UWB M-ary N-Orthogonal PPM Signals in AWGN and Multipath Channels

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***Abstract*— This work studies the performance of N-orthogonal pulse position modulated (PPM) signals for ultra wideband (UWB) communications over both additive white Gaussian noise (AWGN) and multipath channels. We use a pulse position optimization that reduces the detection error probability by exploiting the negative values in the pulse correlation function. Numerical examples are given to illustrate the advantages of N-orthogonal over orthogonal signals.**

I. Introduction

Ultra wideband SSMA communications using time hopping and PPM has been studied extensively [1]- [4]. It also has been proposed for consideration in IEEE standard bodies [5]. The use of M-ary PPM signals allows to increase the number of users supported by the system for a given data transmission rate without degrading the multiple access performance. More importantly for UWB communications, M-ary signals allow to reduce the required transmitter power maintaining the number of users, the data transmission rate and the multiple access performance [6]- [11].

This work studies the performance of N-orthogonal PPM signals for UWB communications over both AWGN and multipath channels. We use a pulse position optimization that reduces the detection error probability by exploiting the negative values in the pulse correlation function. Numerical examples are given to illustrate the advantages of N-orthogonal over orthogonal signals.

II. Signal definition

In this work we consider the generic M-ary PPM signals [6]

$$\Psi_i(t) = \sum_{k=0}^{N_s-1} w(t - kT_f - \rho_i^k), \quad i = 1, 2, 3, \ldots, M. \quad (1)$$

The maximum value of ρ_i^k is denoted $\rho_{\max}$. The T_f is the frame repetition period. The duration of the PPM signals is $Ts = N_s T_f$.

The signal $w(t)$ is the basic UWB pulse used to convey information. It has duration T_w and energy $E_w = \int_{-\infty}^{\infty} [w(t)]^2\, dt$. For simplicity we assume that $T_w + \rho_{\max} < T_f$ (to avoid overlapping between pulses belonging to different frames). The normalized signal correlation function of $w(t)$ is

$$\gamma(\tau) \triangleq \frac{1}{E_w} \int_{-\infty}^{\infty} w(t) w(t-\tau) dt > -1\, \forall \tau.$$

The minimum value of $\gamma(\tau)$ is denoted $\gamma_{\min} \triangleq \gamma(\tau_{\min})$.

The signals $\Psi_i(t)$ in (1) have energy $E_\Psi = \int_{-\infty}^{\infty} [\Psi_i(t)]^2\, dt = N_s E_w$. The normalized correlation between $\Psi_i(t)$ and $\Psi_j(t)$ is

$$\begin{aligned} \alpha_{ij} &\triangleq \frac{1}{E_\Psi} \int_{-\infty}^{\infty} \Psi_i(t) \Psi_j(t) dt \\ &= \frac{1}{N_s} \sum_{k=0}^{N_s-1} \gamma(\rho_i^k - \rho_j^k) \ > -1, \end{aligned}$$

since for $k \neq l$ the pulses are non overlapping.

A. The M-ary N-orthogonal PPM signals

An N-orthogonal signal set consists of $M = NL$ signals, where M, N, L are all positive non-zero integers. The M equal energy, equal time duration signals have the following two properties: [1]

1) The signal set may be divided into L disjoint subsets, each subset containing N non-orthogonal members,
2) Signals from different subsets are orthogonal.

We wish to choose a set of time shift values ρ_i^k to get signals having the correlation properties 1) and 2) above. Equation (2) illustrate one possible choice [14]

$$\begin{aligned} \rho_i^k &\triangleq \rho_{n,l}^k = \tau_n + lT_o, \, \forall k, \\ n &= i - \lfloor \frac{i-1}{N} \rfloor N, \quad n = 1, 2, \ldots, N, \\ l &= \lfloor \frac{i-1}{N} \rfloor, \qquad l = 0, 1, 2, \ldots, L-1, \\ i &= lN + n, \qquad i = 1, 2, \ldots, N, \\ & \qquad\qquad N+1, N+2, \ldots, 2N, \\ & \qquad\qquad \ldots, 3N, \ldots, M, \end{aligned} \quad (2)$$

