Signal Design for Ultra-Wide-Band Communications in Dense Multipath

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***Abstract*—Ultra-wide-band (UWB) wireless communications utilizes information signals characterized by a radiated spectrum with a very wide bandwidth around a relatively low center frequency. In this paper, we formulate the signal design for binary UWB communications taking into consideration the particular characteristics of UWB propagation in a dense multipath channel.**

***Index Terms*—Multipath channels, signal design, ultra-wide-band communications.**

I. Introduction

ULTRA-WIDE-BAND (UWB) wireless communications utilizes information signals characterized by a radiated spectrum with a very wide bandwidth around a relatively low center frequency. A signal has UWB nature if the 3-dB bandwidth of the signal is at least 25% of its center frequency [1]. With UWB signals, the wireless links can be operated with low-power transmission and with reduced fading margin [2], making UWB systems good candidates for short-range high-speed indoor wireless communications [3]–[7].

In this paper, we study signal design for binary UWB communications in a dense multipath channel [8]. Within the context of this work, signal design is the process of finding information signals that have "good distance" properties, and therefore provide good bit error rate (BER) performance [9]. The aim of this work is to formulate the signal design for binary UWB communications, taking into consideration the particular characteristics of UWB propagation in a multipath channel.

Signal design in this paper is based on distance properties between information signals. In particular, we will use the following normalized squared Euclidean distance[1] between signals $\Psi_1(t)$ and $\Psi_2(t)$ used in coherent communications [10]:

$$d^2(\Psi_1(t), \Psi_2(t)) \triangleq \frac{1}{2E}\int_0^T |\Psi_1(t) - \Psi_2(t)|^2 dt \qquad (1)$$

where T and

$$E = \int_{-\infty}^{\infty} [\Psi_i(t)]^2\, dt$$

$i = 1, 2$ are the duration and energy of each signal, respectively. Our job will be to find the signal parameters that maximize this distance and/or minimize the BER.

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[1]Notice that this distance has a value of one for orthogonal signals and a value of two for antipodal signals.

We first analyze signal design for a channel with free-space propagation conditions and additive white Gaussian noise (AWGN), denoted *Gaussian channel*. We then extend the signal design formulation for a channel with dense multipath and AWGN, denoted *multipath channel*. We calculate one numerical example in each case.

II. Signal Design in Gaussian Channel

The UWB signals considered in this analysis are based on subnanosecond pulses $p(t)$, which are generated and controlled relatively easily using current technology [1]. The effect of the Gaussian channel in the UWB pulse $p(t)$ is described now.

The transmitted pulse is

$$p_{\mathrm{TX}}(t) \triangleq \int_{-\infty}^{t} p(\xi)d\xi$$

and the received signal is $p(t) + n(t)$ (we ignore attenuation and delay due to propagation). The effect of the antenna system in the transmitted pulse is modeled as a differentiation operation. The noise $n(t)$ is assumed AWGN with two-sided power density $N_o/2$. The received pulse $p(t)$ has duration T_p (i.e., is different from zero for an interval of time T_p) and energy

$$E = \int_{-\infty}^{\infty} [p(t)]^2\, dt.$$

The normalized signal correlation function of $p(t)$ is defined as the inner product of $p(t)$ with a shifted version $p(t-\tau)$

$$\gamma(\tau) \triangleq \frac{1}{E}\int_{-\infty}^{\infty} p(t)p(t-\tau)dt > -1\ \forall\tau. \qquad (2)$$

The binary information signals under study are defined using pulse position modulation (this modulation has been used in several discussions of UWB communications [2]–[4], [7]). More specifically, the binary transmitted signals are $\Psi_{\mathrm{TX}}^{(1)}(t) = p_{\mathrm{TX}}(t)$ and $\Psi_{\mathrm{TX}}^{(2)}(t) = p_{\mathrm{TX}}(t-\tau)$, for $0 \le t \le T$. Similarly, the binary received signals are $\Psi_1(t) = p(t)$ and $\Psi_2(t) = p(t-\tau)$, for $0 \le t \le T$, with $T_p + \tau < T$.

