Error Probability of Ultra Wideband SSMA in a Dense Multipath Environment

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Abstract— **This work study the error probability of ultra wideband (UWB) spread spectrum multiple access (SSMA) for a system in which all the active users transmit in a channel containing both additive white Gaussian noise and dense multipath. Using M-ary pulse position modulated signals the multiple-access performance is analyzed in terms of the bit error rate for a given number of active users in the system, signal-to-noise ratio, bit transmission rate and number of signals in the M-ary signal set. The results can be applied to the performance analysis of impulse radio modulation.**

Keywords— **Ultra wideband communications, multipath channels, spread spectrum, multiple access, pulse position modulation, impulse radio modulation.**

I. INTRODUCTION

ECENT studies [1] - [12] has shown that UWB SSMA can potentially enable radio communications with low-power transmission and with reduced fading margin, making UWB systems good candidates for short-range high-speed indoor wireless communications In this paper we study the error probability of UWB SSMA in a dense multipath environment. To formulate the problem, we describe two previous results found in the literature.

The first result is the performance analysis of UWB SSMA using time hopping and block waveform encoded (M-ary) pulse position modulated (PPM) signals [8]. For different M-ary PPM signal designs the multiple-access performance in free-space propagation conditions and additive white Gaussian noise (AWGN) was analyzed in terms of the bit error rate (BER) for a given number of active users in the system , signal-to-noise ratio (SNR), bit transmission rate and number of signals in the M-ary signal set. For the bit signal-to-interference ratio (SIR) of the user of interest is

SIRb
MA

where is the bit energy, is the signal correlation function, is the (one sided) AWGN power spectrum density (PSD), and MA is the *effective PSD* of the multiple-access (MA) interference caused by other active users operating in the system. When only the user of interest is present and MA , therefore

SIRb

The second result is the performance analysis of a single-link UWB communications in the presence of AWGN and dense multipath using M-ary PPM signals and filters perfectly matched to the received waveform [7]. For the bit SNR is

SNRb

where is the average received bit energy, is a random variation in the received signal energy, is the random signal correlation value, and denotes a random spatial location of the receiver with respect to the transmitter within the multipath channel under study. The average performance is obtained by taking the expected value of the BER with respect to .

In this paper we study the performance of UWB SSMA using time hopping and M-ary PPM for communications in a channel containing both AWGN and multipath, hence connecting the results in [8] and [7]. We develop an expression for the bit SIR that is conditioned on , which for the case has the form

SIRb
MA

The averaged multipath performance will be obtained by taking the expected value of the BER with respect to .

II. SYSTEM MODEL

The system model is explained in detail in both [8] and [7]. This section includes new elements related to the effects of multipath in both the desired signal and the MA interference.

A. Transmitted signals

Each user utilizes the same type of M-ary time hopping PPM signals to convey information. The *transmitted* signal are described by

TX TX (1)

where the superscript , , indicates user-dependent quantities, the index is the number of time hops that the signal TX has experienced, is the average time between pulse transmissions, and TX is the UWB pulse used to build the transmitted PPM signals.

For a given time shift parameter , the pseudo-random time hopping code provides an additional time shift to the pulse in every frame, each time shift being a discrete time value , with .

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The data sequence is an M-ary symbol stream of equally-likely symbols, . The time shift corresponding to is , with . The data symbol changes only every hops, i.e., the system uses fast time hopping.

To avoid overlapping of pulses belonging to different frames in (1), we assume that , where is defined in section II-B.2.

B. Multipath channel

In the system model under consideration all active users experience the same multipath environment. User one will be the user of interest.

B.1 Multiple-path trajectories

For each active link the corresponding transmitter stays fixed at certain arbitrary position, and the corresponding receiver moves in a spatially random fashion.

In particular, the link between user one's receiver and user 's transmitter defines a multiple-path propagation trajectory that is a function of the relative position of user one's receiver with respect to the position of user 's transmitter. This random trajectory will be identified with the random index . There will be of such trajectories, one for every pair (user 's transmitter, user one receiver), .

When user 's transmitter radiates the signal $_{TX}$, the signal detected by user one's receiver will be represented by , where denotes time. As we move user one's receiver position, these trajectories change. Hence, the received waveforms coming from each of the transmitters also change.