where

$$\Omega_{\mathrm{N}} \triangleq (\tau_1, \tau_2, \ldots, \tau_{\mathrm{N}}) \quad (3)$$

is a set of N time shifts such that $\tau_1 = 0$, $\tau_1 < \tau_2 \leq T_w, \ldots, \tau_{N-1} < \tau_N \leq (N-1)T_w$, $T_o \triangleq \tau_{\mathrm{N}} + T_w$, $\rho_{n,l}^k \leq \rho_{\max} \triangleq \tau_{\mathrm{N}} + (L-1)T_o$, and where $\lfloor \xi \rfloor$ denotes the integer part of ξ, with $N_s \geq L$. In (2) we use the double index (n,l) instead of the index i to emphasize that the signals $\Psi_{n,l}(t)$ with different

[1] N-orthogonal phase-modulated signals are the generalization of bi-orthogonal signals and were first introduced by Reed and Scholtz [12], and later studied by Viterbi and Stiffler [13]. The work in [14] studied N-orthogonal position-modulated signals.

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values of l belong to different orthogonal subsets. Using (2) in (1) we get N-orthogonal signals having correlation matrix $\Gamma_{\text{NO}} \triangleq [\alpha_{ij}]_{M\times M}$ with elements

$$\alpha_{ij}=\begin{cases} 0, & \lfloor\frac{i-1}{N}\rfloor \neq \lfloor\frac{j-1}{N}\rfloor \\ & \text{(signals from different subsets)} \\ 1, & i=j \\ & \text{(signal correlated with itself)} \\ \gamma(\tau_{ij}), & \lfloor\frac{i-1}{N}\rfloor = \lfloor\frac{j-1}{N}\rfloor \\ & \text{(signals within the same subset)} \end{cases},$$

where $\tau_{ij} \triangleq \tau_{i-\lfloor\frac{i-1}{N}\rfloor N} - \tau_{j-\lfloor\frac{j-1}{N}\rfloor N}$ is the difference between two time shifts.

Using (2) with $N = 1$ and $M = L$ we get M-ary PPM orthogonal signals having diagonal correlation matrix Γ_{OR}.

B. Signal optimization

For UWB pulses the $\gamma(\tau)$ has negative values, and we can find sets of negatively correlated signals by choosing the appropriate values of time shifts Ω_{N}. The work in [14] describes three signal optimization criteria:

1) Choose Ω_{N} that minimize the symbol error rate (SER) P_e for shift-coherent communications in the presence of AWGN [15].
2) Calculate Ω_{N} that minimize the union bound on the SER

$$\text{UB}_e\left(\tfrac{E_w}{N_o},\Lambda(\Omega_{\text{N}})\right) = \tfrac{1}{N}\sum_{\substack{i=1\\i\neq j}}^{N}\sum_{j=1}^{N} Q\left(\sqrt{\tfrac{E_w}{N_o}(1-\gamma(\tau_{ij}))}\right),$$

where $Q(\cdot)$ is the Gaussian tail integral, the correlation matrix $\Lambda(\Omega_{\text{N}}) \triangleq [\alpha_{ij}]_{N\times N}$, and where

$$\alpha_{ij}=\begin{cases} 1, & i=j \\ \gamma(\tau_{ij}), & i\neq j, \end{cases},$$

$i, j, = 1, 2, \ldots, N$. The $\Lambda(\Omega_{\text{N}})$ is the correlation matrix of the set of signals

$$w(t-\tau_1), w(t-\tau_2), \ldots, w(t-\tau_{\text{N}}).$$

3) Find Ω_{N} that solves the optimization problem:

$$\underset{\text{all possible } \Omega_{\text{N}}}{\text{minimize}} \quad \max\Big(\begin{array}{l} \gamma(\tau_{12}),\gamma(\tau_{13}),\ldots,\gamma(\tau_{1\text{N}}), \\ \gamma(\tau_{23}),\gamma(\tau_{24}),\ldots,\gamma(\tau_{2\text{N}}), \\ \vdots \\ \gamma(\tau_{(\text{N}-2)(\text{N}-1)}),\gamma(\tau_{(\text{N}-2)\text{N}}),, \\ \gamma(\tau_{(\text{N}-1)\text{N}}) \end{array} \Big).$$