Using the definition in (1), we find the squared distance between *received* signals

$$\begin{aligned} d^2(\tau) =& \frac{1}{2E}\int_0^T |p(t) - p(t-\tau)|^2 dt \\ =& (1-\gamma(\tau)). \end{aligned} \qquad (3)$$

We will illustrate the use of $d^2(\tau)$ in (3) for signal design with an example. Consider a UWB pulse that can be modeled by the

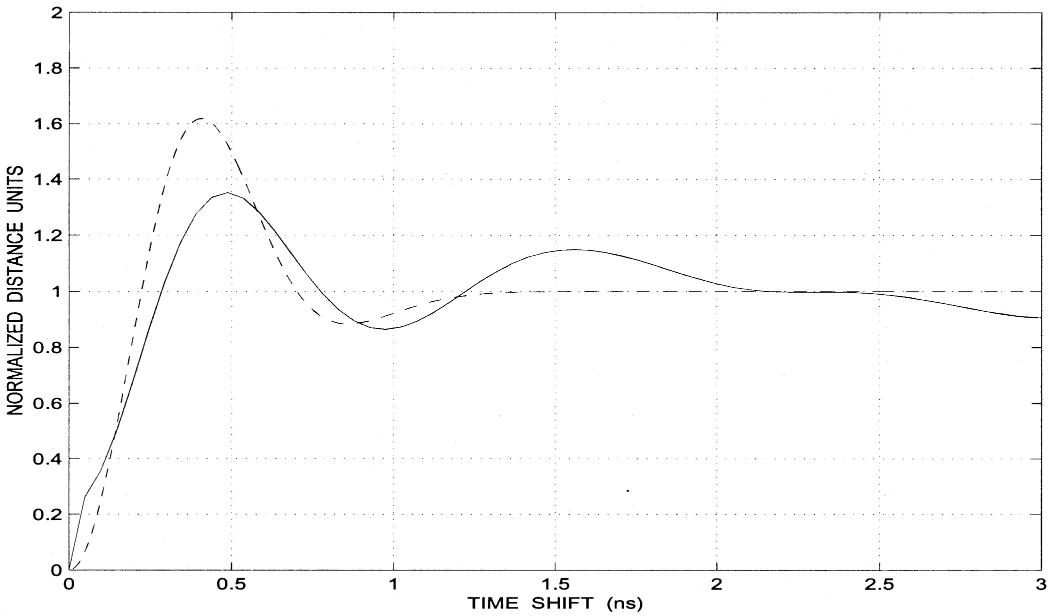

Fig. 1. The squared distance for the Gaussian channel $d^2(\tau)$ (dashed line) and the averaged squared distance for the multipath channel $\overline{d^2}(\tau)$ (solid line) as a function of time shift $0 \le \tau \le 3$ ns.

second derivative of a Gaussian function properly scaled. The transmitted pulse is

$$p_{\rm TX}(t) = t \exp\left(-2\pi \left[\frac{t}{t_n}\right]^2\right)$$

and the received pulse is

$$p(t) = \left[1 - 4\pi \left[\frac{t}{t_n}\right]^2\right] \exp\left(-2\pi \left[\frac{t}{t_n}\right]^2\right)$$

where the pulse parameter $t_n = 0.7531$ ns was used to fit the model $p(t)$ to a measured waveform $p_m(t)$ from a particular experimental radio link [11]. With this value of t_n, the duration of $p(t)$ is $T_p \simeq 2.0$ ns, with a 3-dB bandwidth of about 1 GHz. Furthermore, the spectrum of $p(t)$ is centered approximately at 1.1 GHz, satisfying the definition of UWB signal.

The squared distance is calculated using (2) and (3) to give

$$d^2(\tau) = 1 - \left\{\left[1 - 4\pi\left[\frac{\tau}{t_n}\right]^2 + \frac{4\pi^2}{3}\left[\frac{\tau}{t_n}\right]^4\right] \times \exp\left(-\pi\left[\frac{\tau}{t_n}\right]^2\right)\right\}. \quad (4)$$

Fig. 1 depicts $d^2(\tau)$ in (4).

