Model and Algorithm for Sea Cargo Mix Problem

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Abstract

In this paper, we study a sea cargo mix problem that occurs in the context of container shipment planning and revenue analysis in ocean carriers. We describe structure and characteristics of the cargo mix problem, and formulate as a Multi-Dimension Multiple Knapsack Problem (MDMKP). In particular, the MDMKP is an optimization model that maximizes the total profit in several periods, subject to the limited shipping capacity and the limited number of empty containers in the origin port, etc. Algorithm is proposed to obtain the near optimal solution for the problem. Numerical experiments demonstrate the efficiency of the algorithm.

Track area: SCM & e-logistics/ Operations Planning &Control.
Keywords: sea cargo mix, container shipping industry, MDMKP.

1. Introduction and Problem Description

In the recent economic environment, it is often the case that profits can only be maintained or increased by improving efficiency and cutting costs. This is particularly proverbial in the shipping industry, where it has been seen that the competition is very intensive among container carriers, thus alliances and partnerships are resulting for cost effective services in recent years. In this scenario, effective planning methods are important not only for strategic but also operating tasks, covering their entire transportation business (see [2][4][13][18][19]). Sea cargo mix planning is an important part of the operations of container shipping lines. This paper addresses the problem of sea cargo mix planning for containerized cargoes, to achieve the most profitable, time definite and cost-effective services in a competitive ocean container shipping market.

Shipping industry has started utilizing internet B2B exchanges, a marketplace where many buyer and sellers post buy and sell bids to make trades at dynamically determined market prices. Both shipping lines and shippers are moving towards e-logistics in all dealings. All documents can be received electronically, captured and processed directly into the e-logistics system seamlessly. Online booking request is made available 24 hours a day. Within the e-booking, shipper has to select the cargo from the commodity list provided, specify the size and type of container, and enter port of receipt or delivery on the latest departure and arrival date. For examples, Maersk-Sealand, Hanjin, APL and Hyundai are among the carriers offering such service (see their websites for details). Maersk-Sealand uses digital certificate to prove one’s identity and enable access to protected area to make booking online faster and safer. The shipper suggests the most appropriate vessel sailing from global schedule system displayed on screen and finally save all information. The shipper’s request will then be transmitted to the local Maersk-Sealand office, where confirmation of requirement together with the booking reference number will be sent back to the shipper.

For many containerized cargo carriers, the majority of their bookings are under long-term contracts with shippers. Such long-term agreements help carriers support requests for the large amounts of capital required to run their businesses. These long-term contracts are typically for one to three years. For some carriers, annual contracts all date to the same time of year and may cause a flurry of activity to re-negotiate terms and solidify the next year’s business. Regardless of timing, these contracts form a significant part of the revenue stream for a container carrier and negotiating the right prices and terms can be a major contributor to overall profitability. In practice, booking requests are often made less than 14 days before the freight becomes available for shipment, in which case the carrier may delay moving the freight if insufficient capacity is available. For example, the shipper may request a booking on a voyage that departs 9 days into the future, but if that voyage is already too fully booked, then the carrier may offer to transport the freight on a voyage that departs a week later, thus 16 days into the future. The shipper may then accept the booking offer, or decline the booking offer and request a booking with another carrier. Also, often shippers' cancel previously made bookings. It also happens that freight scheduled to depart on a voyage does not arrive at the port in time for the voyage. Approximately 30% of booked freight ends up not transported as originally booked. The booking control decision maker at the carrier has to decide, whenever a shipper requests a booking, what booking to offer the shipper. For example, the carrier may offer the shipper a booking on the voyage 9 days into the future, or on a voyage scheduled to depart a week later. The decision maker should take into account the revenue specified in the contract with the shipper, the probability of the shipper declining the booking offer, as well as uncertain future booking requests, cancellations and no-shows.

In practical situation, less-than-container-load (LCL) cargo orders come through telephone, email or website from established customers (these usually have their own shipping department), freight forwarders who act for customers or some new customers. Full-container-load (FCL) cargo orders are usually from big firms such as many electronics manufacturing companies. These big customers usually let the carrier know approximately the number of containers they will ship over the next three or four weeks (need to check if this practice still is in place). Agreement is loose and not legally binding.

The carrier will have the following priorities:

1) High paying FCL cargo (e.g., electronics)
2) Medium paying
3) Low paying (e.g., gypsum board)

If times are good and demand exceeds container space, the shipping company wills rollover those low paying cargo (i.e. not ship on schedule but postpone shipment to a later date). Under the contract, customers cannot sue the shipping company. If times are bad, the shipping company will look for low-paying cargo to fill the vessel rather than ship empties.

