

An Inequality

If a, b and c are positive real numbers such that $abc = 1$. Then

$$\frac{ab}{a^5 + b^5 + ab} + \frac{bc}{b^5 + c^5 + bc} + \frac{ac}{a^5 + c^5 + ac} \leq 1$$

Proof: Since $\frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c} = 1$, then we must prove

that $\frac{ab}{a^5 + b^5 + ab} \leq \frac{c}{a+b+c}$, $\frac{bc}{b^5 + c^5 + bc} \leq \frac{a}{a+b+c}$ and

$$\frac{ac}{a^5 + c^5 + ac} \leq \frac{b}{a+b+c}.$$

If $\frac{ab}{a^5 + b^5 + ab} \leq \frac{c}{a+b+c}$ then $\frac{a^5 + b^5 + ab}{ab} \geq \frac{a+b+c}{c}$. This

simplifies into $\frac{a^5 + b^5}{ab} \geq \frac{a+b}{c}$. Since $abc = 1$, then $c = \frac{1}{ab}$. By substituting,

the inequality turns into $\frac{a^5 + b^5}{ab} \geq ab(a+b)$ or

$$a^5 + b^5 \geq a^3 b^2 + b^3 a^2.$$

To prove this, first we prove another argument. (Re-

Arrangement Inequality)

Re-Arrangement Inequality For 2 Terms

If $A, B, C, D \geq 0$ where, $A + C = A_1$ and $B + D = B_1$

Then $A_1 B_1 + AB \geq A_1 B + AB_1$

Proof: $(A + C)(B + D) + AB \geq (A + C)B + (B + D)A$

$$AB + AD + BC + CD + AB \geq AB + BC + AB + AD$$

$CD \geq 0$, which is true.

Therefore, $a^5 + b^5 \geq a^3 b^2 + b^3 a^2$ can be shown to be true by plugging in

$a^3 = A_1$, $b^3 = A$ and $a^2 = B_1$ and $b^2 = B$.

Therefore, $\frac{ab}{a^5 + b^5 + ab} \leq \frac{c}{a + b + c}$. By changing the letters,

$$\frac{bc}{b^5 + c^5 + bc} \leq \frac{a}{a + b + c} \quad \text{and} \quad \frac{ac}{a^5 + c^5 + ac} \leq \frac{b}{a + b + c}$$

can also be shown to be true. Therefore, by combining all inequalities, the conclusion is that

If a , b and c are positive real numbers such that $abc = 1$. Then

$$\frac{ab}{a^5 + b^5 + ab} + \frac{bc}{b^5 + c^5 + bc} + \frac{ac}{a^5 + c^5 + ac} \leq 1$$