III. Performance in Gaussian Channel

A. Channel and signal models

The channel model has free-space propagation conditions. The effect of the antenna system in the UWB transmitted pulse is modeled as a differentiation operation. The transmitted signal is $\Psi_{\text{TX}}(t) \triangleq \int_{-\infty}^{t} \Psi(q)dq$ and the received signal is $A\Psi(t-\tau) + n(t)$. The constants A and τ represent the attenuation and propagation delay, respectively, that the signal experiences over the link path between the transmitter and receiver. The noise $n(t)$ is AWGN with two-sided power density $N_o/2$.

B. Receiver structure and error probability

We consider time-shift-coherent communications. For decoding we use the M-ary correlation receiver with perfect synchronization [16]. This receiver SER is bounded by

$$\text{UB}_e\left(\tfrac{E_\Psi}{N_o},\Gamma_{\text{NO}}\right) = \text{UB}_e\left(\tfrac{E_\Psi}{N_o},\Lambda(\Omega_{\text{N}})\right)+(M-N)Q\left(\sqrt{\tfrac{E_\Psi}{N_o}}\right). \tag{4}$$

IV. Performance in Multipath Channel

A. Channel and signal models

The channel is a slowly varying indoor radio channel. The transmitter is placed at a certain fixed location, and the receiver is placed at a variable location denoted u_o. The transmitted pulse is the same pulse $w_{\text{TX}}(t)$ used in the Gaussian channel case, and the received "pulse" is $\sqrt{E_a}w(u_o,t)$. The $\sqrt{E_a}w(u_o,t)$ is a multipath spread version of $w(t)$ received at position u_o, it has average duration $T_a >> T_w$ and "random" energy $E_w(u_o) \triangleq E_a\beta^2(u_o)$, where E_a is the average energy and $\beta^2(u_o) \triangleq \int_{-\infty}^{\infty}[w(u_o,t)]^2dt$. The pulse has "random" normalized signal correlation

$$\gamma(u_o,\tau) \triangleq \frac{1}{E_w(u_o)}\int_{-\infty}^{\infty} w(u_o,t)w(u_o,t-\tau)dt, > -1\,\forall\tau.$$

Clearly, the multipath effects change for different u_o, and therefore $E_w(u_o)$ and $\gamma(u_o,\tau)$ both changes with u_o [17] [18].

Using the previous pulse definitions, the Mary PPM signals containing the multipath effects can be denoted

$$\Psi_i(u_o,t) = \sum_{k=0}^{N_s-1}\sqrt{E_a}\, w(u_o, t-kT_f-\rho_i^k), \tag{5}$$

$i = 1,2,3,\ldots,M$. We assume that $\Psi_i(u_o,t)$ has fixed duration $T_s \simeq N_sT_f$, provided that $T_a + \rho_{\text{max}} < T_f$. The signals $\Psi_i(u_o,t)$ in (5) have "random" energy $E_\Psi(u_o) = \int_{-\infty}^{\infty}[\Psi_i(u_o,\xi)]^2d\xi \simeq \overline{E}_\Psi\beta^2(u_o)$, where $\overline{E}_\Psi \triangleq N_sE_a$ is the average signal energy. The signals $\Psi_i(u_o,t)$ and $\Psi_j(u_o,t)$ have "random" normalized correlation values

$$\alpha_{ij}(u_o) \triangleq \frac{\int_{-\infty}^{\infty}\Psi_i(u_o,\xi)\,\Psi_j(u_o,\xi)\,d\xi}{E_\Psi(u_o)}.$$

Since the multipath effects change with the particular position u_o, the M-ary set of received signals $\Psi_i(u_o,t)$ in (5) also changes with that particular position u_o. This means that the correlation properties of the signals $\Gamma(u_o) \triangleq [\alpha_{ij}(u_o)]_{M\times M}$ are now position dependent. Hence, M-ary signal originally defined as N-orthogonal or orthogonal in an AWGN channel, won't necessarily preserve their correlation properties in the presence of multipath [17] [18]. However, we still denote their correlation properties as $\Gamma_{\text{NO}}(u_o)$ or $\Gamma_{\text{OR}}(u_o)$.

