[\tfrac{t}{t_n}\right]^2\right), \tag{5}
$$

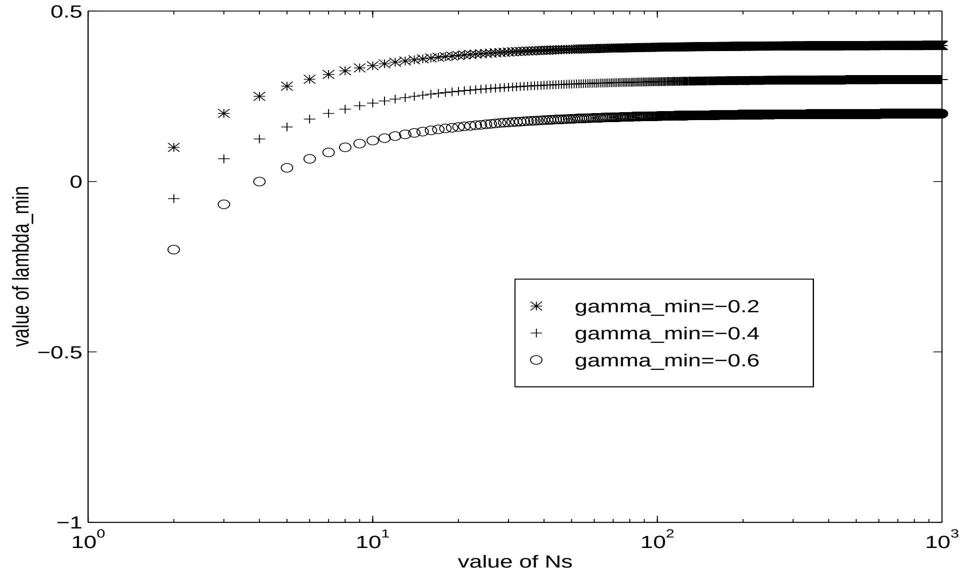

Fig. 1. The value of $\lambda_{\min}$ versus N_s for different values of $\gamma_{\min}$.

where the value $t_n = 0.4472$ ns was used to fit the model $p(t)$ to an impulse $p_m(t)$ measured in a particular radio link. The signal correlation function corresponding to $p(t)$ is

$$\gamma_p(\tau) \quad = \quad \left[1-4\pi\left[\tfrac{\tau}{t_n}\right]^2+\tfrac{4\pi^2}{3}\left[\tfrac{\tau}{t_n}\right]^4\right] \exp\left(-\pi\left[\tfrac{\tau}{t_n}\right]^2\right).$$

In this case $\tau_{\min} = 0.2419$ ns and $\gamma_{\min} = -0.6183$. Both $p(t-\frac{T_p}{2})$ and $\gamma_p(\tau)$ are shown in figure 2. Figure 3 shows the spectrum of the impulse $p(t)$. The bandwidth of the impulse is in excess of 1 Gigahertz.

Figure 4 shows P_b for $M = 8$ and $N_s >> 1$, calculated using the impulse in (5) and $\lambda = \lambda_{\min}$.

V. Receiver simplification

To detect the received signal we need to correlate this signal with M reference signals. For large M this can result in a receiver of considerable complexity. We can take advantage of the structure of the TSK signals to simplify the construction of the receiver.

Let $x(t) = \Psi_i(t) + n(t)$, where $\Psi_i(t)$ is one of the TSK signals in (1) and $n(t)$ is AWGN. In the receiver, each of the M channel correlation output can be written

$$y_j \quad = \quad \int_0^{N_s T_f} x(t)\,\Psi_j(t)\,dt$$

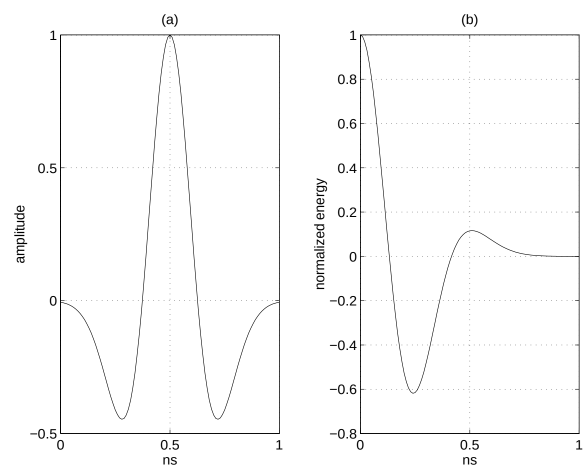

Fig. 2. (a) The impulse $p(t-\frac{T_p}{2})$ as a function of time t. (b) The signal autocorrelation $\gamma_p(\tau)$ as a function of time shift τ.

$$= \quad \sum_{k=0}^{N_s-1} \int_{kT_f}^{(k+1)T_f} x(t)\; p(t-kT_f-\rho_j^k)\,dt.$$

For the EC signals (1), y_j can be written

$$y_j \quad = \quad \sum_{k=0}^{N_s-1} \left[(b_j^k-1)z_1(k) + b_j^k z_2(k)\right],$$

where

$$z_m(k) \triangleq \int_{kT_f}^{kT_f+T_p+\tau_2} x(t)p(t-kT_f-\tau_m)dt, \quad m=1,2.$$