FCL cargo usually has no weight/volume specification because customers always pack to the fullest. Because maximum weight of 20-foot and 40-foot container are 18 MT and 40MT, respectively, so if cargo is volume cargo, you can pick in something like 40 cu meters and yet the cargo weighs less 20 tons. For weight cargo (e.g., steel), 20 tons is easily reached when the container is 1/3 full. You just cannot pack in anymore (legal requirement). Most LCL cargo tends to be volume cargo and the shipping company prefers that because they charge by weight or volume. The shipping company takes all orders but those they can't ship, they will inform the customers after the ship sails.

Sea cargo mix problem arises when a liner operator wants to select cargoes in order to maximize his revenue for some particular trips. Optimal cargo mix technique needs data from the online booking system and container tracking system to provide current status and location of container inventory as well as details of all costs associated with shipments. Historical data is gathered or built up over time to permit forecasting of container flows and inventory levels. From demands, bookings, available capacity and average contributions, it mathematically determines an optimal target contribution to determine whether to accept or reject a proposed booking, or to suggest a surcharge or alternate product or service at a higher/lower price. The result would be used to adjust contribution calculations to favor certain types of freight on certain routes and discourage others.

Each type of cargo has different volume/ weight characteristic, different freight rate and generates different revenue. For example, the tradeoff in FCL and LCL cargo mix is important operational issue for liner operators. Shipping companies offer customers priority, time-bound forwarding for LCL and FCL shipments with fast transits, reliable schedules and competitive price. A FCL of electronics pays higher than a FCL of gypsum board. Furthermore, if the inbound vessel is full, and if this is not balanced by a full outbound, then there is the need to ship empties necessitated by the need to re-position the empties for use by the destination port. This too has to be taken into account in the optimal revenue analysis. The repositioning of empty containers is one of the single largest expenses for most container carriers. Regarding container demand and supply it minimizes empty repositioning costs and maximize vessels utilization and it lead to a pro-active pricing mechanism for improved revenues and reduced costs. In order to optimally use vessels optimally matched where possible according to customer requirements to minimize empty repositioning costs. This takes into account excess empties and will result in optimal utilization of vessels while total transportation costs (for both full and empty containers) are reduced and trade imbalances will be compensated. Many different models have been proposed in the literature to obtain an optimal container repositioning planning. However, most of them did not consider it with transportation of laden containers and revenue management (see [1][3][9]).

In our work, we abstract the FCL and LCL cargo and the empty container as different type of cargo. Further more, in the model proposed in our paper, we consider more detailed information concerned with each type of cargo. Such information include the weight, volume, shipment price and shipment due date requirement, etc.

To our knowledge, up until now, there is no any description and formulation for the multi-period sea cargo mix problem in the literature. Single period sea cargo mix problem is proposed by [5] and it is formulated as linear programming model by [10], when all cargoes are separable for shipment. Literature on related mathematical programming models and algorithms see related Sections.

The rest of the paper is organized as follows. In Section 2, the multi-period sea cargo mix problem is formulated as the Multi-Dimension Multiple Knapsack Problem (MDMKP). Section 3 presents algorithm for the model. The heuristic algorithm with effective gradient and its modified algorithm are given in this section. We demonstrate the algorithm with effective gradient by a simple example in this section, followed in Section 4 by numerical experiments of the algorithm with a wide range of problem instances. Section 5 concludes the paper.

2. Mathematical Formulation

There are several technical constraints relative to how a ship should be loaded. The obvious constraints are that cargo weight and volume should not exceed the ship's capacity. The operational objective is to select the loading parameters in order to maximize some profit criterion subject to the mathematical expression of the weight/volume constraints. If the freight rates are known and the operator can refuse or delay to carry cargoes, then the multi-period sea cargo mix problem can be formulated as MDMKP. The objective of the problem is to maximize a weighted sum of the quantities transported. The relative coefficients for the cargoes might reflect their unit profits.

In this section, we develop a mathematical model for the problem under consideration. For the ease of our discussion, we consider that one time period represents one day or one week and we use the term t to represent the t th time period.

