

## Some Inequalities Regarding Right Triangles by E.V.

**Theorem 1:** In a right triangle, twice the product of the measure of the legs ( $2ab$ ) does not exceed the measure of the hypotenuse squared ( $c^2$ )

**Proof:** By Pythagorean Theorem,  $a^2 + b^2 = c^2$ . From here, we can subtract  $c^2$  from both sides to obtain,  $a^2 - c^2 + b^2 = 0$ . Here, if  $c^2$  exactly equals  $2ab$ , then we can factor it to  $(a - b)^2 = 0$  which has real solutions. If we say  $c^2 < 2ab$ , we can change  $a^2 - c^2 + b^2 = 0$  to  $a^2 - 2ab + b^2 + q = 0$  for some positive  $q$ . This turns into  $(a-b)^2 + q = 0$ . Which is impossible since  $(a-b)^2 \geq 0$  and  $q > 0$ . Therefore,  $c^2$  cannot be less than  $2ab$ .

Which means,  $c^2 \geq 2ab$

**Corollary:** The sum of the measure of the legs ( $a + b$ ) does not exceed  $\sqrt{2}$  times the measure of the hypotenuse ( $c$ ).

**Proof:** By theorem above, we know that  $c^2 \geq 2ab$ . We also know that  $a^2 + b^2 = c^2$ . Adding the inequality to both sides, we get  $2c^2 \geq a^2 + 2ab + b^2$  or  $2c^2 \geq (a + b)^2$ . By taking the square root of both sides, we conclude that  $c\sqrt{2} \geq a + b$ .