B. Receiver structure and error probability

For performance evaluation purposes we shall assume that the signals $\Psi_i(u_o,t)$ are used in a time-shift-coherent communication system transmitting through a multipath channel, and are decoded using a Rake receiver ideally matched to the received signal and perfectly synchronized with the transmitter [19]. A value of $T_f > T_a + \rho_{\max}$ ensures that intersymbol interference due to multipath can be ignored.

Performance is calculated by averaging the SER over the multipaths effects, i.e, by taking the expected value $\mathbf{E}_u\{\cdot\}$ over all values of u_o

$$\overline{\mathrm{UB}}_e\left(\tfrac{\overline{E}_\Psi}{N_o}\right) = \mathbf{E}_u\{\mathrm{UB}_e\left(\tfrac{\overline{E}_\Psi \beta^2(u)}{N_o}, \Gamma_{\mathbf{NO}}(u)\right)\}, \quad (6)$$

where $\left(\frac{\overline{E}_\Psi}{N_o}\right)$ is the average received symbol SNR. Calculation of $\overline{\mathrm{UB}}_e\left(\frac{\overline{E}_\Psi}{N_o}\right)$ in (6) can be approximated by the sample mean value

$$\overline{\mathrm{UB}}_e\left(\tfrac{\overline{E}_\Psi}{N_o}\right) \approx \frac{1}{u_*}\sum_{u_o=1}^{u_*} \mathrm{UB}_e\left(\tfrac{\overline{E}_\Psi \beta^2(u_o)}{N_o}, \Gamma_{\mathbf{NO}}(u_o)\right). \quad (7)$$

computed using an ensemble of UWB pulse responses $\{w(u_o,t)\}$, $u_o = 1, 2, \ldots, u_*$. [2]

V. EXAMPLE WITH GAUSSIAN PULSES

A. Signal Design

In this example the received pulse is the second derivative of a Gaussian pulse

$$w(t) = \left[1 - 4\pi\left[\frac{t}{t_n}\right]^2\right] \exp\left(-2\pi\left[\frac{t}{t_n}\right]^2\right), \quad (8)$$

$-T_w/2 \le t \le T_w/2$, where $t_n = 0.7531$ ns to get a pulse duration $T_w \simeq 2.0$ ns, with $E_w = 3t_n/8$. The spectrum of $w(t)$ is centered at 1.1 GHz, with a 3 dB bandwidth of about 1.2 MHz, satisfying the traditional definition of UWB signal stating that the 10 dB bandwidth of the signal should be at least 20 percent of its center frequency [20].

The signal correlation function corresponding to $w(t)$ in (8) is

$$\gamma(\tau) = \left[1 - 4\pi\left[\frac{\tau}{t_n}\right]^2 + \frac{4\pi^2}{3}\left[\frac{\tau}{t_n}\right]^4\right] \times \exp\left(-2\pi\left[\frac{\tau}{t_n}\right]^2\right), \quad (9)$$

with $\gamma_{\min} = -0.6181$ and $\tau_{\min} = 0.4068$.

The optimization criterion 2) was applied using a computer program as described in [14]. Tables I show results for $N = 4$.

B. Performance in AWGN channel

Performance in AWGN is independent of the specific values of N_s and T_f, as long as $T_w + \rho_{\max} < T_f$, and $N_s \ge L$. The SER is calculated using (4). Performance for different values $\left(\frac{E_\Psi}{N_o}\right)$ is calculated utilizing the optimized time shift Ω_N^{opt} corresponding to that $\left(\frac{E_\Psi}{N_o}\right)$ value.

