Question 2

Explain what fundamental theories of physics and engineering underlie the myoelastic-aerodynamic theory of phonation. Can glottal airflow, tissue vibration, and acoustic wave propagation be derived from the same basic equation? Carefully describe all the typical assumptions that are made in classical speech and voice science to simplify glottal aerodynamics, soft tissue mechanics (including muscle mechanics), and one-dimensional acoustics of the vocal tract.

Note: The appendices were added so not to clutter up the manuscript with more equations than are already present. References to the appendices are made, and when referring to an equation, the letter of the appendix where the equation is located precedes the equation number, i.e., B4 = equation 4 in Appendix B.

INTRODUCTION

Many speech and voice textbooks have attempted to describe the basics of vocal fold oscillation. These attempts usually ignore complexity of the interaction between the vocal folds and the air flow driving them by attributing them simply to symmetrical Bernoulli effect (Fant, 1960). However, in the past two decades, research has demonstrated differently showing the probable energy transfer between the aerodynamics (fluid flow) and vocal fold tissue vibration (elasticity). This coupling between aerodynamics and tissue vibration for phonation has been called the *myoelastic-aerodynamic theory of phonation* (MATP). According to Alipour and Titze (1996), a biophysical model of voice production, based on the myoelastic-aerodynamic theory of phonation, requires at least three submodels: laryngeal aerodynamics, tissue mechanics, and vocal-tract acoustics.

Since all of these submodels are based on engineering principles, they all should have a common theoretical ancestor in the realm of mechanics and dynamics. These submodels must be based on the same fundamental physical interactions, yet they are often considered separate and kept in different fields of study. The goal of this paper is to explain the fundamental theories of physics and engineering that underlie the myoelastic-aerodynamic theory of phonation. This will be done by deriving the equations of motion used for glottal airflow, tissue vibration and acoustic waves in the MATP. These come respectively from aerodynamics, muscle and soft tissue mechanics, and one-dimensional acoustics. Each of these disciplines can be explained from the same basic equation and as extensions of general continuum mechanics. Descriptions of the typical assumptions that are made in classical speech and voice science to facilitate these derivations were clarified.

I. CONTINUUM MECHANICS

Continuum mechanics, or the mechanics of deformable bodies, are the basic theory underlying non-point mass mechanics. Continuum mechanics are just a general Newtonian mechanic's description of the movement and deformation of bodies. An important part t of continuum mechanics, when describing the mechanics of a system, is a constitutive equation. Zupkas and Fung (1985) said, "the first step in the analysis of the biomechanics of any organ is to obtain its constitutive equation." In order to obtain the constitutive equations for glottal airflow, tissue vibration and acoustic waves from aerodynamics, the basics of continuum mechanics were needed. It will be shown that with a general equation from Newtonian mechanics, equation of motion and constitutive equations can be derived for each subdisipline of interest.

A. General Continuum Mechanics Relations

Appendix A contains a general treatment of continuum mechanics beginning with the deformation of an arbitrary field element. In the appendix some definitions of interest that will be needed in later discussions are the Armani stress tensor (Appendix A; eqn. 15),

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial q_j} + \frac{\partial u_j}{\partial q_i} + \frac{\partial u_k}{\partial q_i}\frac{\partial u_l}{\partial q_j}\right) \quad \textbf{(1)}$$

,

which is also represented in many texts by the Greek symbol TAU (τ_{ij}) , the Green-St. Venant stain tensor (A19)

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial p_j} + \frac{\partial u_j}{\partial p_i} - \frac{\partial u_k}{\partial p_i}\frac{\partial u_l}{\partial p_j}\right) \quad \textbf{(2)}$$

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which is also represented in many texts by the Greek symbol EPSILON ($\in_{ij}$) , and the generalized constitutive relation (A20)

$$\begin{pmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} S_{11} & \cdot & \cdot & \cdot & \cdot & S_{16} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{61} & \cdot & \cdot & \cdot & \cdot & S_{66} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{pmatrix} . \qquad \textbf{(3)}$$

