MOORE PARK, SURRY HILLS

S·H·S

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 1998

MATHEMATICS

4 UNIT

Time allowed: 3 Hours (plus five minutes reading time)

Total Marks: 120

Examiner: C.Kourtesis

DIRECTIONS TO CANDIDATES

ALL questions may be attempted.

All necessary working should be shown in every question. Full marks may not be awarded for

Standard integrals are provided. Approved calculators may be used.

Each question attempted is to be returned on a separate answer sheet. Each answer sheet must show your name.

Additional answer sheets may be obtained from the supervisor upon request.

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject

Question 1. (15 marks) (Start a new answer sheet)

(a) Find

(i) [illegible]

(ii) $\int \frac{1}{x}(1+\ln x)^3 dx$ **3**

(b) Evaluate $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$ by using the substitution $u = 1 + \ln x$. **3**

(c) Evaluate $\int_0^5 \frac{x}{\sqrt{x+4}} dx$. **3**

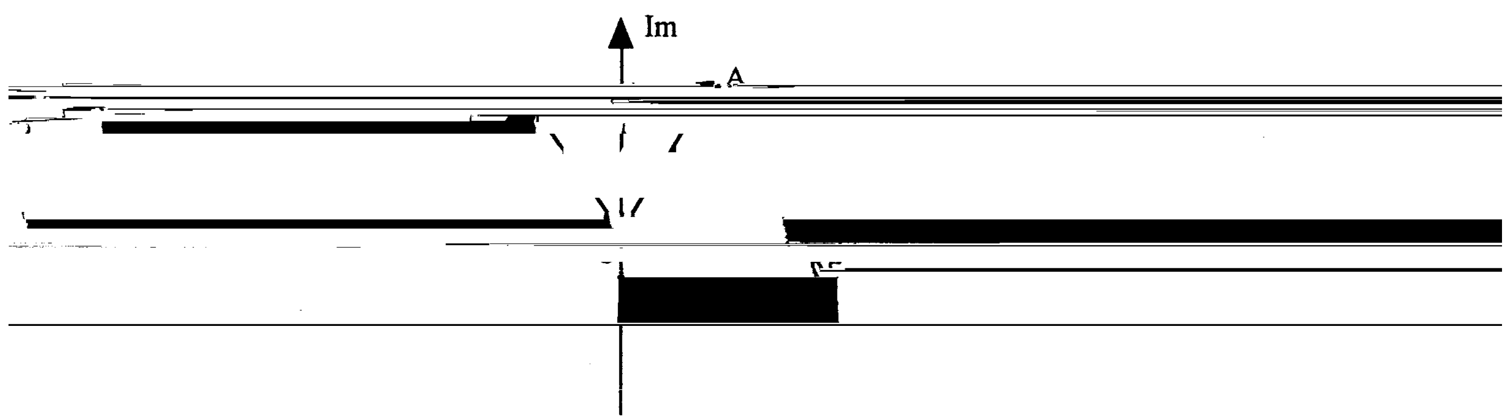
(d) Evaluate $\int_0^2 \frac{8}{(x+2)(x^2+4)} dx$. 4

Question 2. (15 marks) (Start a new answer sheet.)

(a) (i) Express $z = 2 + 2i$ in modulus-argument form. **2**

(ii) Hence write z^8 in the form $a + ib$ where a and b are real **2**

(b)



In the Argand diagram point A corresponds to the complex number $1 + i\sqrt{2}$. If the origin, A, and B are the vertices of an equilateral triangle what complex number corresponds to the vertex B? **2**

(c) Find the locus of z if $\text{Re}(z) = |z|$. **2**

(d) If a, b, c, d are real, and $ad > bc$, show that $\text{Im}\left(\dfrac{a+ib}{c+id}\right) < 0$. **2**

(e) If P represents the complex number z, where z satisfies

$$|z-2| = 2 \text{ and } 0 < \arg z < \frac{\pi}{2}:$$

(i) Show that $|z^2 - 2z| = 2|z|$.