In order to present the mathematical formulation of the model for a given planning time horizon, we introduce some notation:

Index Sets

$\tilde{T}$ set of time periods $\{1, 2, \cdots, t, \cdots, T\}$

$\tilde{J}$ set of ports $\{1, 2, \cdots, j, \cdots, J\}$

$\tilde{K}_t$ set of all cargoes received in period t, i.e., $\tilde{K}_t = \{1, 2, \cdots, k, \cdots, K_t\}$

Parameters

r_{tkdj_k} per volume revenue of cargo k which is received in period t and shipped to port j_k in period d

c_{tkdj_k} per volume cost of cargo k which is received in period t and shipped to port j_k in period d

h_{tk} per volume inventory cost of cargo k which is received in period t and delayed to carry in next period

τ_k due date of cargo k

E_t total volume of available empty containers at origin port in period t

V_{tj} total available volume capacity of shipment to port j in period t

W_{tj} maximum allowable weight capacity of shipment to port j in period t

$v_{tk\tau_k j_k}$ volume of cargo k received in period t ready for shipment to port j_k before its due date τ_k

$w_{tk\tau_k j_k}$ weight of cargo k received in period t ready for shipment to port j_k before its due date τ_k

Decision Variables

x_{tkdj_k} binary variable, i.e., $x_{tkdj_k} = 1$, if cargo k is received in period t and is ready for shipment to port j_k in period d before its due date τ_k, 0, otherwise.

In this paper, we assume that there are one origin port and J destination ports. All cargoes are received at origin port in a certain period of time horizon T. The shipping company decide which cargoes carry in the current period and which cargoes are delay to carry in the other period within their due date and which cargoes are refused to carry in the time horizon T.

Let $\bar{r}_{tkdj_k} = r_{tkdj_k} - c_{tkdj_k} - (d-t)h_{tk}$, $\forall\, t \in \tilde{T}$, $k \in \tilde{K}_t$, $j_k \in \tilde{J}$, $t \le d \le \tau_k$, our multi- period sea cargo mix problem can then be formulated as follows:

$$\text{(M1): } Maximize\ z = \sum_{t=1}^{T} \sum_{k\in\tilde{K}_t} \sum_{d=t}^{\tau_k} v_{tk\tau_k j_k} \bar{r}_{tkdj_k} x_{tkdj_k} \quad (1)$$

subject to

$$\sum_{t=1}^{d} \sum_{\substack{k\in\tilde{K}_t \\ \tau_k \ge d}} v_{tk\tau_k j_k} x_{tkdj_k} \le E_d \quad \forall\, d \in \tilde{T} \quad (2)$$

$$\sum_{t=1}^{d} \sum_{\substack{k\in\tilde{K}_t \\ \tau_k \ge d \\ j_k = j}} v_{tk\tau_k j_k} x_{tkdj_k} \le V_{dj} \quad \forall\, d \in \tilde{T}, \quad j \in \tilde{J} \quad (3)$$

$$\sum_{t=1}^{d} \sum_{\substack{k\in\tilde{K}_t \\ \tau_k \ge d \\ j_k = j}} w_{tk\tau_k j_k} x_{tkdj_k} \le W_{dj} \quad \forall\, d \in \tilde{T}, \quad j \in \tilde{J} \quad (4)$$

$$\sum_{d=t}^{\tau_k} x_{tkdj_k} \le 1 \quad \forall\, t \in \tilde{T}, \quad k \in \tilde{K}_t \quad (5)$$

$$x_{tkdj_k} \in \{0, 1\}\ \forall\, t \in \tilde{T}, k \in \tilde{K}_t,\ t \le d \le \tau_k \quad (6)$$

The objective function (1) maximizes the total profit in the time horizon T. Constraints (2) ensure that the demand for empty containers at origin port is less than the number of available empty containers at origin port in each period. Constraints (3) ensure that the total volume of cargoes which will be carried to port j in period d is less than the total available volume capacity of shipment to port j in period d. Constraints (4) ensure that the total weight of cargoes which will be carried to port j in period d is less than the total available weight capacity of shipment to port j in period d . Constraints (5) impose that each cargo may be carried in a certain period before their due date or be refused to carry in the time horizon T . Constraints (6) assure that each cargo can be either accepted at its total quantity or be turned down.

We may notice that the inequality sign in (5) is replaced by equality, if all received cargoes have to be carried in the time horizon T . In this case, the MDMKP (M1) may have not feasible solution, because the total available capacity of shipment may be less than the amount of demands in the time horizon T . On the other hand, MDMKP (M1) always has the optimal solution, because it has a trivial feasible solution $x_{tkdj_k} = 0$, for all $t \in \tilde{T}$, $k \in \tilde{K}_t$, $d \in \{t, t+1, \cdots, \tau_k\}$, and its LP relaxation optimal value is an upper bound of its optimal value.