B. Equation of Motion for point-mass Newtonian and Continuum Mechanics

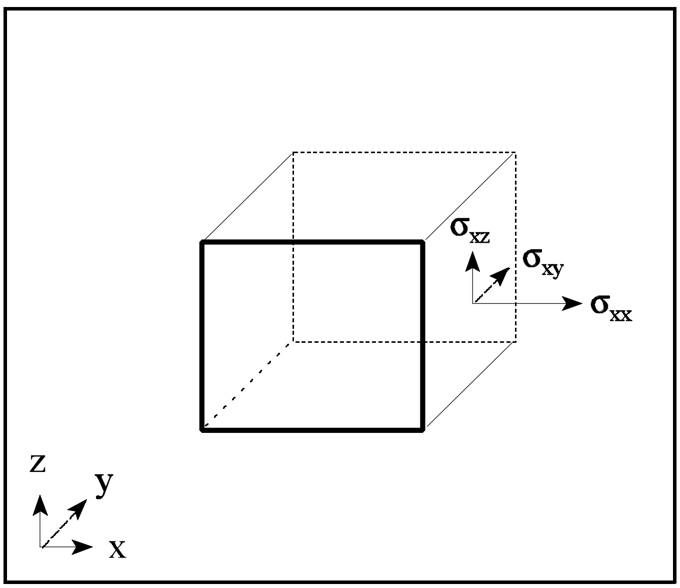

Figure 1. The stresses on the *x* plane of an element of material.

Figure 1 depicts a rectangular element, of a larger body, with dimensions x, y, and z with stresses (normal and shear) labeled. This element is set into motion by the sum of all the forces from neighbor interaction. The total forces in the *x* direction from parallel sides can be written as:

$$\begin{aligned} F_x &= \Delta y \Delta z [\sigma_x(x+\Delta x, y, z) - \sigma_x(x, y, z)] \\ &\quad + \Delta x \Delta z \left[\tau_{xy}(x, y+\Delta y, z) - \tau_{xy}(x, y, z)\right] \\ &\quad + \Delta y \Delta x [\tau_{xz}(x, y, z+\Delta z) - \tau_{xz}(x, y, z)] \\ &= \Delta x \Delta y \Delta z \, \rho \frac{\partial^2 \xi}{\partial t^2} \end{aligned} \quad \textbf{(4)}$$

where *ñ* is the density of the tissue. Force equations can be written for all three directions. Dividing equation 4 and the respective *y* and *z* force equations by the element's volume and implying continuous derivatives, as the volume becomes infinitesimally small, result in the element's equations of motion. Equation 5 depicts the equation of motion in the *x* direction.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho \frac{\partial^2 \xi}{\partial t^2} \quad \textbf{(5)}$$

The three equations of motion can be generally in vector notation as

$$\nabla ó + \boldsymbol{f} + ñ \frac{\boldsymbol{v}}{t} = 0 \quad . \quad (6)$$

In equation 6; *f* was included to represent any distributed body forces (N/m^3), σ is the stress tensor (equation 1), ρ is the density of the material, and *v* is the velocity.

II. DERIVATIONS

Deriving the basics of glottal airflow, tissue vibration and acoustic waves from aerodynamics, muscle and soft tissue mechanics, and one-dimensional acoustics require the definitions of specific constitutive equations and material fields that are assumption based. The differences each of these impose on the general equation is in the definition of the material field and the stress. For tissue mechanics, the primary dynamic variable is the displacement vector *ø*, with its planar components *î* and *æ*, where its constitutive equation is between material stress and strain. For the aerodynamics, the primary dynamic variable is particle velocity v, where its constitutive equation is between fluid stress and the velocity field.

In this section, the basic equations used to describe tissue and muscle mechanics, glottal airflow, and one-dimensional vocal tract acoustics in the myoelastic-aerodynamic theory of phonation were derived, in that general order. The assumptions needed in the derivations will be used and clarified. Each derivation ended with a summary of the assumptions.

A. Passive Tissue Mechanics

For the analysis of the deformation and motion of soft tissues, models for their mechanical characteristics must be provided. This involves developing constitutive relationships for the passive behavior for soft tissues and mechanical models for muscular contraction. Equations of motion and constitutive equation, in tissue mechanics, use space or distance and material stress and strain. Most tissues involved in phonation can be assumed to be of two types, isotropic (completely uniform in all directions), and transversely isotropic (tissue fibers and transverse plane being perpendicular to one another). Of these, there can be two types, passive tissues (cartilage, collagen, elastin, etc.), active tissues (muscles).

1. Isotropic

Isotropic materials have the feature of having uniform tissue properties in every direction. Muscles are sometimes assumed to have the passive property of being isotropic, yet the vocal folds, because of the vocal ligament, are not. To find the specific equation of motion for isometric tissues from equation 6, the stresses represented by σ, must be found. Stresses are functions of the strain as given by the general constitutive relation (equation 3). The particular stiffness matrix $\mathbf{S}$ for isotropic materials, where the shears are symmetric, can be found in many engineering books (Cook *et al*, 1989).

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} [{}^{3}s] & [{}^{3}0] \\ [{}^{3}0] & [{}^{3}s'] \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{pmatrix} \tag{7}$$

Where

$$[{}^{3}s] = \frac{2\mu}{1-2\nu} \begin{pmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{pmatrix} \tag{8}$$

$$[{}^{3}s'] = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix} \tag{9}$$

with μ representing the shear modulus, and ν the Poisson's ratio. For example, two of the six stresses are:

$$\tau_{yz} = \mu \epsilon_{yz} \qquad \sigma_y = \frac{2\mu}{1-2\nu}\left[\nu \epsilon_{xx} + (1-\nu)\epsilon_{yy} + \nu \epsilon_{zz}\right] \tag{10}$$

where the others are of similar form (see B4, for all six). Before the stresses are used in equation 6, the strains need to be discussed.

