Bloch Oscillation

One of the quantum phenomena that might be studied in an Optical lattice is the one of Bloch Oscillation.

This phenomenon arises in a "tilted lattice", that is a periodic lattice to which a slope is added (that is, a constant force F- see Fig 4). A quantum particle placed in such a potential does not climb the slope down (as would do a classical particle) even if its energy is greater than V_0. Instead, it performs an oscillation of limited spatial extension, which is the Bloch oscillation. The frequency of the oscillation, the Bloch frequency, can be interpreted as the basic Bohr frequency of the system $\Delta E/h = Fd/h$.

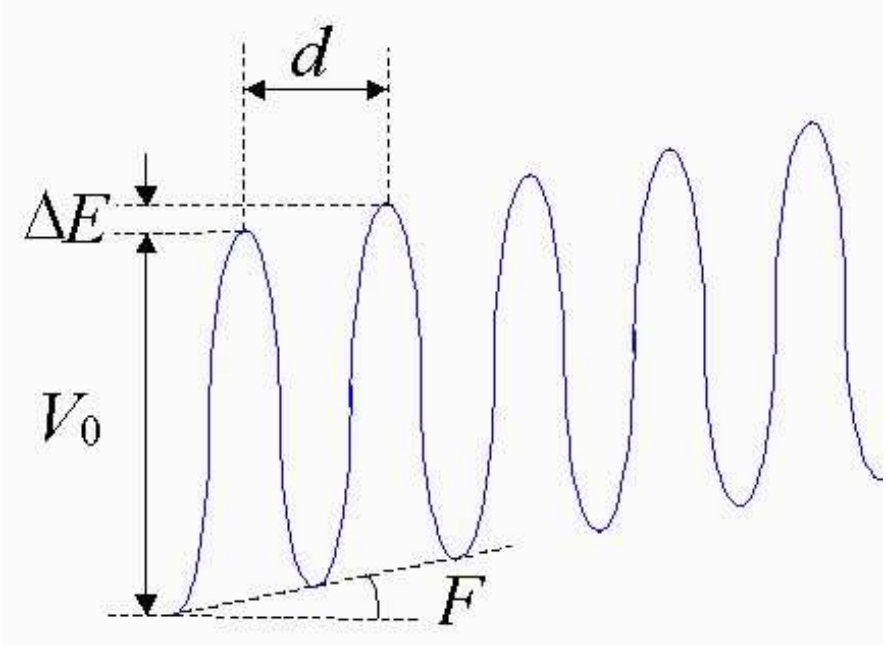

Fig 4

For those who are more interested they may continue read.

THE BLOCH OSCILLATION IN THE WANNIER-STARK DESCRIPTION

Bloch oscillation (BO) is a well known phenomenon discovered by Zener while studying the quantum properties of an electron in a (perfect) crystal submitted to a constant electric field. The BO arises when a small spatial tilt is added to the lattice: Quantum particles do not fall along the slope of the potential, but perform a periodic, bounded, oscillation. BO is thus a strictly quantum behaviour. We shall call the untilted potential the ‘‘lattice,’’ and the sum of the lattice and the tilted potential the ‘‘tilted lattice.’’ The BO is usually described in the basis of the so-called *Bloch states*, i.e., the eigenstates of the lattice. In this paper, we use another framework corresponding to the eigenstates of the tilted potential. While the lattice is invariant under spatial translations by a multiple of the spatial period d, the tilted lattice has a more complicated symmetry: it is invariant under simultaneous spatial translation by a lattice period d and energy translation by $\omega_B = Fd$ (ω_B is the ‘‘Bloch frequency’’).

One then expects the eigenenergies to form ‘‘ladder’’ structures separated by ω_B , the so-called Wannier-Stark ladders, introduced by Wannier in connection with the problem of electrons in a crystal submitted to a homogeneous electric field Each element of the ladder corresponds to eigenfunctions (the Wannier-Stark states) centred at a given well, and thus separated by an integer multiple of d. The form and even the existence of these eigenstates has been the object of a long controversy, which was settled only recently. In the present paper, we consider a spatially limited lattice, extending over many periods, and limited by an infinite-height box. This changes only very slightly the ‘‘bulk’’ properties of the system, and the eigenenergies and eigenstates obtained numerically display (to a very good approximation) the expected ladder structure described above. In this framework, the Wannier-Stark states (WSS’s) are the eigenfunctions of the system. The relation to the case of an infinite tilted lattice is no problem if the corresponding states (Wannier-Stark resonances) have long enough lifetimes compared to the experimental times. The existence of these states was evidenced in 1988 in a semiconductor superlattice, and in 1996 with cold atoms in an optical lattice. Note that, being spatially localized, Wannier-Stark states provide an ideal tool for the description of the wave function of a cold atom [as produced by a ‘‘Sisyphus-boosted’’ magneto-optical trap (MOT)] placed in the potential, whose de Broglie wavelength (around $\frac{\lambda_L}{3}$) is of the order of the lattice period $\frac{\lambda_L}{2}$. Consider the properties of the time-independent Hamiltonian