3. Heuristic Algorithm for MDMKP

The 0-1 or binary integer programming is an integer programming whose all decision variables are 0-1 values. An important class of binary integer programming problems is the Knapsack Problem (KP). The Single Knapsack Problem (SKP) is one kind of KP where only one knapsack needs to be filled and each item may be chosen at most once; If the items should be chosen from disjoint classes and exactly one item from each class, we get the Multiple-Choice Knapsack Problem (MCKP); The Multiple Knapsack Problem (MKP) is also a kind of KP where several knapsacks are to be packed simultaneously. The more general kind of KP is the Multi-Dimensional Knapsack Problem (MDKP), also known as Multi-Constrained Knapsack Problem, where there are more than one limited resources, i.e. the resources are multi-dimensional for the knapsack. And the MDMKP is the combination of MDKP and MKP. All Knapsack problems belong to the *NP-hard* family (see [6]), therefore it is impossible that polynomial time algorithms can be devised for them.

There are different algorithms for solving variants of KPs (see [14][15][16][17][11]). Generally, there are two methods of finding their solutions: one is a method for finding exact solutions and the other is heuristic. Finding exact solutions is NP hard. Using the branch and bound with linear programming technique, Kolesar ([9]), Shih ([20]), Nauss ([12]) and Khan ([7]) presented exact algorithms for KP, MDKP, MCKP and MDKP respectively. A greedy approach has been proposed by Khan et al ([7]), Martello and Toh ([11]) and Toyoda ([21]) to find near optimal solutions of KPs.

3.1 Heuristic Algorithm with Effective Gradient

MDMKP (M1) is different with all kinds of KPs mentioned in the literature and it needs to develop efficient algorithms for it. In this section, we give an efficient heuristic algorithm with the effective gradient for it.

Let $\bar{v}_{tkdj_k} = v_{tk\tau_k j_k} / V_{dj_k}$, $\bar{w}_{tkdj_k} = w_{tk\tau_k j_k} / W_{dj_k}$, then we can rewritten (M1) as follows

$$\text{(M2): } \mathit{Maximize}\ z = \sum_{t=1}^{T} \sum_{k \in \tilde{K}_t} \sum_{d=t}^{\tau_k} v_{tk\tau_k j_k} \bar{r}_{tkdj_k} x_{tkdj_k} \qquad (7)$$

subject to

$$\sum_{t=1}^{d} \sum_{\substack{k \in \tilde{K}_t \\ \tau_k \geq d}} v_{tk\tau_k j_k} x_{tkdj_k} \leq E_d \quad \forall d \in \tilde{T} \qquad (8)$$

$$\sum_{t=1}^{d} \sum_{\substack{k \in \tilde{K}_t \\ \tau_k \geq d \\ j_k = j}} \bar{v}_{tkdj_k} x_{tkdj_k} \leq 1 \quad \forall d \in \tilde{T}, \quad j \in \tilde{J} \qquad (9)$$

$$\sum_{t=1}^{d} \sum_{\substack{k \in \tilde{K}_t \\ \tau_k \geq d \\ j_k = j}} \bar{w}_{tkdj_k} x_{tkdj_k} \leq 1 \quad \forall d \in \tilde{T}, \quad j \in \tilde{J} \qquad (10)$$

$$\sum_{d=t}^{\tau_k} x_{tkdj_k} \leq 1 \quad \forall t \in \tilde{T}, \quad k \in \tilde{K}_t \qquad (11)$$

$$x_{tkdj_k} \in \{0,1\} \forall t \in \tilde{T}, k \in \tilde{K}_t, t \leq d \leq \tau_k \qquad (12)$$

When each of the candidate items (i.e. cargoes) involves only one limited resource, it is easy to pick up more profitable items. When there exists more than one limited resource in the candidate items, it is difficult to evaluate the amount of limited resources used. Toyoda (see [21]) introduced the concept of penalty vector, which results in a single index based on the many limited resources. Toyoda's heuristic for MDKP starts with no items, and adds one item at a time iteratively as long as the solution is feasible. At any iteration, the heuristic picks the item which provides the maximum value per unit of aggregate resource or the effective gradient among the not-yet-picked items. Computational experiments show that this heuristic provides very good near optimal solutions to the MDKP using very short computation time, and it can be applied to large problem instances, such as problems with more than a thousand variables (see [21]).

In this section, we propose a heuristic algorithm HAEG to provide a near-optimal solution to MDMKP (M2) by means of concepts such as penalty vector and effective gradient in [21].

The heuristic algorithm for MDMKP can be presented as follows.

Algorithm HAEG

Step 1: Initialization.

Step 1.1: Let $K_U \leftarrow \phi$, where K_U is the set of accepted items.

Step 1.2: Assign all items to $K_D = K - K_U$, where K_D is the set of items not in K_U and $K = \{k : k \in \tilde{K}_t, t \in \tilde{T}\}$.