The strain equation defined in equation 1 is a nonlinear equation because of the product term. The full version of equation for $\in_{yy}$ was labeled equation 11 (or B5).

$$\begin{aligned} \epsilon_{yy} = \frac{1}{2}\Bigg(& \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial y} + \frac{\partial \xi}{\partial y}\frac{\partial \xi}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial \xi}{\partial y} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial y} + \frac{\partial \xi}{\partial y}\frac{\partial \psi}{\partial y} + \\ & \frac{\partial \psi}{\partial y}\frac{\partial \psi}{\partial y} + \frac{\partial \zeta}{\partial y}\frac{\partial \psi}{\partial y} + \frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial \zeta}{\partial y} + \frac{\partial \zeta}{\partial y}\frac{\partial \zeta}{\partial y}\Bigg) \end{aligned} \tag{11}$$

Substituting these strains into the constitutive equation 7 allows substitution into equation 6 for the equation of motion. Using equation 11 (B5) in anything would be tedious and was not done here. The product terms in the stains can be neglected if deformations or vibration in a material is assumed small.

For small deformations, the product terms are assumed to be negligible, resulting in strains of the form in equation 12 (B6). The small deformation assumption is often used for small amplitude vibratory motion. In speech science this assumption is usually motivated by the fact that vocal fold vibration usually occurs around an equilibrium position at small. For laryngeal posturing, such as adduction, or large muscle mechanics and even large amplitudes of vibration, this assumption cannot be used.

$$\epsilon_{yy} = \frac{\partial \psi}{\partial y} \qquad \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial \psi}{\partial z} + \frac{\partial \zeta}{\partial y}\right) \tag{12}$$

The equations of motion for an isotropic material, assuming small deformation without body forces, would be obtained by getting the stresses from equation 7 using the small deformation strains like equation 12. This results in the stresses of the form in equation 13 (B7). Substituting the stresses into the equations of motion, equation 6, results in equation 14 (B8). The full version of each of these steps and equations can be found in Appendix B.

$$\tau_{zx} = \frac{1}{2}\mu\left(\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial z}\right) \qquad \sigma_z = \frac{2\mu}{1-2\nu}\left[\nu\frac{\partial \xi}{\partial x} + \nu\frac{\partial \psi}{\partial y} + (1-\nu)\frac{\partial \zeta}{\partial z}\right] \tag{13}$$

$$\frac{\partial}{\partial x}\left\{\frac{2\mu}{1-2\nu}\left[(1-\nu)\frac{\partial \xi}{\partial x} + \nu\frac{\partial \psi}{\partial y} + \nu\frac{\partial \zeta}{\partial z}\right]\right\} + \frac{\partial}{\partial y}\left\{\frac{1}{2}\mu\left[\frac{\partial \psi}{\partial z} + \frac{\partial \zeta}{\partial y}\right]\right\} + \frac{\partial}{\partial z}\left\{\frac{1}{2}\mu\left[\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial z}\right]\right\} = \rho\frac{\partial^2 \xi}{\partial t^2} \tag{14}$$

Equation 14 and the other two like it (see equation B8) for all three-dimensions, define the small amplitude deformation of an element in an isotropic tissue. These equations of motion can be analytically solved for specific cases but numerical methods are otherwise used. Note that if the assumption of small deformations was not made, the process for obtaining the equations of motion would be the same except that the full strains in equation 11 would be used instead the ones in 12. Seeing how the equations of motion for this case would have more terms than could fit on this page, they were not shown.

2. Transversely Isotropic

When tissues are fibrous like muscles and the vocal folds (vocal ligament), they are often assumed to be transversely isotropic with direction of the fiber is perpendicular to the transverse plane. With this assumption, the equations of motion for such a tissue can be obtained by following the same basic procedure as for the isotropic tissue case. The particular stiffness matrix **S** where the shears are symmetric for transverse isotropy has been worked out (Cook *et al*, 1989). Starting again with equation 3, in which equations 15 and 16 are substituted in, result in the constitutive equation.