The best signal design is the one that maximizes the squared distance. The distance $d^2(\tau)$ in (4) is maximized by choosing $\tau = 0.410 \triangleq \tau_{\rm best}$ ns to get a maximum squared distance $d^2_{\max} = d^2(\tau_{\rm best}) = 1.6181$.

Therefore, the best pair of binary transmitted signals is $\Psi^{(1)}_{\rm TX}(t) = p_{\rm TX}(t)$ and $\Psi^{(2)}_{\rm TX}(t) = p_{\rm TX}(t - \tau_{\rm best})$ because they result in the best pair of binary received signals $\Psi_1(t) = p(t)$ and $\Psi_2(t) = p(t - \tau_{\rm best})$. Notice that these signals minimize the binary BER

$$P_e(\tau) = Q\left(\sqrt{\frac{\lambda \cdot d^2(\tau)}{2}}\right) \quad (5)$$

for all signal-to-noise ratio (SNR) values λ [9]. The $Q(\cdot)$ in (5) denotes the Gaussian tail integral. For example, Fig. 2 shows that $P_e(\tau_{\rm best}) = 2.224 \cdot 10^{-3}$ is indeed the minimum BER value.

III. Signal Design in Multipath Channel

In this section, we analyze signal design for UWB binary communications in a dense multipath channel with AWGN. The channel can be, for example, a slowly varying indoor radio channel.

In this analysis, we will assume that the transmitter is placed at a certain fixed location and the receiver is placed at a variable location denoted u_o. The transmitted pulse is the same pulse $p_{\rm TX}(t)$ used in the Gaussian channel case, and the received signal is $\tilde{p}(u_o,t)+n(t)$. The pulse $\tilde{p}(u_o,t)$ is a multipath spread version of $p(t)$ received at position u_o. It has average duration $T_a \gg T_p$ and "random" energy

$$\tilde{E}(u_o) \triangleq \int_{-\infty}^{\infty} [\tilde{p}(u_o,t)]^2 dt.$$

The pulse has normalized signal correlation

$$\tilde{\gamma}(u_o,\tau) \triangleq \frac{1}{\tilde{E}(u_o)} \int_{-\infty}^{\infty} \tilde{p}(u_o,t)\tilde{p}(u_o,t-\tau)dt, > -1\, \forall \tau. \quad (6)$$

The binary transmitted signals are the same $\Psi^{(1)}_{\rm TX}(t) = p_{\rm TX}(t)$ and $\Psi^{(2)}_{\rm TX}(t) = p_{\rm TX}(t-\tau)$, $0 \le t \le \tilde{T}$, used in the Gaussian channel case. In the absence of noise, the binary received signals are $\tilde{\Psi}_1(u_o,t) = \tilde{p}(u_o,t)$ and $\tilde{\Psi}_2(u_o,t) = \tilde{p}(u_o,t-\tau)$, $0 \le t \le \tilde{T}$. The signal $\tilde{\Psi}_i(u_o,t)$, $i = 1,2$, is a multipath spread version of $\Psi_i(t)$ received at