Figure 1(a) shows $\mathrm{UB}_e\left(\frac{E_\Psi}{N_o}, \Gamma_{\mathbf{NO}}\right)$ and $\mathrm{UB}_e\left(\frac{E_\Psi}{N_o}, \Gamma_{\mathbf{OR}}\right)$ for different N, L, and M. Notice that for $M = 2$ with $N = 2$, $L = 1$, N-orthogonal signals have a SNR advantage of $10\log_{10}(1 - \gamma_{\min}) = 2.09$ dB over orthogonal signals. For $M = 4$ the SNR advantage is about 0.25 dB. For $M = 8$ the SNR advantage is about 0.05 dB.

[2]A detailed explanation of this method of performance calculation can be found in [21]

C. Performance in multipath channel

For this example we use the same signal parameters used in the AWGN example. To characterize the multipath channel we use an ensemble formed with experimental channel pulse responses $w(u_o,t)$[3] as described in [21].

The measured $w(u_o,t)$ has $T_a \simeq 300$ ns. By selecting $T_f = 500$ ns we make sure that $T_f > T_a + \rho_{\max}$. A total of $u_* = 294$ channel pulse responses $w(u_o,t)$ are used. An equal number of $\beta^2(u_o)$, $\Gamma_{\mathbf{NO}}(u_o)$ and $\Gamma_{\mathbf{OR}}(u_o)$ are calculated. These u_* sets of values are then used to compute (7).

Figure 1(b) shows the averaged symbol error rate $\mathbf{E}_u\{\mathrm{UB}_e\left(\frac{\overline{E}_\Psi \beta^2(u)}{N_o}, \Gamma_{\mathbf{NO}}(u)\right)\}$ and $\mathbf{E}_u\{\mathrm{UB}_e\left(\frac{\overline{E}_\Psi \beta^2(u)}{N_o}, \Gamma_{\mathbf{OR}}(u)\right)\}$ for different N, L, and M. For every value of $\left(\frac{E_\Psi}{N_o}\right)$, the same Ω_N^{opt} value corresponding to $\left(\frac{\overline{E}_\Psi}{N_o}\right) = 12$ dB is used. Notice that for $M = 2$ N-orthogonal signals have a SNR advantage of 1.30 dB over orthogonal signals. For $M = 4$ there is no SNR advantage, and for $M = 8$ there is a disadvantage of 0.2 dB.

VI. EXAMPLE WITH PULSED SINEWAVES

A. Signal Design

The UWB signals considered in this example are based on pulsed sinewaves. The received pulse is

$$w(t) = \begin{cases} \sin(2\pi \frac{Q}{T_w} t), & 0 \le t \le T_w, \\ 0, & \text{otherwise}, \end{cases} \quad (10)$$

where Q is a positive integer. The duration of $w(t)$ is $T_w = 2.0$ ns, the spectrum of $w(t)$ is centered at $\frac{Q}{T_w}$ GHz, with a 10 dB bandwidth of about 700 MHz, satisfying the new definition of UWB signal stating that the 10 dB bandwidth of the signal should be at least 500 MHz [20]. The signal correlation function corresponding to $w(t)$ in (10) is

$$\gamma(\tau) = \begin{cases} \frac{1}{E_w}\frac{T_w - |\tau|}{T_w}\cos(2\pi \frac{Q}{T_w}\tau), & -T_w \le \tau \le T_w, \\ 0, & \text{otherwise}, \end{cases},$$

where $E_w = \left(\frac{T_w}{2}\right)$.

The optimization criteria 3) was calculated using a computer program as described in [14] . Table II show the results of the minimization for $N = 3$.

B. Performance in AWGN channel

Using the same T_f and N_s as before, performance in AWGN is calculated using the optimized time shift Ω_N^{opt}. Figure 2(a) shows $\mathrm{UB}_e\left(\frac{E_\Psi}{N_o}, \Gamma_{\mathbf{NO}}\right)$ and $\mathrm{UB}_e\left(\frac{E_\Psi}{N_o}, \Gamma_{\mathbf{OR}}\right)$ for different N, L, and M. Using $Q = 10$ and $M = 2$ the N-orthogonal signals have a SNR advantage of 2.9 dB over orthogonal signals. For $M = 3$ and $M = 6$ the SNR advantage is about 1.31 dB and 0.16 dB, respectively.