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} [{}^3 s] & [{}^3 0] \\ [{}^3 0] & [{}^3 s'] \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{pmatrix} \qquad \textbf{(3) or (7)}$$

$$[{}^3 s] = \frac{E'}{k} \begin{pmatrix} \frac{\alpha - \nu'^2}{\alpha(1+\nu)} & \nu' & \frac{\alpha\nu - \nu'^2}{\alpha(1+\nu)} \\ \nu' & \alpha(1-\nu) & \nu' \\ \frac{\alpha\nu - \nu'^2}{\alpha(1+\nu)} & \nu' & \frac{\alpha - \nu'^2}{\alpha(1+\nu)} \end{pmatrix} \qquad \textbf{(15)}$$

$$\left[{}^{3}s' \right] = \begin{pmatrix} \mu' & 0 & 0 \\ 0 & \mu' & 0 \\ 0 & 0 & \mu \end{pmatrix} \qquad \textbf{(16)}$$

where for simplification purposes,

$$k = \alpha(1 - \nu) - 2\nu'^{2} \qquad \textbf{(17)},$$

$$\alpha = \frac{E'}{E} = \frac{E'}{2\mu(1 + \nu)} \qquad \textbf{(18)} \quad .$$

Similar too before, μ' is the shear modulus and ν' represents the Poisson's ratio along the longitudinal axis (direction of fiber) while μ is the shear modulus and ν represents the Poisson's ratio in the transverse plane. *E'* and *E* are the two Young's moduli respectively. Note that the upper bound on ν for transversely isotropic materials is given by: ν 1-2 $\nu'^{2}E/E'$ (Lempriere, 1968). After the matrix multiplication and stress substitution into equation 6, the equations of motion for a transversely isotropic material can be obtained. Equations of motion can be solved for both assumed small- and large- deformations.

Often in speech science, the vibration of the vocal folds are assumed to take place in the transverse plane on transversely isotropic material. Also, it is assumed that the small deformation assumption can be applied because the vibrations are small. This assumption is usually motivated by the fact that normal mode vibration usually occurs around an equilibrium position at small amplitudes, as based on observations of trajectories of vocal fold tissue points during self-oscillation (Fukuda *et al*. 1985). Vibratory patterns tend to be in the coronal plane, perpendicular to the vocal ligament. This assumption also makes all motion along the fiber negligible. Applying these assumptions to the equations of motion results in just two, the two for the transverse plane:

$$\frac{}{z}\left[\frac{2\mu}{(1-\text{í})}\left(\frac{\text{æ}}{z}+\text{í}\frac{\text{î}}{x}\right)\right]+\frac{}{y}\left[\mu'\frac{\text{æ}}{y}\right]+\frac{}{x}\left[\mu\left(\frac{\text{î}}{z}+\frac{\text{æ}}{x}\right)\right]=\text{ñ}\frac{^{2}\text{æ}}{t^{2}} \qquad \mathbf{(19)}$$

$$\frac{}{x}\left[\frac{2\mu}{(1-\text{í})}\left(\frac{\text{î}}{x}+\text{í}\frac{\text{æ}}{z}\right)\right]+\frac{}{y}\left[\mu'\frac{\text{î}}{y}\right]+\frac{}{z}\left[\mu\left(\frac{\text{æ}}{x}+\frac{\text{î}}{z}\right)\right]=\text{ñ}\frac{^{2}\text{î}}{t^{2}} \quad . \qquad \mathbf{(20)}$$

(These equations were the same as derived in I.R. Titze's, unpublished book Myoelastic-aerodynamic Theory of Phonation).

On the other hand, for laryngeal posturing (e.g., adduction), muscle mechanics, and even large amplitudes of vibration, this assumption cannot be used and would result in three equations of motion, including the transverse motion terms. Although these two ideas seem to contradict (small amplitude, no transverse motion vs. large deformation, 3-D motion) since they take place on the same tissues, it can be justified by assuming that posturing is a slow movement compared with phonatory vibration and that many vibratory cycles happen during a small posturing movement. Note that large amplitude vibrations do happen, and for those times, this assumption is no longer applicable.

Assumptions for Tissue Mechanics (vocal folds) in the MATP:

1. Small amplitude vibrations even during large deformation posturing movements,
2. Vibrations only happen in transverse plane,
3. Vocal folds are a transversely isotropic material (vocal ligament),
4. Negligible gravitation.

B. Active Tissue Mechanics

Developing a constitutive equation for active tissue means that passive behavior of the muscle and the mechanical models for muscular contraction have to be developed. Gielen (1998) and others (Fung, 1993; Wilhelms-Tricarico, 1994, 1998) have assumed that normal stress in muscle, assuming single fiber direction, to be the superposition of two stresses; 3-dimensional passive stresses σ_p, and one-dimensional active stress generated by the contracting tissues. This assumption allows the muscle to have the characteristic of passive tissue when not activated, then a combination of passive stress and a force component when activated. Note that active stresses are assumed to be combinations of normal stresses.