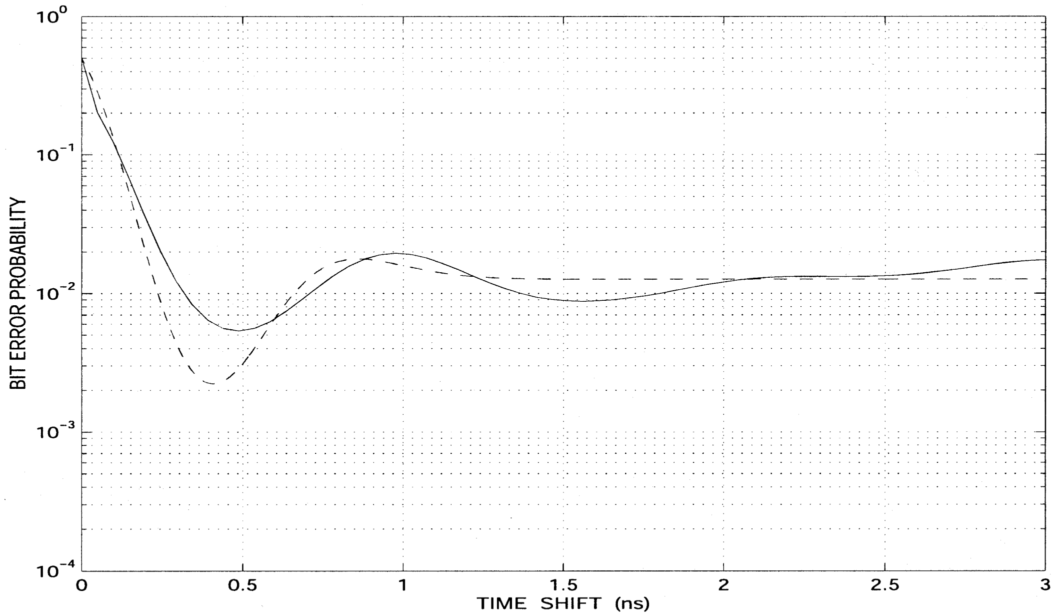

Fig. 2. The bit error probability for the Gaussian channel $P_e(\tau)$ (dashed line) and the averaged bit error probability for the multipath channel $\overline{P_e}(\tau)$ (solid line) as a function of time shift $0 \leq \tau \leq 3$ ns, for $\lambda = 10$ dB.

position u_o. It has duration $\tilde{T}$ with $T_a + \tau < \tilde{T}$.[2] The assumption that the multipath channel varies slowly with respect to the duration of the symbol waveform ensures that $\tilde{\Psi}_2(t)$ is a shifted version of $\tilde{\Psi}_1(t)$.

Using the definition in (1), we find the squared distance between *received* signals

$$\begin{aligned}\tilde{d}^2(u_o, \tau) =& \frac{1}{2\tilde{E}(u_o)} \int_0^{\tilde{T}} |\tilde{p}(u_o, t) - \tilde{p}(u_o, t-\tau)|^2 dt \\ =& (1 - \tilde{\gamma}(u_o, \tau))\end{aligned} \tag{7}$$

for $0 \leq \tau \leq T < \tilde{T}$. Obviously, the multipath effects change with the particular position u_o, and therefore the squared distance $\tilde{d}^2(u_o, \tau)$ between $\tilde{\Psi}_1(u_o, t)$ and $\tilde{\Psi}_2(u_o, t)$ also changes with that particular position u_o. The average squared distance can be obtained by taking the expected value

$$\overline{d^2}(\tau) = \mathbf{E}_u\{\tilde{d}^2\,(u_o, \tau)\} \tag{8}$$

over all values of u_o. Similarly, the variance of the squared distance can be obtained by taking the expected value

$$\sigma_d^2(\tau) = \mathbf{E}_u\{[\tilde{d}^2\,(u_o, \tau) - \overline{d^2}(\tau)]^2\} \tag{9}$$

over all values of u_o.

In general, the time shift that maximizes the mean of $\tilde{d}^2(u_o, \tau)$ does not necessarily minimize the variance of $\tilde{d}^2(u_o, \tau)$. In this sense, the UWB signal design in multipath is a problem that is different from the traditional signal design in AWGN. The approach taken in this paper is to find directly the signal design that minimizes the average binary BER

$$\overline{P_e}(\tau) = \mathbf{E}_u \left\{ Q\left(\sqrt{\frac{\lambda \cdot \tilde{d}^2\,(u_o, \tau)}{2}} \right) \right\}. \tag{10}$$

[2]The duration of $\Psi_{\mathrm{TX}}^{(i)}(t) = p_{\mathrm{TX}}(t), i = 1, 2$, is longer than the delay spread to avoid intersymbol interference

Notice that the degradation caused by total SNR fluctuations due to fading is not considered here; hence degradation in $\overline{P_e}(\tau)$ will be mainly caused by the signal correlation function distortions due to multipath. This is a reasonable approach for a slowly varying channel because we expect the energy of the signal to be a function of u_o but not a function of τ [12].