(ii) Find the value of k (a real number) if $\arg(z-2) = k \arg(z^2 - 2z)$. **3**

Question 3. (15 marks) (Start a new answer sheet.)

(a) The equation $2x^3 + 5x + 1 = 0$ has roots α, β, γ. Evaluate $\alpha^3 + \beta^3 + \gamma^3$. **2**

(b) Given the polynomial $P(x) = 2x^3 - 4x^2 + mx + n$ where m and n are real numbers:

[illegible]

(ii) Find the zeros of $P(x)$. **1**

(c) A monic cubic polynomial when divided by $x^2 - 9$ leaves a remainder of $x + 8$ and when [illegible] **3**

(d) (i) By letting $c = \cos\theta$, show that the equation $\cos 4\theta = \cos 3\theta$ can be expressed in the form $8c^4 - 4c^3 - 8c^2 + 3c + 1 = 0$. **2**

(ii) Show that $\theta = \dfrac{2n\pi}{7}$, where n is an integer, satisfies the equation $\cos 4\theta = \cos 3\theta$. **2**

(iii) Using parts (i) and (ii) above, find the equation whose roots are **2**

$\cos\dfrac{2\pi}{7}$, $\cos\dfrac{4\pi}{7}$, $\cos\dfrac{6\pi}{7}$, expressing your answer in polynomial form.

Question 4. (15 marks) (Start a new answer sheet.)

(a) If $f(x)=\dfrac{2-x}{2+x}$ sketch the graphs of:

(i) $y=f(x)$ **2**

(ii) $y=[f(x)]^2$ ~~by finding~~ the turning points. **3**

(iii) $y=\sqrt{f(x)}$ **2**

(iv) $y=\ln[f(x)]$ **2**

(b) (i) On the same set of axes shade in the region satisfying both **2**

$$x^2+y^2\le 1 \text{ and } x^2\le\frac{8}{3}y.$$

$y \geqslant \dfrac{3x^2}{8}$

(ii) The area in part (i) is rotated about the y axis through one complete revolution. Using the cylindrical shell method find the volume of the solid generated. **4**

Question 5. (15 marks) (Start a new answer sheet.)

(a) An ellipse has equation $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$.

(i) Find the eccentricity, co-ordinates of the foci S and S', and the equations of the **3**

(ii) Find the equation of the tangent to the ellipse at a point $P(3\cos\theta, 2\sin\theta)$ on it, where θ is the auxiliary angle. **2**

(iii) The ellipse meets the y axis at the points A and B. The tangents to the ellipse at A and B meet the tangent at P at the points C and D respectively. **4**

Prove that $AC.BD=9$.

(b) (i) If ω is the root of $z^5-1=0$ with the smallest positive argument, find the real **4**

(ii) Given that $z=X+iY$ and $w=x+iy$ where $z=w^n$ for positive integers n, prove that $X^2+Y^2=\left(x^2+y^2\right)^n$. **2**

Question 6. (15 marks) (Start a new answer sheet.)

(a) Prove that the curve $\sqrt{x}$ [illegible] $\sqrt{y}$ [illegible] $= 1$ touches the y-axis (x and y are positive constants) **2**

(b) A body of unit mass is projected vertically upwards against a constant gravitational force g and a resistance $\frac{v}{[illegible]}$, where v is the velocity of the projectile at a given time t. The initial [illegible]

(i) [illegible] $\frac{dv}{[illegible]}$ [illegible] v [illegible] **1**

(ii) [illegible]

(iii) [illegible]

$$H = 2000 - 100g\left[1 + \ln\left(\frac{20}{g}\right)\right].$$

(iv) The particle falls to its original position under gravity and under the same law of resistance.

(α) What is its terminal velocity? **2**

(β) Will the time taken to reach the maximum height be greater or less than the time taken to fall to the original position from the maximum height? (Give reasons for your answer.) **2**

(Hint: Consider the magnitude of acceleration at any time t for each of the upward and downward motions of the particle.)

Question 7. (15 marks) (Start a new answer sheet.)