[3]These UWB pulses are taken from the Time Domain Corporation Indoor Channel Database, available at USC's ULTRA-LAB WEB site at $http://click.usc.edu/New-Site/database.html$.

$\left(\frac{E_w}{N_o}\right)$ (dB)	$\Omega_4^{\text{opt}} = (\tau_1^{\text{opt}}, \tau_2^{\text{opt}}, \tau_3^{\text{opt}}, \tau_4^{\text{opt}})$ (ns)	$\{\alpha_{21},\ \alpha_{31},\ \alpha_{41},\ \alpha_{32},\ \alpha_{42},\ \alpha_{43}\} \in \Lambda(\Omega_4^{\text{opt}})$	$\text{UB}_e\left(\frac{E_w}{N_o}, \Lambda(\Omega_4^{\text{opt}})\right)$ (prob. of error)
2	$(0.0000, 0.3264, 0.5945, 0.9209)$	$\{-0.4967, -0.2429, 0.1061, -0.2561, -0.2429, -0.4967\}$	$2.4003952e^{-1}$
4	$(0.0000, 0.3211, 0.5902, 0.9113)$	$\{-0.4800, -0.2548, 0.1087, -0.2608, -0.2548, -0.4800\}$	$1.1726925e^{-1}$
6	$(0.0000, 0.3153, 0.5862, 0.9015)$	$\{-0.4604, -0.2659, 0.1111, -0.2698, -0.2659, -0.4604\}$	$4.1460574e^{-2}$
8	$(0.000, 0.3109, 0.5842, 0.8951)$	$\{-0.4445, -0.2714, 0.1124, -0.2821, -0.2714, -0.4445\}$	$9.1811626e^{-3}$
10	$(0.0000, 0.3211, 0.5966, 0.9178)$	$\{-0.4799, -0.2372, 0.1070, -0.2929, -0.2370, -0.4801\}$	$1.0597741e^{-3}$
12	$(0.0000, 0.2536, 0.5058, 0.7594)$	$\{-0.1790, -0.4827, 0.0749, -0.1714, -0.4826, -0.1790\}$	$4.4588128e^{-5}$
14	$(0.0000, 0.2452, 0.4901, 0.7352)$	$\{-0.1319, -0.5186, 0.0489, -0.1304, -0.5187, -0.1317\}$	$3.2869771e^{-7}$
16	$(0.0000, 0.2452, 0.4901, 0.7352)$	$\{-0.1049, -0.5378, 0.0306, -0.1045, -0.5379, -0.1046\}$	$1.55400285e^{-10}$
18	$(0.0000, 0.2374, 0.4746, 0.7119)$	$\{-0.0867, -0.5500, 0.0168, -0.0858, -0.5501, -0.0866\}$	$9.3470225e^{-16}$

TABLE I

OPTIMIZED VALUES Ω_N^{OPT}, $\Lambda(\Omega_N^{\text{OPT}})$, AND $\text{UB}_e\left(\frac{E_w}{N_o}, \Lambda(\Omega_{\mathbf{N}}^{\text{OPT}})\right)$ FOR $N = 4$ AND DIFFERENT $\frac{E_w}{N_o}$ CALCULATED USING THE PULSE IN (8) TO SOLVE THE MINIMIZATION PROBLEM 2).

Q	$\Omega_3^{\text{opt}} = (\tau_1^{\text{opt}}, \tau_2^{\text{opt}}, \tau_3^{\text{opt}})$	$\{\alpha_{21},\ \alpha_{31},\ \alpha_{32}\} \in \Lambda(\Omega_3^{\text{opt}})$ (ns)	$\gamma(\tau_{\text{min}}) = \gamma_{\text{min}}$	$10\log_{10}(1-\gamma_{\text{min}})$ (dB)
1	$(0.00, 0.62, 1.24)$	$\{-0.2540, -0.2770, -0.2540\}$	$\gamma(0.91) = -0.5234$	1.8
2	$(0.00, 0.33, 0.66)$	$\{-0.4023, -0.3594, -0.4023\}$	$\gamma(0.48) = -0.7540$	2.4
3	$(0.00, 0.22, 0.44)$	$\{-0.4288, -0.4179, -0.4288\}$	$\gamma(0.33) = -0.8346$	2.6
4	$(0.00, 0.16, 0.32)$	$\{-0.3917, -0.5354, -0.3917\}$	$\gamma(0.25) = -0.8750$	2.7
10	$(0.00, 0.06, 0.12)$	$\{-0.2997, -0.7605, -0.2997\}$	$\gamma(0.1) = -0.9500$	2.7

