<u>An Infinite Sequence Representation of Pi</u>

$$0 + \frac{1}{2 * 3^2} + \frac{2}{3 * 4^2} + \ldots = \frac{\pi^2 - 9}{3}$$

Proof: The above sequence can also be expressed as $\sum_{j=1}^{\infty} \frac{j - 1}{j(j+1)^2}$

This can be changed into partial fractions.

$$\frac{j - 1}{j(j+1)^2} = \frac{A}{(j+1)^2} + \frac{B}{j+1} + \frac{C}{j}$$

Therefore, $j - 1 = A(j) + b(j+1)(j) + C(j+1)^2$

$= A(j) + Bj^2 + Bj + Cj^2 + 2Cj + C$

From here, C must equal to -1, so by substituting, we get

$$j - 1 = A(j) + Bj^2 + Bj - j^2 - 2j - 1$$

there must only be 1 positive j^2 to neutralize the single negative j^2. So B=1.

Again, by substituting, we get

$$j - 1 = A(j) + j^2 + j - j^2 - 2j - 1$$

From here, A's value can be determined to be 2.

So the original expression can be shown as $\sum_{j=1}^{\infty} \frac{2}{(j+1)^2} + \frac{1}{j+1} - \frac{1}{j}$

Since $\sum_{j=1}^{\infty} \frac{1}{j+1} - \frac{1}{j}$ = -1 the expression can be further simplified to

$$\left[2 \sum_{j=1}^{\infty} \frac{1}{(j+1)^2}\right] - 1 \quad \text{or} \quad \left[2 \sum_{j=1}^{\infty} \frac{1}{j^2}\right] - 3$$

Since, $\sum_{j=1}^{\infty} \frac{1}{j^2} = \frac{\pi^2}{6}$ then $\left[2 \sum_{j=1}^{\infty} \frac{1}{j^2}\right] - 3 = \frac{\pi^2 - 9}{3}$