$$\mathrm{H}_0,\ H_0 = \frac{P^2}{2m} + V_0 \cos(2\pi x) + Fx \qquad (1)$$

where F is a constant force. The eigenfunctions of H_0 are Wannier-Stark states forming an energy ladder whose separation
is ω_0=Fd , $E_{mn}=E_m+nFd$ The BO can be advantageously described by using the Wannier-Stark state localized inside a given individual lattice well, corresponding to the lowest energy of the states associated with this well (see Fig. 5).

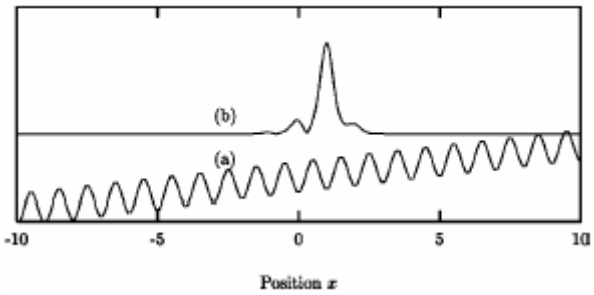

Fig5

(a) Periodic potential with a tilt. (b) T WSS state ψ_n is localized in the well of index n=1 and has appreciable overlap with neighbour lattice site n=0,2. This eigenfunction is obtained for V_0=2.5 and F=0.5.

Note that considering only the ground state of each well is equivalent to a restriction to the first Bloch band in the description based on Bloch states. We also choose strong enough F and V_0 to produce well localized WSS’s. The WSS associated with the lattice well labelled n is denoted $\psi_n(x)$ (supposed real) and the corresponding eigenenergy is E_n (we drop the index *m*). The symmetries of the potential discussed above then imply

$$\psi_{n+p}(x) = \psi_n(x - pd) \qquad (2)$$

and

$$E_{n+p} = E_n + pFd \qquad (3)$$

Let us describe the atomic wave function by a superposition of WSS’s:

$$\Psi(x,t) = \sum_n c_n(t)\psi_n(x) \qquad (4)$$

with $c_n(t) = c_n e^{-iE_n t}$,c_n being the amplitude at t=0. The dynamical quantities of the atom can be easily calculated. For instance, the mean value of the atomic position operator is

$$\langle x \rangle = \sum_n X_{n,n} |c_n|^2 + \sum_{n<m} (X_{n,m} c_n^* c_n e^{i(E_n - E_m)t} + c.c.) \qquad (5)$$

where $X_{n,m} \equiv \langle \psi_n | x | \psi_m \rangle$. As long as the WSS ψ_n is well localized in the respective well, we can keep only nearestneighbor contributions (n=m+1) and derive a simplified expression:

$$\langle x \rangle = \bar{x} + X_{n,n+1} (\sum_n c_n^* c_{n+1} e^{i\omega_B t} + c.c.) \quad (6)$$

where $\bar{x} = \sum_n X_{n,n} |c_n|^2$ is the mean position of the wave packet and $X_{n,n+1} = X_{0,1} = X_{0,-1} = X_{n,n-1}$ is independent of n. This result evidences a counterintuitive property of the quantum motion in a tilted lattice: instead of falling along the slope, the atom performs an oscillation with frequency ω_B (the Bloch frequency). The amplitude of this Bloch oscillation is proportional to $X_{0,1}$ and grows with the overlap between neighbour WSS's, i.e., for a small slope F and small lattice depth V_0. The physical origin of the BO appears here clearly as an interference effect between neighbour sites, as $c_n^* c_{n+1}$ is the coherence between the sites n and n+1. Note the lack of oscillations if the atom is localized in only one well. Equation (5) also predicts the occurrence of harmonics with frequency $p\omega_B$ (p integer) but with smaller amplitudes since they involve the coupling strength $X_{n,n+p}$. The description of the BO given here is very different from the usual ''solid-state'' approach. There, the Bloch states (eigenstates of the untilted lattice) are taken as the reference basis. The oscillation is described in a semiclassical frame as the periodic evolution of the atom's quasimomentum Ft (in the first Brillouin zone) with period $T = \frac{2\pi}{Fd}$. Although natural, this approach does not make clear the role of quantum interference produced by the periodic lattice structure as the basic mechanism underlying the Bloch oscillation, which is evidenced in our approach.