B.2 Channel effect in the UWB pulse

The UWB signal $_{TX}$ is the basic pulse used to convey information. Typically an UWB signal is defined as any signal in which the 3 dB bandwidth of the signal is at least percent of its center frequency [2]. The pulse under consideration is characterized by a radiated spectrum with a very wide bandwidth (a few Gigahertz) around a relatively low center frequency (one or two Gigahertz). The duration of the pulse $_{TX}$ is denote and is in the order of a few nanoseconds.

In an indoor multipath channel, transmission of the pulse $_{TX}$ results in a received pulse . The pulse is a multipath spread version of . The average duration of is denoted , and can be in the order of a few hundreds of nanosecond, hence . We will assume that is the mean delay spread of the channel.

The pulse has random energy , where is the average received energy, and is the normalized energy. The pulse has normalized signal correlation

The normalized signal cross correlation corresponding to pulses received with two different trajectories and is

where

C. Received signals

The time hopping PPM *received* signal are

(2)

The signal in (2) is a multipath spread version of $_{TX}$ in (1) received with trajectory . Here we have assumed that the channel is slowly time invariant, therefore the PPM signal is composed of shifted version of the same multipath pulse. We will further assume that has fixed duration .

To define signal energy and correlation, lets make , with . The signals in (2) have random energy

for all , where is the average symbol energy. The signals have normalized correlation values

for all .

When asynchronous transmitters are active, the received signal at user one's receiver position will be modeled as

(3)

where represent time asynchronisms between the clock of user 's transmitter and user one's receiver, is the ratio of average power used by user 's transmitter with respect to the average power used by user one's transmitter (with), and represents non MA interference modeled as AWGN.

D. Statistical averages

The signal in (3) is a random process that depends on the random noise and three other types of random variables: The random time delays, denoted by the vector ; the random time hopping codes, denoted by the vector , where each code for all ; and the random multiple-path trajectories indexes, denoted by and the vector . The , and for all , are deterministic because the receiver will be assumed to be perfectly synchronized.

Performance computation is based on SIR ratios and BER rates averaged over all random variables. For a given value of , we first calculate the conditioned expected value of SIR with respect to , , and . The final answer is the expected value of BER SIR with respect to . To facilitate our analytical treatment, the following assumptions are made

1. The statistical behavior of the multipath channel depends only on the relative position of the receiver with respect to the transmitter. The spatial distribution of the users is uniform, and every belongs to the set of all possible trajectories in the channel. This assumptions means that we can treat , , as independent, identically distributed (i.i.d.) random variables, with each uniformly distributed over its range.
2. The receiver is able to perfectly match user one's received signal, i.e., is perfectly estimated.
3. The expected values with respect to can be approximated with sample averages based on parameters calculated using measured received waveforms.
4. The elements for and for all , are i.i.d random variables. Each is uniformly distributed on the interval .
5. The transmission time differences , for , are i.i.d random variables, with mod being uniformly distributed on .
6. The d .
7. The $\delta_{d(\nu}$ for 3 and all . This requires that the maximum time shift produced by the data is limited to $<<$; and similarly $<<$.
8. The maximum time shift produced by the time hopping code is limited to $<$ ϵ, where $\epsilon \triangleq$ $_a$ is two times the average duration of .

With these assumptions the net effect of the multiple access interference at the output of the demodulation circuit can be modeled as a zero mean Gaussian random variable with *random (position-dependent)* variance.

III. Demodulation and performance

Performance analysis in the absence of multipath was studied in [8]. Those results can be used to analyze performance in the presence of multipath, provided that we use the right interpretation.

To simplify notation, in the following analysis we will drop the super-index from A , x_j , , , and d . When necessary, we will make the distinction between time shift used in γ , the propagation delay , and the data-dependant time shift $\triangleq$.

We will present results for equally correlated PPM signals. The analysis can be generalized to signal sets with different correlation properties.