(a) Assume that ten distinct points are drawn on a number plane, no three of which are collinear. Find:

[illegible]

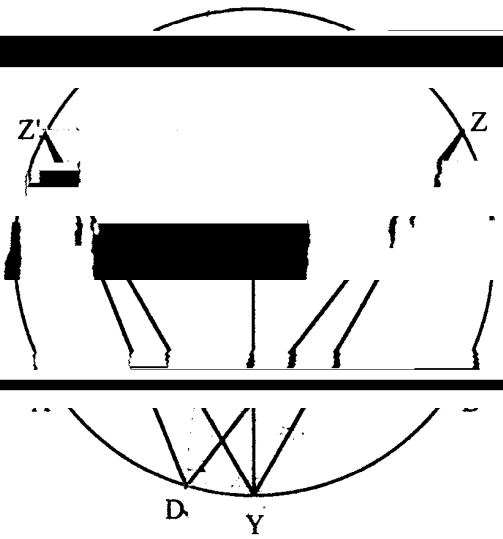
(ii) How many diagonals a convex decagon would have if these ten points were the vertices of the decagon? **2**

(b) If the roots of the polynomial equation $x^n - 1 = 0$ are $1, \alpha_1, \alpha_2, \ldots \alpha_{n-1}$ prove that **3**

$$(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)\ldots(1-\alpha_{n-1}) = n.$$

(c) Show that if $x > 0$, then $\int_0^x \frac{t^{n-1}}{1+t}\,dt < \frac{x^n}{n}$, (give reasons). **3**

(d)



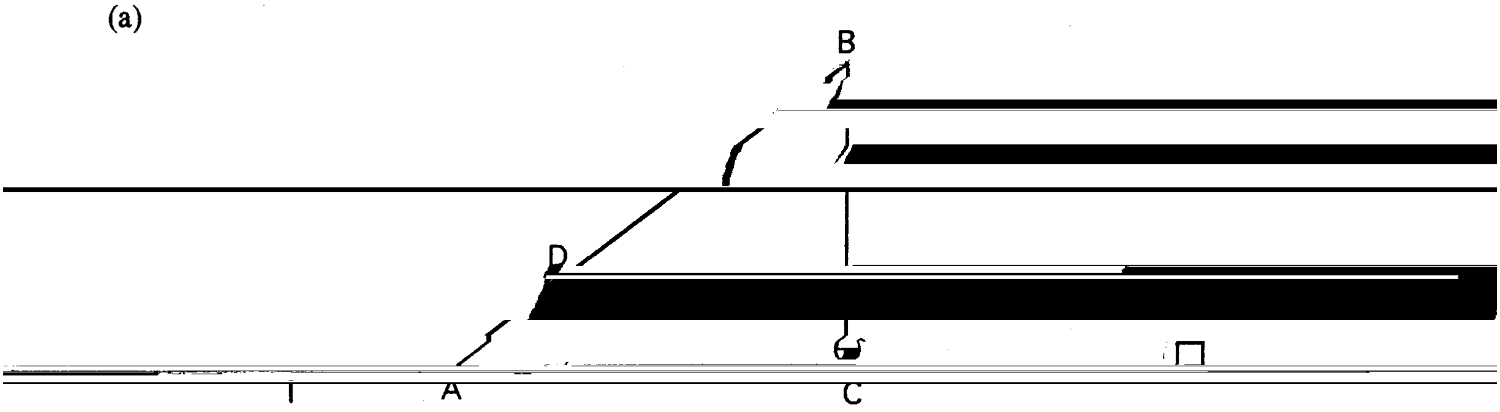
[illegible]

(i) Prove that C', X, Y, D are concyclic.

(ii) Prove that $CY \geq XD$.

Question 8. (15 marks) (Start a new answer sheet.)

(a)



[redacted]

T on the ground below is 30° and from a point D, three-quarters of the way down the slope the angle of depression of the point T is 15°.

[redacted]

(b) (i) State the binomial theorem for $(1+x)^n$ where n is a positive integer. **2**

(ii) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots$ **2**

(iii) [redacted]

(iv) Deduce that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = N$ where $2 < N < 3$. **3**

This is the end of the paper.