Step 1.3: Let $A_U^{dj} \leftarrow (0, 0)$, where A_U^{dj} is the total quantity vector of accepted items shipping to port j in period d.

Step 1.4: Let the objective value be zero, i.e., $z \leftarrow 0$.

Step 1.5: Let $x_{tkdj_k} \leftarrow 0,$ for

$$\forall\ t \in \tilde{T}, k \in \tilde{K}_t, d \in \{t, t+1, \cdots, \tau_k\}.$$

Step 2: Let $K_C \leftarrow \{k : k \in K_D, \exists d \le \tau_k$

$$s.t. (\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k}) \le (1,1) - A_U^{dj_k},$$

$$v_{tk\tau_k j_k} \le E_d - \sum_{\substack{k' \in K_U \\ \tau_{k'} \ge d}} v_{tk'\tau_k j_k}\},$$

where K_C is the set of candidate items.

Step 3: Check K_C. If K_C is empty, the procedure terminates. Otherwise, proceed to the next step.

Step 4: Let $\bar{K}_C = \{(k,d) : k \in K_D,$

$$(\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k}) \le (1,1) - A_U^{dj_k},$$

$$v_{tk\tau_k j_k} \le E_d - \sum_{\substack{k' \in K_U \\ \tau_{k'} \ge d}} v_{tk'\tau_k j_k}\}.$$

Compute effective gradients for the items in K_C as follows.

Step 4.1: If $A_U^{dj_k}$ is a zero vector, then we set

$$G_{kd} \leftarrow \frac{\sqrt{2}\, v_{tk\tau_k j_k} \bar{r}_{tkdj_k}}{\bar{v}_{tkdj_k} + \bar{w}_{tkdj_k}}, \text{ for } (k,d) \in \bar{K}_C$$

Step 4.2: Otherwise, we set

$$G_{kd} \leftarrow \frac{v_{tk\tau_k j_k} \bar{r}_{tkdj_k} \sqrt{(\sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}})^2 + (\sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}})^2}}{\bar{v}_{tkdj_k} \sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}} + \bar{w}_{tkdj_k} \sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}}},$$

for $(k,d) \in \bar{K}_C$, where

$K_U^d = \{k' : k' \in K_U, x_{tk'dj_{k'}} = 1\}$.

Step 5: Find that item k whose effective gradient is the largest in a period, i.e.,

$$G_{kd} \leftarrow \max\{G_{k'd'} : (k',d') \in \bar{K}_C\}.$$

Step 6: Accept k. Let $K_U \leftarrow K_U + \{k\}$,

$$A_U^{dj_k} \leftarrow A_U^{dj_k} + (\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k}),$$

$$z \leftarrow z + v_{tk\tau_k j_k} \bar{r}_{tkdj_k},\ K_D \leftarrow K_D - \{k\},$$

$x_{tkdj_k} \leftarrow 1$. Then, goto **step 2**.

We may notice that the solution of HAEG is a feasible solution of MDMKP (M2) and the objective function value is a lower bound of the optimal solution value of it. The numerical experiments in Section 4 show that the solution of HAEG is very close to the optimal solution of MDMKP (M2).

3.2 A Simple Example

Let $\hat{r}_{tkdj_k} = v_{tkdj_k} \bar{r}_{tkdj_k}$, $\tilde{T} = \{1, 2\}$, $\tilde{J} = \{1, 2\}$, $\tilde{K}_1 = \{1, 2, 3, 4, 5\}$, $\tilde{K}_2 = \{6, 7\}$; $j_k = 1$, for $k = 1, 3, 5, 6$; $j_k = 2$, for $k = 2, 4, 7$; $\tau_1 = 1, \tau_k = 2$, for $2 \le k \le 7$; $(E_1, E_2) = (40, 32)$,

$k = (V_{12}, V_{22}) = (13, 21)$, $(V_{11}, V_{21}) = (39, 30)$,

$k = (W_{11}, W_{21}) = (35, 25)$, $(W_{12}, W_{22}) = (12, 38)$,

$(w_{1111}, w_{1321}, w_{1521}, w_{2621}) = (10, 13, 5, 9)$,

$(w_{1222}, w_{1422}, w_{2722}) = (13, 12, 8)$;

$(\hat{r}_{1111}, \hat{r}_{1311}, \hat{r}_{1511}) = (340, 660, 60)$,

$(\hat{r}_{1212}, \hat{r}_{1412}) = (182, 77)$,

$(\hat{r}_{1321}, \hat{r}_{1521}, \hat{r}_{2621}) = (506, 70, 170)$,

$(\hat{r}_{1222}, \hat{r}_{1422}, \hat{r}_{2722}) = (70, 231, 200)$.