TABLE II

OPTIMIZED VALUES OF Ω_N^{OPT} AND $\Lambda(\Omega_N^{\text{OPT}})$ FOR $N = 3$, AND VALUES OF τ_{MIN}, γ_{MIN}, AND SNR ADVANTAGE $10\log_{10}(1-\gamma_{\text{MIN}})$. VALUES ARE CALCULATED USING THE PULSE IN (10) WITH $Q = 1, 2, 3, 4, 10$, TO SOLVE THE MINIMIZATION PROBLEM 3).

C. Performance in multipath channel

For this example we use the same signal parameters used in the AWGN example. To characterize the multipath channel we use an autoregressive channel model [22] [23] to form and ensemble of modeled channel pulse responses $w(u_o, t)$ as described in [24].

The simulated $w(u_o, t)$ has $T_a \simeq 160$ ns. By selecting $T_f = 500$ ns we make sure that $T_f > T_a + \rho_{\text{max}}$. A total of $u_* = 294$ channel pulse responses $w(u_o, t)$ are used. An equal number of $\beta^2(u_o)$, $\Gamma_{\mathbf{NO}}(u_o)$ and $\Gamma_{\mathbf{OR}}(u_o)$ are calculated. These u_* sets of values are then used to calculate (7)

Figure 2(b) shows the averaged symbol error rate $\mathbf{E}_u\{\text{UB}_e\left(\frac{\overline{E}_\Psi \beta^2(u)}{N_o}, \Gamma_{\mathbf{NO}}(u)\right)\}$ and $\mathbf{E}_u\{\text{UB}_e\left(\frac{\overline{E}_\Psi \beta^2(u)}{N_o}, \Gamma_{\mathbf{OR}}(u)\right)\}$ for different N, L, and M. For every value of $\left(\frac{E_\Psi}{N_o}\right)$, the same Ω_N^{opt} value is used. Using $Q = 10$ and $M = 3$ the N-orthogonal signals have a SNR advantage of 1.78 dB over orthogonal signals. For $M = 6$ there is an SNR advantage of about 0.24 dB.

VII. CONCLUSIONS

This work studies the performance of N-orthogonal PPM signals for UWB communications over both AWGN and multipath channels.

Numerical examples are given to illustrate the advantages of N-orthogonal over orthogonal signals. Table III summarizes the results for different values of M using Gaussian and sine pulses.

It is well known that in AWGN channels the SNR advantage using M-ary signals decreases as M increases [16]. In our study, the SNR advantage of the N-orthogonal set over the orthogonal set vanishes for $M > 8$ with $L > 2$. Remember that L determines how many orthogonal dimensions exists in the N-orthogonal set. When both M and L grow, the second term in the left hand side of (4) (accounting for errors caused by wrong decision along ortogonal dimensions) dominates over the first term (accounting for errors caused by wrong decision among the N-signals sets).

The above observation is also true for the multipath channel, with the addition that the random variations in the signal correlation values accumulate more severely as L increases. [4]

More studies needs to be done with other types of pulses and channels to further explore the possibilities of N-orthogonal PPM signals.

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	Gaussian pulse	Gaussian pulse	Sine pulse	Sine pulse
M	SNR gain (dB) (AWGN)	SNR gain (dB) (Multipath)	SNR gain (dB) (AWGN)	SNR gain (dB) (Multipath)
2	2.09	1.30	2.90	
3			1.31	1.78
4	0.25	0.0		
6			0.16	0.24
8	0.05	-0.2		

TABLE III

VALUES OF SNR ADVANTAGE FOR DIFFERENT M IN AWGN AND MULTIPATH CHANNELS FOR THE UWB PULSES USED IN THE EXAMPLE.

