Proof of the Infinitude of Primes

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Theorem: There is a prime between every n **and** $n!$ **where** $n! = n(n-1)(n-2)...2$ **and** $\{n \geq 3 : n \in \mathbb{N}\}$**.**

Proof: Take the number $n!-1$. $n!$ and $n!-1$ are relatively prime because if there is a greatest common factor for them, it must divide their difference, which is 1. So the greatest common factor is 1. Therefore, 1 is the only integer less than or equal to n divides that divides $n!-1$. Otherwise, if an integer other than 1, less than or equal to n divides $n!-1$ this will make $n!-1$ and n! have a greatest common factor larger than 1. n!-1 >1 since $n \geq 3$ so n! - 1 =pk for some prime p , $k \geq 1$. Therefore, p > n. So $n! > n!-1 \geq p > n$. ■

There are infinitely many primes

Proof: Because of the previous theorem, we now know that there exists a prime between all n and n! for $n \geq 3$. Therefore, there is a prime between n and n!, n! and (n!)!, (n!)! and ((n!)!)! ...