Usually, evaluation of $\overline{P_e}(\tau)$ in (10) would require precise statistical characterization of the channel. However, we can make our calculations based on the received binary waveforms properly characterized using the ensemble of pulse responses

$$\{\tilde{p}(u_o, t)\}, \quad u_o = 1, 2, \ldots, u_* \tag{11}$$

recorded in a measurement experiment in the multipath channel of interest. Calculation of $\overline{P_e}(\tau)$ in (10) for the indoor channel of interest can be approximated by using (7) in the sample mean value

$$\overline{P_e}(\tau) \approx \frac{1}{u_*} \sum_{u_o=1}^{u_*} Q\left(\sqrt{\frac{\lambda \cdot \tilde{d}^2\,(u_o, \tau)}{2}} \right). \tag{12}$$

We will illustrate the use of $\overline{P_e}(\tau)$ in (12) for signal design with an example.

Let the transmitted pulse be the same $p_{\mathrm{TX}}(t)$ that was used in the numerical example for the Gaussian channel case. Let the received pulse be represented by the ensemble in (11). This ensemble is formed with channel pulse responses $\tilde{p}(u_o, t)$, as described in [2]. The measured $\tilde{p}(u_o, t)$ has $T_a \simeq 300$ ns. A total of $u_* = 294$ channel pulse responses $\tilde{p}(u_o, t)$ were stored. An equal number of squared distance functions were calculated using (6) and (7). These $u_* = 294$ distances were then used in (12). The resulting $\overline{P_e}(\tau)$ is depicted in Fig. 2.

The best signal design is the one that minimizes the average BER. The probability $\overline{P_e}(\tau)$ in (12) is minimized by choosing $\tau = 0.487 \triangleq \tilde{\tau}_{\text{best}}$ ns to get a minimum BER $\overline{P_e}(\tilde{\tau}_{\text{best}}) =$

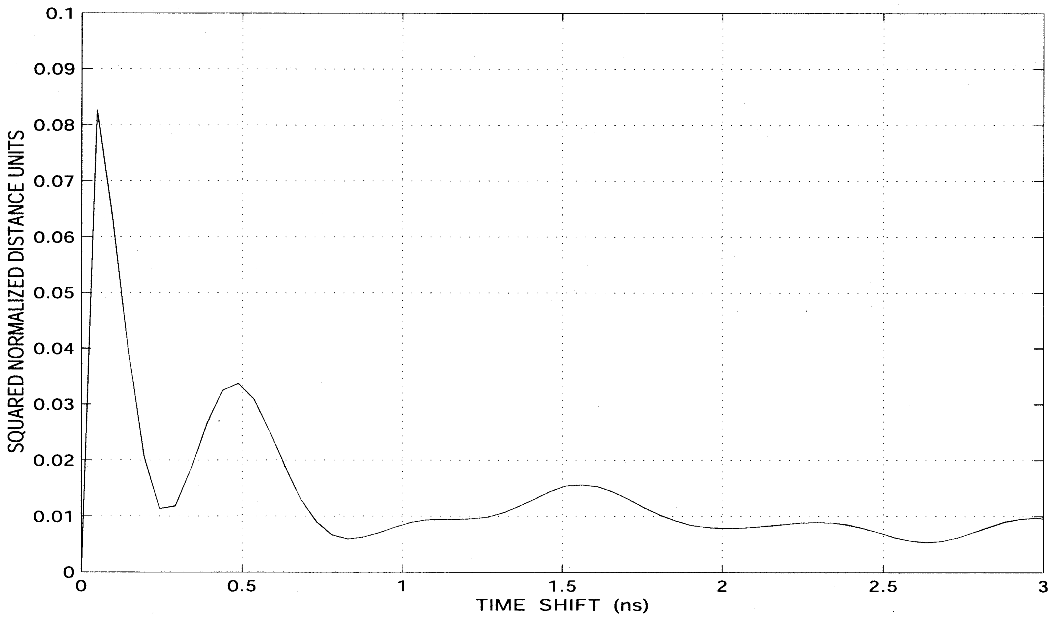

Fig. 3. The sample variance of the squared distance for the multipath channel $\tilde{d}^2(u_0, \tau)$ as a function of time shift $0 \leq \tau \leq 4$ ns.