We solve it by the heuristic algorithm with effective gradient HAEG given in Section 3.1. the procedure is briefly given as follows.

Since $\bar{v}_{tkdj_k} = v_{tk\tau_k j_k} / V_{dj_k}$, $\bar{w}_{tkdj_k} = w_{tk\tau_k j_k} / W_{dj_k}$, we have

$(\bar{v}_{1111}, \bar{v}_{1311}, \bar{v}_{1511}) = (0.4359, 0.5641, 0.1282)$,

$(\bar{v}_{1222}, \bar{v}_{1422}, \bar{v}_{2722}) = (0.3333, 0.5238, 0.8095)$,

$(\bar{w}_{1111}, \bar{w}_{1311}, \bar{w}_{1511}) = (0.2857, 0.3714, 0.1429)$,

$(\bar{w}_{1212}, \bar{w}_{1412}) = (1.0833, 1)$,

$(\bar{w}_{1222}, \bar{w}_{1422}, \bar{w}_{2722}) = (0.3421, 0.3158, 0.2105)$,

$(\bar{v}_{1212}, \bar{v}_{1412}) = (0.5385, 0.8462)$,

$(\bar{v}_{1321}, \bar{v}_{1521}, \bar{v}_{2621}) = (0.7333, 0.1667, 0.3333)$,

$(\bar{w}_{1321}, \bar{w}_{1521}, \bar{w}_{2621}) = (0.52, 0.2, 0.36)$.

Step 1. Let $K_U \leftarrow \phi$, all items be assigned to

$$K_D = K - K_U,\ z \leftarrow 0,\ A_U^{dj} \leftarrow (0, 0),$$

$$x_{tkdj_k} \leftarrow 0, \forall\ t \in \tilde{T}, k \in \tilde{K}_t, t \le d \le \tau_k.$$

Step 2. Let $K_C \leftarrow \{k : k \in K_D, \exists d \le \tau_k$

$$s.t. (\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k}) \le (1,1) - A_U^{dj_k},$$

$$v_{tk\tau_k j_k} \le E_d - \sum_{\substack{k' \in K_U \\ \tau_{k'} \ge d}} v_{tk'\tau_k j_k}\},.$$

Step 3. Since $K_C \ne \phi$, proceed to the next step.

Step 4. Since $A_U^{dj_k} = (0, 0)$, the effective gradients are calculated by formula:

$$G_{kd} = \frac{\hat{r}_{tkdj_k} \sqrt{2}}{\bar{v}_{tkdj_k} + \bar{w}_{tkdj_k}}, \text{ for } (k,d) \in \bar{K}_C,$$

we have $G_{11} = 666.33$, $G_{31} = 997.7$, $G_{51} = 313.04$, $G_{32} = 666.33$, $G_{52} = 269.99$,

$G_{62} = 346.75$, $G_{41} = 58.98$, $G_{22} = 146.56$, $G_{42} = 389.09$, $G_{72} = 277.28$.

Step 5. The largest effective gradient is $G_{31} = 997.7$.

Step 6. Accept item 3. Let $K_U \leftarrow K_U + \{3\}$,

$$A_U^{11} \leftarrow A_U^{11} + (\bar{v}_{1311}, \bar{w}_{1311}),\ z \leftarrow z + \bar{r}_{1311},$$

$K_D \leftarrow K_D - \{3\}$, $x_{1311} \leftarrow 1$. Then, go to step 2.

After it is repeated several times, K_C becomes empty and we obtain the final solution:

$z = 1471$, $x_{1111} = x_{1311} = x_{1521} = x_{2621} = x_{1422} = 1$, $x_{1511} = x_{1321} = x_{1212} = x_{1412} = x_{1222} = x_{2722} = 0$.

This solution is the same as the optimal one obtained by perfect enumeration.

3.3 Modified Heuristic Algorithm

In this section, we modify the effective gradient G_{ij} which introduced in Section 3.1 to increase accuracy of the heuristic algorithm.

Algorithm MHA

Step 1: Initialization.

Step 1.1: Let $l \leftarrow 0$, $\hat{z} \leftarrow 0$, $\hat{x}_{tkdj_k} \leftarrow 0$,

$$\forall\, t \in \tilde{T}, k \in \tilde{K}_t, t \le d \le \tau_k .$$

Step 1.2: Let $K_U \leftarrow \phi$, where K_U is the set of accepted items.

Step 1.3: Assign all items to $K_D = K - K_U$, where K_D is the set of items not in K_U and $K = \{k : k \in \tilde{K}_t, t \in \tilde{T}\}$.