A. Signal detection

Let's assume that the receiver wants to demodulate user one's signal. When d j, the received signal r in (3) can be rewritten

$$r \quad A x_j \quad n_{\text{TOT}} \in \mathcal{T}$$

where $\mathcal{T} \triangleq$ $_s$, and

$$n_{\text{TOT}} \triangleq \sum_{=2}^{N_u} A \quad x_{d(\nu} \quad g$$

For the time being, let's assume that user one's receiver is static at one place, and that user one's transmitter is at a fixed position, i.e, is kept fixed.

In the present analysis signal detection is achieved using a Rake receiver [13]. In this work we generalize the Rake concept to consider a perfectly synchronized receiver that is able to perfectly match a signal consisting of the superposition of a number of pulses, each pulse with a different amplitude, time delay, phase and frequency content.

We will assume that the analysis in [8] is valid for every finger of the Rake receiver. We further assume that the signal-dependent self-noise produced by the cross-correlation of the signals at the input of two different fingers is negligible.[2] This requires that the average time between pulse transmissions be larger that the delay spread of the channel, i.e., $>$ $_a$. Since $_s$ is usually large, the duration of the signal $s >> {}_a$, and intersymbol interference can be neglected. Hence, the performance results should be considered as lower bound, i.e., performance of an ideal Rake receiver.

With these assumptions, the output of the Rake receiver is approximately equivalent to the output of a filter perfectly matched to the signal, and the detection problem [16] can be analized as the time-shift-coherent detection of M equal-energy, equally-likely signals in the presence of Gaussian interference plus noise using a M-ary correlation receiver. Since the MA interfence is modeled as Gaussian noise, this correlation receiver is sub-optimum, the optimum receiver being some form of multi-user detector [13].

B. Performance conditioned on

In this example we use M-ary PPM equi-correlated signals [17]. These signals are defined by the time shift values δ_j a_j $_{\text{EC}}$, where a_j is a pattern representing the j^{th} cyclic shift of an m-sequence of length $_s$ [18]. The time shift value $_{\text{EC}} \in$ $_p$ is chosen such that the signal correlation $\beta_{j,i}$ β_{EC} $\frac{-\gamma \; \xi, \tau_{\text{EC}}}{2}$ is minimized in an average sense.

The union bound [16] on symbol error probability (SER) for time-shift-coherent detection of equi-correlated signals is denoted UBPe $|$, indicating that the probability of error depends on and, for the time being, is conditioned on . The expression for this probability of error is

$$\text{UBPe} \quad | \quad \frac{M}{} \int_{\sqrt{\log_2 M \; \text{SIRb}_{\text{EC}} \; N_u|\xi}} \frac{\exp \quad \rho^2}{\sqrt{\quad \pi}} \, d\rho \tag{4}$$

[1] In contrast with this UWB channel model [14], narrowband communications traditionally uses a channel model in which a multipath signal is represented as a superposition of a number of pulses, each pulse with a different amplitude, time delay and phase, but same frequency content [15].

[2] This assumption is reasonable considering the order of diversity achieved with UWB signals (approximately T_a/T_p). For PPM signals the cross correlations of the signals at the input of two different fingers is proportional to the sample cross correlation of the channels coefficients of the two Rake fingers.

where $Q(\cdot)$ denotes the Gaussian tail integral,

$$\text{SIRb}_{\text{EC}}(N_u|\xi) = \frac{(A^{(1)})^2 \overline{E}_b \alpha^2(\xi)}{N_o + N_{\text{MA}}^{(\text{EC})}(\xi)}[1 - \beta_{\text{EC}}(\xi)], \quad (5)$$

is the output bit SIR observed in the presence of $N_u - 1$ other users,

$$N_{\text{MA}}^{(\text{EC})}(\xi) \triangleq \sum_{\nu=2}^{N_u} \frac{(A^{(\nu)})^2 \overline{E}_b \alpha^2(\xi)}{(\mu_{\text{EC}}(\xi)/T_f)/R_b}[1 - \beta_{\text{EC}}(\xi)],$$

is the *equivalent power spectral density level* of the total MA interference, $R_b = \log_2(M)/T_s$ is the bit transmission rate,

$$\mu_{\text{EC}}(\xi) = \frac{m_p^2(\xi, \xi, 0, 0, \tau_{\text{EC}})}{2\sigma_p^2(\xi, 0, \tau_{\text{EC}})}$$