For simplicity, the passive stress of muscles has often taken on an isotropic flavor. To obtain the equation of motions for muscle tissue, first start with the passive stresses for the cases previously discussed (e.g., equations 13 for the small deformation isotropic case). For muscle mechanics, the small deformation assumption is almost assuredly invalid but it will be used here to show methodology with previously derived items.

Mechanical models for active muscle stress have been modeled in a variety of methods, where each is an attempt to relate muscle force to the deformation and activation of the muscle fibers (Schneck, 1992; Zupkas and Fung, 1985; Fung, 1993, Baskin and Paolini, 1965; Hill, *et al*. 1975; Gielen, 1998; Wilhelms-Tricarico, 1994, 1998; Zahalak, 1996; Zahalak and Ma, 1990; Zahalak and Motabarzadeh, 1996; Dang and Honda, 1998). One of the most popular is Huxley's force model based on the cross-bridge theory.

Most of these models assume muscles mechanics are dependent on strain, strain rate, current history and amount of activation or desired force.

$$\sigma_a(\epsilon_y, \epsilon_y, a(t), t) \qquad \textbf{(21)}$$

One internal muscle description by Titze (unpublished) has the internal muscle stress generalized to have dependance on strain, strain rate, time and muscle activation (0 to 1). Titze's model for the active contraction is based on work by Hill (1938). In this work active stress contains activation and maximum stress,

$$\sigma_a = a\,\sigma_m\, f(\epsilon_y)\, g(\dot{\epsilon}_y) \qquad \textbf{(22)}$$

where *a* being the activation multiplier of the maximum active stress σ_m. The function *g* is dependent on the strain rate and is empirically determined. For this case the stress-velocity (*g*) is a piecewise function written as:

$$g(\dot{\epsilon}_y) = \begin{cases} \max\left[0, \dfrac{\frac{\dot{\epsilon}_y}{\dot{\epsilon}_m} + 1}{1 - 3\,\frac{\dot{\epsilon}_y}{\dot{\epsilon}_m}}\right] & \dot{\epsilon}_y \le 0 \\ \dfrac{9\,\frac{\dot{\epsilon}_y}{\dot{\epsilon}_m} + 1}{5\,\frac{\dot{\epsilon}_y}{\dot{\epsilon}_m} + 1} & \dot{\epsilon}_y > 0 \end{cases} \qquad \textbf{(23)}$$

and function *f*, from experimental data, is:

$$f(\epsilon_y) = \max\left[0, 1 - b\left(\epsilon_y - \epsilon_m\right)^2\right] \qquad \textbf{(24)}$$

For the 3-D case, the subscript *y* will denote the muscle fiber direction, or magnitude of strain (if using the cross-bridge theory).

With the active muscle stress defined, the shear stresses are not changed but the normal stress constitutive equations have the active stresses added (equation 25):

$$\sigma_y = \frac{2\mu}{1 - 2\nu}\left[\nu\,\epsilon_{xx} + (1 - \nu)\,\epsilon_{yy} + \nu\,\epsilon_{zz}\right] + \sigma_{a\,y} \qquad \textbf{(25)}$$

Notice that only the normal stress in the *y* direction is shown, *x* and *z* components are similar. This revised stress would result in three equations of motion, for the active small-deformation isometric tissue (in the *z* direction):

$$\frac{\partial}{\partial x}\left\{\frac{1}{2}\mu\left[\frac{\partial \zeta}{\partial x}+\frac{\partial \xi}{\partial z}\right]\right\}+\frac{\partial}{\partial y}\left\{\frac{1}{2}\mu\left[\frac{\partial \psi}{\partial z}+\frac{\partial \zeta}{\partial y}\right]\right\}$$
$$+\frac{\partial}{\partial z}\left\{\frac{2\mu}{1-2\nu}\left[\nu\frac{\partial \xi}{\partial x}+\nu\frac{\partial \psi}{\partial y}+(1-\nu)\frac{\partial \zeta}{\partial z}\right]+\sigma_{z,z}\right\}=\rho\frac{\partial^2 \zeta}{\partial t^2} \qquad \textbf{(26)}$$

where other variations; including transverse isotropy and/or large deformations, would use the same method but with the appropriate relations as given above.

Assumptions for Muscle Mechanics (posturing muscles) in the MATP:

1. Transversely-Isotropic material (sometimes),
2. Cannot make small-deformation assumption (be prepared for large equations),
3. Stress is a superposition of passive tissue stress and an active (normal) stress component,
4. Active component here based on Hill’s muscle model and force-length curves,
5. Negligible gravitation effect.

Note that there are more physiologically exact muscle constitutive relations being developed based on Huxley’s cross-bridge theory expanded to that of the Distribution-Moment model. The newest of these take into account of off-axis stresses and encompassing contraction, activation, energetics, and metabolism. Though these show great promise, they are reportedly not ready for implementation into muscle models (Gielen, 1998; Zahalak and Ma, 1990; Zahalak and Motabarzadeh, 1996; Zahalak, 1996).

