



Problema bidimensional:

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$

Por separación de variables se obtiene la solución de la forma [2]:

$$\varphi(x, y) = (\tilde{A} e^{\alpha x} + \tilde{B} e^{-\alpha x}) (C e^{i\alpha y} + D e^{-i\alpha y}) + \tilde{a} xy + \tilde{b} x + c y$$

$$\alpha \geq 0$$

Condiciones de frontera:

$$\varphi(x, a) = 0 \quad ; \quad \varphi(0, y) = V_1$$

$$\varphi(x, 0) = 0 \quad ; \quad \varphi(b, y) = V_2$$

Se sigue:

$$\varphi(x, 0) = (\tilde{A} e^{\alpha x} + \tilde{B} e^{-\alpha x}) (C + D) + \tilde{b} x = 0$$

$x$  y  $e^{\pm \alpha x}$  son linealmente independientes en consecuencia:

$$\begin{cases} C + D = 0 \\ D = -C \end{cases} \quad \text{y} \quad \tilde{b} = 0$$

Se sigue:

$$\varphi(x, y) = (\tilde{A} e^{\alpha x} + \tilde{B} e^{-\alpha x}) C (e^{i\alpha y} - e^{-i\alpha y}) + \tilde{a} xy + c y$$

$$\varphi(x, y) = (A e^{\alpha x} + B e^{-\alpha x}) \sin(\alpha y) + \tilde{a} xy + c y \quad ; \quad \begin{aligned} \text{Aquí } A &= \tilde{A} C (2i) \\ B &= \tilde{B} C (-2i) \end{aligned}$$

Ahora:

$$\varphi(x, a) = (A e^{\alpha x} + B e^{-\alpha x}) \sin(\alpha a) + \tilde{a} x a + c a = 0 \quad ; \quad a \neq 0$$

$$\sin(\alpha a) = 0, \quad \tilde{a} = 0, \quad c = 0$$