is a normalized SIR parameter defined in terms of the data modulation time shift τ_{EC} and the pulses $p(\xi, t)$ and $p(\xi^{(\nu)}, t)$, with

$$\sigma_p^2(\xi, 0, \tau_{\text{EC}}) \triangleq \mathbf{E}_{(\xi^{(\nu)}|\xi)}\{T_f^{-1} \int_{-\infty}^{\infty} m_p^2(\xi, \xi^{(\nu)}, \varsigma, 0, \tau_{\text{EC}})\, d\varsigma\} \quad (6)$$

and

$$\begin{aligned} & m_p(\xi^{(1)}, \xi^{(\nu)}, \varsigma, \theta, \eta) \\ & \triangleq \int_{-\infty}^{\infty} p(\xi^{(\nu)}, \varrho - \varsigma) \ \left[p(\xi^{(1)}, \varrho - \theta) - p(\xi^{(1)}, \varrho - \eta)\right] d\varrho \\ & = \begin{cases} \alpha^2(\xi^{(1)}) \cdot \\ \left[\gamma(\xi^{(1)}, \varsigma - \theta) - \gamma(\xi^{(1)}, \varsigma - \eta)\right], & \text{for } \nu = 1 \\ \tilde{\alpha}^2(\xi^{(1)}, \xi^{(\nu)}) \cdot \\ \left[\tilde{\gamma}(\xi^{(1)}, \xi^{(\nu)}, \varsigma - \theta) - \tilde{\gamma}(\xi^{(1)}, \xi^{(\nu)}, \varsigma - \eta)\right], & \text{for } \nu \neq 1 \end{cases} \end{aligned}$$

The $\mathbf{E}_{(\xi^{(\nu)}|\xi)}\{\cdot\}$ in (6) is the expected value operator with respect to $\xi^{(\nu)}$, conditioned on ξ. To calculate (5) we have used the assumption that $\xi^{(\nu)}$,$\nu = 1, 2, \ldots, N_u$, are i.i.d., hence the expected values in (6) are the same for all ν.

C. Averaged performance

The averaged performance can be obtained by taking the expected value $\mathbf{E}_\xi\{\cdot\}$ of (4) over all values of ξ to get

$$\overline{\text{UBPe}}\left(\frac{\overline{E}_b}{N_o}\right) = \mathbf{E}_\xi\{\text{UBPe}\,(N_u|\xi)\}, \quad (7)$$

where

$$\left(\frac{\overline{E}_b}{N_o}\right) \triangleq \mathbf{E}_\xi \left\{\text{SIRb}_{\text{EC}}(N_u|\xi)\right\}$$

is the averaged received bit SNR.

As in [7], we can make our calculations based on the *received* waveforms properly characterized. This is possible since the expected value in (7) is taken with respect to $\tilde{\alpha}^2(\xi)$, $\tilde{\beta}(\xi)$ and $\mu_{\text{EC}}(\xi)$; and the expected value in (6) is taken with respect to $\int_{-\infty}^{\infty} m_p^2(\xi, \xi^{(\nu)}, \varsigma, 0, \tau_{\text{EC}})\, d\varsigma$. Hence, we can calculate histograms of these quantities for a particular indoor channel environment and get a first approximation using the sample mean values

$$\overline{\text{UBPe}}\left(\frac{\overline{E}_b}{N_o}\right) \approx \frac{1}{\xi_*}\sum_{\xi=1}^{\xi_*} \text{UBPe}\,(N_u|\xi)\,,$$

and

$$\sigma_p^2(\xi_0, 0, \tau_{\text{EC}}) \approx \tfrac{1}{(\xi_* - 1)} \sum_{\substack{\xi=1 \\ \xi \neq \xi_0}}^{\xi_*} T_f^{-1} \int_{-\infty}^{\infty} m_p^2(\xi_0, \xi, \varsigma, 0, \tau_{\text{EC}})\, d\varsigma$$

IV. CONCLUSIONS

This performance analysis provides a theoretical matched filter bound for the best performance attainable when the multipath channel effects in user one's link are perfectly estimated.

Since we are not using a specific channel model, confidence limits can not be established properly. However, this calculation represents a first approximation to the performance of UWB SSMA in the presence of dense multipath in a particular indoor radio environment.

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