$5.346 \cdot 10^{-3}$. Therefore, the best pair of binary transmitted signals is $\Psi_{\rm TX}^{(1)}(t) = p_{\rm TX}(t)$ and $\Psi_{\rm TX}^{(2)}(t) = p_{\rm TX}(t - \tilde{\tau}_{\rm best})$ because it results in the best pair of binary received signals in an average sense.

IV. Discussion

The example for the Gaussian channel shows a best parameter value $\tau_{\rm best} = 0.410$ ns to get a maximum squared distance $d^2(\tau_{\rm best}) = d^2_{\rm max} = 1.6181$ and a minimum BER $P_e(\tau_{\rm best}) = 2.224 \cdot 10^{-3}$.

The example for the multipath channel shows a best parameter value $\tilde{\tau}_{\rm best} = 0.487$ ns to get a minimum average BER $\overline{P_e}(\tilde{\tau}_{\rm best}) = 5.346 \cdot 10^{-3}$. Notice that in this example, $\tilde{\tau}_{\rm best}$ also maximizes the sample average of the squared distance

$$\overline{d^2}(\tau) \approx \frac{1}{u_*} \sum_{u_o=1}^{u_*} \tilde{d}^2(u_o, \tau) \tag{13}$$

where $\overline{d^2}_{\rm max} \triangleq \overline{d^2}(\tilde{\tau}_{\rm best}) = 1.351$; hence maximization of the Euclidean distance or minimization of BER provide similar results.[3] The $\overline{d^2}(\tau))$ is depicted in Fig. 1.

These examples show that the UWB signal design best suited for the Gaussian channel is different from the UWB signal design best suited for the dense multipath channel. The discrepancy in the signal design results can be explained by considering the particular characteristics of UWB propagation.

Recall that in narrow-band communications, the nonresolvable multipath signal is a superposition of a number of pulses, each with a different amplitude, time delay, and phase, but with no change in the frequency content [14].

In contrast, in wide-band communications, the quasi-resolvable multipath signal is a superposition of a number of pulses, each with a different amplitude, time delay, phase, and frequency content (the higher frequencies are likely to be attenuated more than lower frequencies) [15]. In UWB communications over a dense multipath channel, this effect is more notorious. Each resolvable component has different frequency content since a UWB pulse suffers frequency distortions as it propagates through walls and other obstacles.

Hence, in a frequency-selective channel, the UWB signal provides more diversity, but the pulse in each resolvable path is slightly distorted depending on the particular propagation trajectory (the last pulses to arrive are more likely to be more distorted than the first ones). These frequency distortions translate into changes in the Euclidean distance between signals, as can be appreciated by looking at the sample variance of the squared distance

$$\sigma_b^2(\tau) \approx \frac{1}{u_*} \sum_{u_o=1}^{u_*} (\tilde{d}^2(u_o, \tau) - \overline{d^2}(\tau))^2. \tag{14}$$

The $\sigma_b^2(\tau)$ is depicted in Fig. 3. By comparing Figs. 1 and 3, we notice that indeed, the time shift that minimizes the mean $\overline{d^2}(\tau)^2$ does not minimize the variance $\sigma_b^2(\tau)$.

V. Conclusion

We formulated signal designs for a Gaussian channel and for a multipath channel. In these signal designs, the squared distance depends on the time shift parameter τ. Hence, the BER also depends on τ.

The signal design for multipath channel formulated here assumes that the receiver is able to perfectly match the received signals. However, it can be modified to design mismatched signals that are an approximation to the received signals.

The sample average distance calculation in this paper, although just an approximation, has the virtue that once the experimental data is processed, it can generate different signal designs in just a few seconds.

[3]Estimation of $\mathbf{E}_u\{z(\mathbf{u})\}$ by $z(\mathbf{E}_u\{\mathbf{u}\})$, where $z(\cdot)$ is a smooth function, is studied in [13].

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