Appendix C: Transversely Isotropic Materials in Continuum Mechanics

Transversely isotropic materials are materials where the material properties in a single direction are different than in the perpendicular plane. Fibrous tissues like the vocal fold ligament and muscles, can be assumed to be transversely isotropic with fiber direction perpendicular to the transverse plane. The particular stiffness matrix **S** where the shears are symmetric for transverse isotropy has been worked out (Cook *et al*, 1989). Equations B1 and B2 are replaced with C2 and C3 for transversely isotropic materials.

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} [s] & [0] \\ [0] & [s'] \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{pmatrix} \quad \textbf{(1)}$$

$$[s] = \frac{E'}{k} \begin{pmatrix} \frac{\alpha - \nu'^2}{\alpha(1+\nu)} & \nu' & \frac{\alpha\nu - \nu'^2}{\alpha(1+\nu)} \\ \nu' & \alpha(1-\nu) & \nu' \\ \frac{\alpha\nu - \nu'^2}{\alpha(1+\nu)} & \nu' & \frac{\alpha - \nu'^2}{\alpha(1+\nu)} \end{pmatrix} \quad \textbf{(2)}$$

$$[s'] = \begin{pmatrix} \mu' & 0 & 0 \\ 0 & \mu' & 0 \\ 0 & 0 & \mu \end{pmatrix} \quad \textbf{(3)}$$

With simplification definitions in equations 4 and 5.

$$k = \alpha(1-\nu) - 2\nu'^2 \quad \textbf{(4)}$$

$$\alpha = \frac{E'}{E} = \frac{E'}{2\mu(1+\nu)} \qquad \textbf{(5)}$$

Where μ' is the shear modulus and ν' is the Poisson's ratio along the longitudinal axis (direction of fiber) while μ is the shear modulus and ν is the Poisson's ratio in the transverse plane. *E'* and *E* are the two Young's modulus respectively. Note that the upper bound on ν for transversely isotropic materials is given by: $\nu \leq 1\text{-}2\,\nu'^2 E/E'$ (Lempriere, 1968). After the matrix multiplication and stress substitution results in the equations of motion for a transversely isotropic material assuming small deformation without body forces. These equations were not submitted here but are available upon request. Using the same method as in the isotropic case (Appendix B), the equations of motion can be solved for both small and large deformations.