C. Aerodynamics

In the development of the fundamental equations of motion and the continuum mechanics to this point, the dynamics have been primarily based on the deformation and movement of a spacial body and mechanical stress. In aerodynamics, the previous fundamental developments still apply but we have to adjust the dynamics to particle velocity as our primary variable and its relation to fluid stress. First, glottal flow will be addressed followed by the one-dimensional acoustics of a vocal tract. For both discussions, their derivation from the general equation of motion will be given.

1. Glottal Flow

Glottal airflow is described by several fluid dynamic equations, Euler's equation of motion for steady incompressible in-viscid flow, Navier-Stokes equation, and Bernoulli's equation. Each of these have special assumptions made in speech science. The first equation derived from the general equation above, equation 6; [div(σ)+f-ρ dv/dt], will be Euler's equation.

a. Euler's equation of fluid motion

Euler made the assumption of flow incompressibility. If a flow is incompressible, then the divergence of the velocity vector vanishes everywhere. Next, Euler assumed that the flow was in-viscid, or that there is no friction between adjacent fluid particles. The in-viscid assumption can be sometimes justified in glottal flow by showing that a typical Reynolds number in the glottis is about 1800. This number is not necessarily high and suggest that a strong velocity field keeps most of the viscosity and shear at the boundary. The final assumption for Euler's equation in the glottis is that the flow can be assumed as steady flow. Flow in the glottis can be assumed steady even under oscillatory flow if the Strouhal number is low. For fundamental frequencies of range 100-1000 Hz and for typical glottis geometries, the Strouhal number is well within the range for validity of the assumption (see Titze, unpublished). Note that at the moment of glottal opening and closing, this assumption is not valid, so glottal flow is termed *quasi-steady*. Euler's equation, valid for glottal quasi-steady flow, is written as:

$$\frac{p}{s} + \tilde{n} g \frac{z}{s} + \tilde{n} v \frac{v}{s} = 0 \quad . \qquad (27)$$

where p represents pressure, $\tilde{n}$ is the fluid density, v is particle velocity and the s denotes the stream line of the flow. The second term in equation 27 is the contribution from gravity. The glottis is only 3-5 mm in length in the vertical direction. This length only allows gravity to contribute less than 0.1 Pa, which when compared with normal interglottal pressures of 100-1000 Pa, it can be neglected.

Rewriting equation 27 in vector notation and with the gravity term neglected, results in

$$\nabla P \;+\; \tilde{n}(\boldsymbol{v}\;\nabla)\boldsymbol{v} \;+\; \tilde{n}\frac{\boldsymbol{v}}{t} \;=\; 0 \qquad . \tag{28}$$

which is also equation 6, the general equation, when assuming normal stress is hydrostatic pressure, and that force comes from the velocity field.

Assumptions for Euler's Equation derivation:

1. Incompressablility in the glottis,
2. Viscous effects restricted to a thin boundary layer only (core flow),
3. Steady (quasi-steady) flow,
4. Negligible gravitation affects in the glottis.

b. Bernoulli's equation

A special case of Bernoulli's equation has been used in the MATP for showing a condition of the theory; the system must be self-oscillating (Story and Titze, 1994). Bernoulli's equation can be derived from equation 27 by integrating along the stream line, *s*, to obtain

$$p \;+\; \tilde{n}gz \;+\; \tilde{n}\frac{v^2}{2} \;=\; \textit{constant} \tag{29}$$

which is the general Bernoulli equation. This equation could be derived from equation 28 if gravity were neglected, which it can be when comparing pressure at two points in the glottis. Bernoulli's equation can be used in the glottis if one takes Prandtl's assumption; which is that flow follows Bernoulli's law, with a corrective energy-loss term, from the subglottic to the detachment point of the flow (near the top of the glottis).

Assumptions for Bernoulli's equation derivation:

1. Incompressibility in the glottis,
2. Viscous effects restricted to a thin boundary layer only,
3. Steady flow (along the stream line),
4. Negligible gravitation affects in the glottis,
5. Dependent on Prandtl's assumption.