Step 1.4: Let $A_U^{dj} \leftarrow (0, 0)$, where A_U^{dj} is the total quantity vector of accepted items shipping to port j in period d.

Step 1.5: Let the objective value be zero, i.e., $z \leftarrow 0$.

Step 1.6: Let $x_{tkdj_k} \leftarrow 0, \forall\, t \in \tilde{T},\ k \in \tilde{K}_t$, $t \le d \le \tau_k$.

Step 2: Let

$$K_C \leftarrow \{k : k \in K_D, \exists d \le \tau_k \ s.t. (\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k}) \le (1,1) - A_U^{dj_k},\ v_{tk\tau_k j_k} \le E_d - \sum_{\substack{k' \in K_U \\ \tau_{k'} \ge d}} v_{tk'\tau_k j_k} \},$$

where K_C is the set of candidate items.

Step 3: Check K_C. If K_C is empty, goto **step 7**. Otherwise, proceed to the next step.

Step 4: Let

$$\bar{K}_C = \{(k,d) : k \in K_D, (\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k}) \le (1,1) - A_U^{dj_k},\ v_{tk\tau_k j_k} \le E_d - \sum_{\substack{k' \in K_U \\ \tau_{k'} \ge d}} v_{tk'\tau_k j_k} \}.$$

Compute effective gradients for the items in K_C as follows.

Step 4.1: If $A_U^{dj_k}$ is a zero vector, then we set

$$G_{kd} \leftarrow \frac{\sqrt{2}\, v_{tk\tau_k j_k} \bar{r}_{tkdj_k}}{\bar{v}_{tkdj_k} + \bar{w}_{tkdj_k}},\ \text{for } (k,d) \in \bar{K}_C$$

Step 4.2: Otherwise, let $K_U^d = \{k' : k' \in K_U, x_{tk'dj_{k'}} = 1\}$, for $l = 0, 1, 2$, we set

$$G_{kd} \leftarrow \max\Bigg(\frac{v_{tk\tau_k j_k} \bar{r}_{tkdj_k} \sqrt{(\sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}})^2 + (\sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}})^2}}{\bar{v}_{tkdj_k} \sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}} + \bar{w}_{tkdj_k} \sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}}} - \alpha \cdot \max(\sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}}, \sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}}), 0\Bigg),$$

for $(k,d) \in \bar{K}_C$, where $\alpha = 0, 0.2, 0.9$ respectively, for $l = 0, 1, 2$;

for $l = 3$, we set

$$G_{kd} \leftarrow \max\Bigg(\frac{v_{tk\tau_k j_k} \bar{r}_{tkdj_k} \sqrt{(\sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}})^2 + (\sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}})^2}}{\bar{v}_{tkdj_k} \sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}} + \bar{w}_{tkdj_k} \sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}}} - (\max(\sum_{k' \in K_U^d} \bar{v}_{tk'dj_{k'}}, \sum_{k' \in K_U^d} \bar{w}_{tk'dj_{k'}}))^2, 0\Bigg),$$

for $(k,d) \in \bar{K}_C$; we set

$$G_{kd} \leftarrow \frac{\sqrt{\alpha}\, v_{tk\tau_k j_k} \bar{r}_{tkdj_k}}{\bar{v}_{tkdj_k} + \bar{w}_{tkdj_k}},\ \text{for } (k,d) \in \bar{K}_C,$$

where $\alpha = 0.125, 0.25, 2, 1.2, 1$ respectively, for $l = 4, 5, 6, 7, 8$.

Step 5: Find that item k whose effective gradient is the largest in a period, i.e.,

$$G_{kd} = \max\{G_{k'd'} : (k', d') \in \bar{K}_C\}.$$

Step 6: Accept k. Let $K_U \leftarrow K_U + \{k\}$, $A_U^{dj_k} \leftarrow A_U^{dj_k} + (\bar{v}_{tkdj_k}, \bar{w}_{tkdj_k})$, $z \leftarrow z + v_{tk\tau_k j_k} \bar{r}_{tkdj_k}$, $K_D \leftarrow K_D - \{k\}$, $x_{tkdj_k} \leftarrow 1$. Then, goto **step 2**.

Step 7: If $z > \hat{z}$, then $\hat{z} \leftarrow z$, $\hat{x}_{tkdj_k} \leftarrow x_{tkdj_k}$, for $t \in \tilde{T}$, $k \in \tilde{K}_t$, $d \in \{t, t+1, \cdots, \tau_k\}$.