From the expression for $y_j,\;\; j = 1, 2, \ldots, M$, it is clear that the receiver needs only 2 correlators and

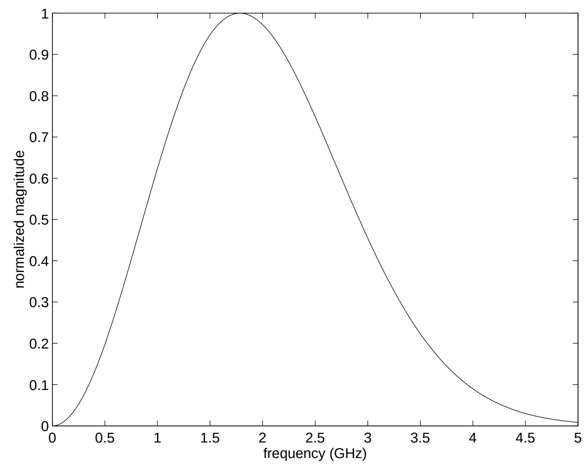

Fig. 3. The magnitude of the spectrum of the impulse $p(t)$.

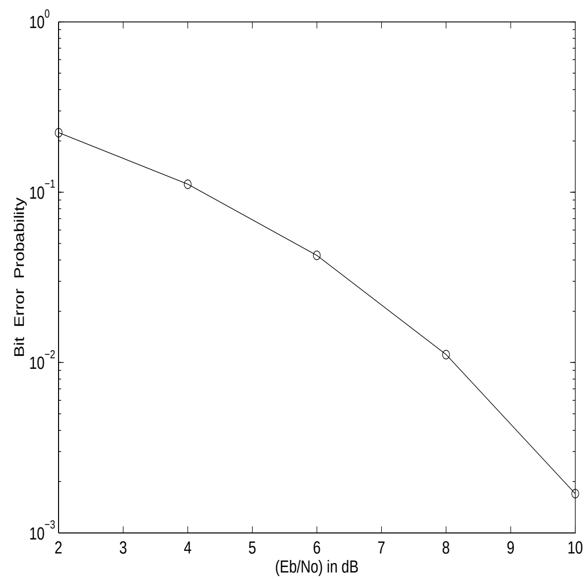

Fig. 4. The P_e for $M = 8$.

M store and sum circuits. The decision variables $\{y_j\}$ can be calculated while $x(t)$ is received and no symbol delay occur. This is illustrated in figure 5.

VI. Conclusion

This paper described the construction of M-ary time-shift-keyed equicorrelated signal sets for ultra-wideband impulse radio modulation. The construction is based on cyclic Hadamard matrices. Since the time-shift-keyed signals are linearly independent, it is not possible to reduce the dimensionality M of the signal set. Nevertheless, we can take advantage of the structure of the time-shift-keyed signals to reduce the complexity of the receiver and use less than M correlators combined with M store and sum circuits to calculate the decision variables without introducing a symbol delay.

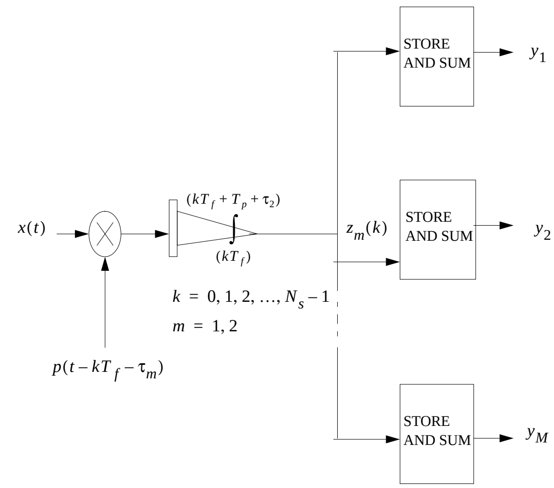

Fig. 5. This diagram shows one of the two correlators and the M store and sum circuits that are needed in the simplified receiver for signal set 2.

References

[1] B. Noel, editor, *Ultra-Wideband Radar: Proceedings of the First los Alamos Symposium,* CRC Press, 1991.

[2] R. A. Scholtz, *"Multiple Access with Time Hopping Impulse Modulation,"* invited paper, Proceedings of Milcom'93, Dec. 1993.

[3] H. Hashemi, *"The Indoor Radio Propagation Channel,"* Proceedings IEEE Vol. 81, No. 7, July 1993.

[4] R. Price and P. E. Green Jr., *"A Communication Technique for Multipath Channels,"* Proc. IRE, March 1958, pp. 555-570.

[5] S. W. Golomb, *"Construction of signals with favorable correlation properties,"* in Surveys in Combinatorics, London Mathematical Society Lecture Notes Series 166, Cambridge University Press, 1991.

[7] R. M. Gagliardi, *Introduction to Telecommunications Engineering,"* John Wiley and Sons, 1988.