c. Navier-Stokes equation

In this discussion, the fluid was assumed to be isotropic and a Newtonian fluid. Consider figure 1 again but where now this is a fluid element. Assume that the fluid is subject only to shear stresses, no normal stresses. This is done by assuming hydrostatic pressure is a constant. Thus, the change in pressure normal to the element in space is zero. So, the force on that element in the *x* direction would be equation 4 but with the normal forces as zero. This results in a force equation as shown in equation 30 and, by inspection, 31 and 32

$$\begin{aligned} F_x &= \Delta x \Delta z[\tau_{xy}(x, y + \Delta y, z) - \tau_{xy}(x,y,z)] \\ &+ \Delta y \Delta x[\tau_{xz}(x,y,z+\Delta z) - \tau_{xz}(x,y,z)] \\ &= \Delta x \Delta y \Delta z\left(\frac{\tau_{xy}}{y} + \frac{\tau_{xz}}{z}\right) \end{aligned} \tag{30}$$

and

$$F_y = \Delta x \Delta y \Delta z\left(\frac{\tau_{yz}}{z} + \frac{\tau_{yx}}{x}\right), \quad \textbf{(31)} \qquad F_z = \Delta x \Delta y \Delta z\left(\frac{\tau_{zx}}{x} + \frac{\tau_{zy}}{y}\right). \tag{32}$$

Assuming a Newtonian fluid, according to Newton's law of viscosity, shear stress is assumed linear, thus the product terms in the stress are negligible. So for fluid stress, we start again with the *Almani stress tensor* but we make a few adjustments. First, we need a velocity gradient, not a displacement gradient. Next, normal stresses are zero. Finally, by Newton's law of viscosity, shear stress on a fluid is assumed linear and the product terms are zero. This results in shear stress that can be written as:

$$\hat{o}_{xy} \;=\; \mu \left(\frac{v_x}{y} + \frac{v_y}{x} \right) \,, \qquad (33)$$

$$\hat{o}_{yz} \;=\; \mu \left(\frac{v_y}{z} + \frac{v_z}{y} \right) \,, \qquad \mathbf{(34)} \qquad\qquad \hat{o}_{zx} \;=\; \mu \left(\frac{v_z}{x} + \frac{v_x}{z} \right) \,, \qquad (35)$$

where μ is the viscosity of the fluid. Having those stress terms allow the force equation to be written as a volume, viscosity, and gradient of particle velocity (assuming that the hydrostatic pressure was constant in all directions):

$$\boldsymbol{F} \;=\; \ddot{A}x\ddot{A}y\ddot{A}z\,\mu\nabla^2\boldsymbol{v} \quad . \qquad (36)$$

Now it is time to derive the Navier-Stokes equation. Euler's equation of motion does not take into account the shear stresses from neighboring elements (in-viscid assumption). If this assumption is false, and fluid is viscid, then the opposing shear forces must be accounted for (equation 36). To derive the Navier-Stokes equation, it is assumed that the force term *f* in our general equation not only contains the *ñ (v·▽)v* force like in Euler's equation but also the shear force from equation 36. The shear force in 36 must be put into the same form (similar to going from equation 4 to 5) by dividing it by Ä*x* Ä*y* Ä*z*. Now the Navier-Stokes equation can be written in its normal form from the general equation of motion:

$$\nabla P + \tilde{n}(\boldsymbol{v}\,\nabla)\boldsymbol{v} - \mu\nabla^2\boldsymbol{v} + \tilde{n}\frac{\boldsymbol{v}}{t} \;=\; 0 \qquad (37)$$

Assumptions for Navier-Stokes equation derivation:

1. Newtonian fluid,
2. Fluid is at most a linear function of strain rate,

3. Fluid is isotropic,
4. Will reduce to hydrostatic pressure when strain rate = 0,
5. Incompressibility in the glottis,
6. Non-steady flow,
7. Viscid,
8. Negligible gravitation affects in the glottis.

Note that the use of Navier-Stokes in glottal airflow dynamics is extensive and further assumptions simplify the equation even further. Those assumptions are now briefly discussed.

To account for rotation about fluid lines, the Navier-Stokes equation will produce the Helmholtz equation of hydrodynamics. This new equation leads to the potential flow assumption when vorticity is zero. In the glottis, the assumption is made of potential flow. This allows vorticity to be treated only near the walls of the glottis and not in the main flow, with the exception at the glottal exit where the boundary layer expands.

Navier-Stokes can also be simplified for a boundary layer using Prandtl's assumptions: 1) the pressure gradient in the boundary layer (BL) is constant across the BL thickness, and is the same as the pressure gradient for core flow, and (2) flow direction is nearly parallel with the wall. With these assumptions, the Navier-Stokes equation can describe the motion of fluid particles in the boundary layers.

Other work has been done to see at what point the flow detaches from the glottis (Guo and Scherer, 1993). Knowing where this happens, lends knowledge to where assumptions of such things as core and laminar flow have limits. It has been done where loss terms have been added into Bernoulli's equation to account for these losses and limitations.

2. One-dimensional acoustics of a vocal tract

One-dimensional acoustic theory for use in the vocal tract uses the same idea as that found in plane wave acoustics for ducts or tubes. The equations that describe acoustics in a duct are solutions of the one-dimensional acoustic wave equation. In this section, the one-dimensional acoustic wave equation was derived from the general equation of motion form of equation 6.