Step 8: If $l = 8$, then the procedure terminates. Otherwise, let $l \leftarrow l + 1$, goto **step 1.1**.

From the steps of the modified heuristic algorithm, the solution of MHA is closer to the optimal solution than HAEG, because we take the maximum value among solutions given by nine methods including the method used in HAEG. We may easily notice that the computation time of MHA is more than four times of HAEG.

4. Numerical Experiments

In this section, we implement the modified heuristic algorithm MHA and give comparisons among heuristic solutions and optimal solutions or LP optimal solutions (as the up bound of optimal solutions). The algorithm has been coded in C++ and run under Windows 2000 Professional using a notebook computer (Pentium IV with 1794MHz and 256MB in RAM). CPU times were obtained through the C++ function clock(). To conduct our experiments we used randomly generated instances. For simplicity of implementation, we assumed that destination port j_k of each cargo k is decided by:

$j_k = j$, if $\text{floor}(K/J) \times (j-1) \le j_k < \text{floor}(K/J) \times j$, for $j = 1, 2, \cdots, J-1$;

$j_k = J$, if $\text{floor}(K/J) \times (J-1) < j_k \le J$.

For each set of parameters J, T and K, we generated 10 random MDMKP instances. We tested heuristic solutions and optimal solutions or LP optimal solutions on all 10 instances, and tabulated the average relative gap and average computation time. In table 1, the relative gap between heuristic solution and optimal solution g_O is computed as

$$\frac{\text{total profit of optimal solution - total profit of heuristic solution}}{\text{total profit of optimal solution}} \times 100\%$$

In table 1 and table 2, the relative gap between heuristic solution and LP relaxation optimal solution g_L is computed as

$$\frac{\text{total profit of LP relaxation optimal solution - total profit of heuristic solution}}{\text{total profit of LP relaxation optimal solution}} \times 100\%$$

Table 1. Results for small test problems

T	*J*	*K*	*Number of variables*	*Number of constraints*	*Instances tested*	Average relative gap (%)	
						g_O	g_L
2	2	37	74	84	10	1.62	2.82
2	2	57	114	124	10	1.27	2.65
3	4	27	81	108	10	2.31	3.29
4	8	23	92	160	10	2.85	9.41

Table 2. Results for Medium Scale problems

T	*J*	*K*	*Number of variables*	*Number of constraints*	*Instances tested*	Average relative gap g_L (%)	*Average CPU time (sec)*	
							Heuristic	*LP*
2	2	271	542	552	10	5.79	0.54	5.02
4	8	800	3200	3268	10	6.18	26.43	1167.53
4	40	500	2000	2324	10	7.79	46.32	298.79
5	70	700	3500	4205	10	8.74	110.23	1982.67
5	9	900	4500	4595	10	7.65	41.58	2139.04
5	30	990	4950	5255	10	8.41	160.23	2531.45

Table 3. Computation Time for Large Scale Problems

T	*J*	*K*	*Number of variables*	*Number of constraints*	*Instances tested*	*Average CPU time (sec)*
5	70	1200	6000	6705	10	367.02
3	4	4000	12000	12027	10	273.46
6	7	2500	15000	15090	10	158.85
4	4	5500	22000	22036	10	503.49
5	10	5000	25000	25105	10	1310.34
6	8	4500	27000	27102	10	631.82

Table 1 shows the results obtained for a set of small test problems. Test problems 1-2 have 2 periods, 2 destination ports and 74, 114 items (cargoes) respectively; test problem 3 have 3 periods, 4 destination ports and 27 items; test problem 4 have 4 periods, 8 destination ports and 23 items. For comparison, the optimal solution has been computed using CPLEX 8.0. As can be seen from table 1, the obtained results seem to be encouraging. The gap between the optimal solution and the heuristic solution is small and the computation time is very short. Table 2 shows the results obtained for a set of medium scale problems. Table 3 gives the computation times for a set of large scale problem. It show that the computation time of the heuristic algorithm is very short even for very large scale problems with tens of thousands of decision variables.

We believe that heuristic algorithm will be a very good candidate for solving MDMKP in time critical or real-time applications such as sea cargo mix problems where a near optimal solution is acceptable, and fast computation is more important than guaranteeing optimal value.

5. Conclusions

We formulated the multi-period sea cargo mix problem as the MDMKP model and presented an effective heuristic algorithm for it.

The heuristic algorithm gives fast and near optimal solution for the multi-period sea cargo mix problem. We presented experimental results to evaluate the algorithm by a wide range of problem instances. The heuristic algorithm is a very effective for time critical operations level decisions where a near optimal solution is acceptable, and fast computation is more important than guaranteeing optimal value.

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