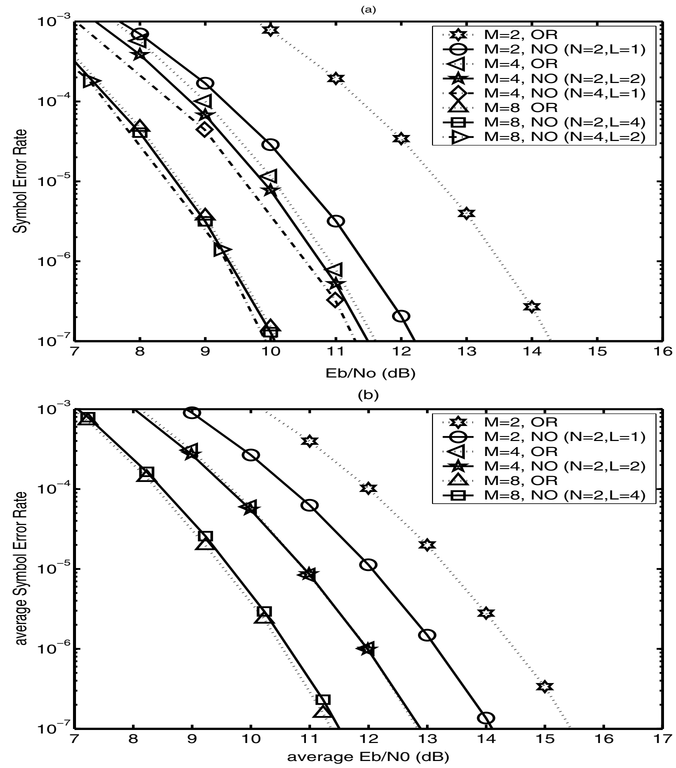

Fig. 1. SER for N-orthogonal vs. SER for orthogonal for $M = 2, 4, 8$; $N = 2, 4$; $L = 1, 2, 4$. (a) Gaussian channel, and (b) Multipath channel. The UWB pulse is the second derivative of a Gaussian pulse in (8).

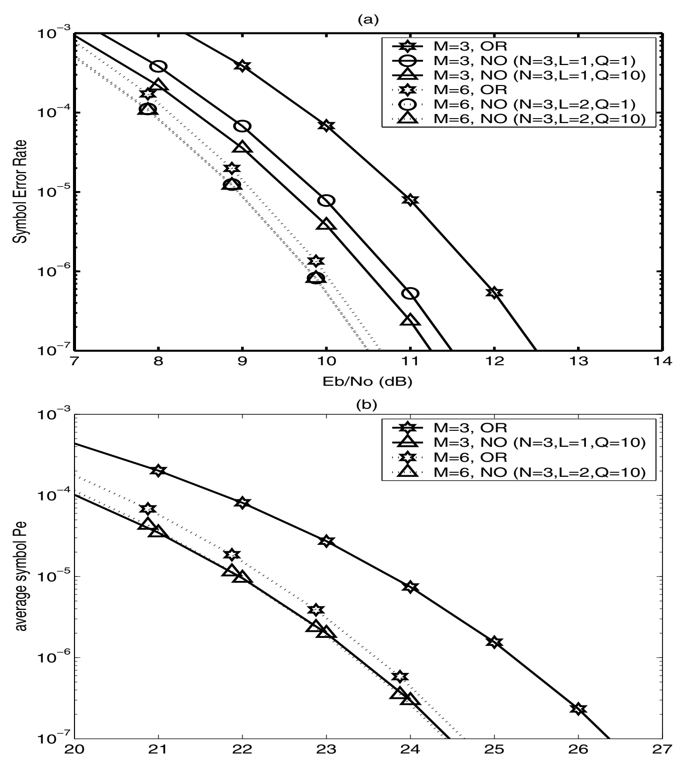

Fig. 2. SER for N-orthogonal vs. SER for orthogonal with $M = 3, 6$; $N = 3$; $L = 1, 2$. (a) Gaussian channel, and (b) Multipath channel. The UWB pulse is the pulsed sinewave in (10) with $Q = 1, 10$.