Rewriting equation 6 as was done for Euler's equation where it was assumed linear, in-viscid fluid, results in:

$$\nabla P \; + \; \tilde{n}_0 (\boldsymbol{v}\,\nabla)\boldsymbol{v} \; + \; \tilde{n}_0 \frac{\boldsymbol{v}}{t} \;\; = \;\; 0 \qquad . \tag{38}$$

Now assume there is flow in the direction of ***v*** and mass is conserved. This means that the rate of influx of mass is equal to the rate of change in density within an element.

$$\frac{\tilde{n}}{t} \; + \; \tilde{n}_0 \nabla \boldsymbol{v} \;\; = \;\; 0 \qquad . \tag{39}$$

Let ρ be the instantaneous density at any point and ρ_0 be the constant equilibrium density of the fluid.

Assume that the particle velocity ***v*** is in the *y* direction. Equations 38 and 39 can then be rewritten as:

$$\frac{P}{y} \; + \; \tilde{n}_0 \frac{v_y}{t} \;\; = \;\; 0 \quad , \qquad \textbf{(40)} \qquad\qquad \frac{\tilde{n}}{t} \; + \; \tilde{n}_0 \frac{v_y}{y} \;\; = \;\; 0 \quad . \tag{41}$$

Another piece of information is needed. That is the one-dimensional linearized constitutive equation, $\in_y \mu$=P. Note that the definition of strain in the *y* direction is a condensation in fluid and can be approximated as shown in equation 42.

$$\mathring{a}_y \;\; = \;\; -\frac{\text{ø}}{y} \qquad -\frac{\tilde{n} - \tilde{n}_0}{\tilde{n}_0} \qquad . \tag{42}$$

First thing to do is differentiate equation 40 with respect to *y*:

$$\frac{\partial}{\partial y}\frac{\partial P}{\partial y} = -\frac{\partial}{\partial y}\left(\rho_0 \frac{\partial v_y}{\partial t}\right) = -\rho_0 \frac{\partial}{\partial y}\frac{\partial v_y}{\partial t} = -\rho_0 \frac{\partial}{\partial t}\frac{\partial v_y}{\partial y} \tag{43}$$

(Note that it was assumed that the time and space derivatives were not order dependent). Now solve for instantaneous density ρ from equation 42 to obtain equation 44:

$$\rho = -\rho_0 (1 + \varepsilon_y) \tag{44}$$

which in turn can be substituted into equation 41. Using the new equation 41 with equation 43, followed with the use of the constitutive equation, yields the one-dimensional wave equation.

$$\begin{aligned} \frac{\partial^2}{\partial y^2} P &= -\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial(-\rho_0(1+\varepsilon_y))}{\partial t}\right) \\ &= \rho_0 \frac{\partial^2}{\partial t^2}\varepsilon_y \\ &= \frac{\rho_0}{\mu}\frac{\partial^2}{\partial t^2} P \end{aligned} \tag{45}$$

If the speed of propagation is $(\rho_0/\mu)^{1/2}$, then the wave equation can be written in its most usual form:

$$\frac{\partial^2}{\partial y^2} P = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} P \tag{46}$$

The average particle velocity *v* across a tube can be obtained from a single plane-wave representation of the equation of motion:

$$\frac{\partial p}{\partial y} = -\rho \frac{\partial v}{\partial t} \quad . \tag{47}$$

This equation has a relatively simple solution of both leftward and rightward going waves. At the junctions of a series of tubes with various diameters that can be assumed to model the vocal tract, we can assume that the volume velocity and the pressure are continuous across the junction, which also simplifies the overall solution.

Assumptions for one-dimensional wave acoustics in the vocal tract:

1. Homogeneous, isotropic fluid,
2. Conservation of mass,
3. Small amplitude (steady flow),
4. In-viscid,
5. Small strain $\in$,
6. Plane pressure waves,
7. Volume velocity and pressure are continuous at tube junctions.

CONCLUSIONS

The goal of this paper was to explain what fundamental theories of physics and engineering underlie the myoelastic-aerodynamic theory of phonation. Zupkas and Fung (1985) said, “the first step in the analysis of the biomechanics of any organ is to obtain its constitutive equation.” According to Alipour and Titze (1996), a biophysical model of voice production, based on the myoelastic-aerodynamic theory of phonation, requires at least three submodels: laryngeal aerodynamics, tissue mechanics, and vocal-tract acoustics. To obtain the constitutive equations for glottal airflow, tissue vibration and acoustic waves from aerodynamics, certain assumptions were needed to be applied to a general constitutive equation. It was shown that with general Newtonian mechanics and using the idea of continuum mechanics a general constitutive equation could be found.

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