Medium-Size Macroeconomic Model for the Brazilian Economy*

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Abstract

This paper presents a medium-scale macroeconomic model for the Brazilian economy with more than 30 equations. The potential output is derived from a Cobb-Douglas production function and the demand side is divided in estimated equation for: consumption of the families, investment in machinery and construction, government spending and net exports. The estimated Phillips curve has an interesting feature: a step dummy variable captures the macroeconomic break in the pass-through occurred after the change of the exchange rate regime in 1999. An algorithm to run the model with a model consistent forward-looking term in the Phillips curve is implemented. There are long-run equilibrium conditions for the external and fiscal debts and also for the real interest rate. External and supply shocks hit the medium-size model in order to generate impulse responses.

Area: Macroeconomia Aplicada

JEL Classification Numbers: E12, E27, F43, F47

Key words: macroeconomic model, interest rate equilibrium, and potential output.

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1. Introduction

Small-scale macroeconomic models are very useful to forecast in the short run, but they are not very useful to anchor the key variables in the long run. They are not able to answer questions about the macro equilibrium of the economy, nor to establish fiscal or external constraints. Bigger macroeconomic models are better to provide interaction among stabilization and growth in the medium run. Questions concerning technology, investment, labor markets and current account balance are better addressed by a more comprehensive model. The micro-founded models are also able to present long run properties consistent with the optimal agent behavior. In the other hand Keynesian models are important because they can be used to determine the levels of output, employment, inflation, current account, rate of investment and fiscal balance simultaneously. However the long-run equilibrium of some variables, as interest rate and exchange rate, are not endogenously determined.

Many Central Banks have built micro founded structural models. Among them, we can quote the QPM from Bank of Canada, the FRB/US from the Federal Reserve, the MM (*Macroeconomic Model*) from Bank of England and also the Multimod from the IMF. They are in general divided in two parts. A steady-state part assures the long-run equilibrium based in the optimal behavior of the agents and a dynamic part, which describes the equilibrium path using error correction framework.

The steady-state model of the QPM is an overlapping-generation model with only one good. It relies on indirect tax and share of imported consumption to include inflation and exchange rate in the model. The Multimod is very similar and for the first five years uses the outcome from the World Economic Outlook as a baseline. The dynamic part brings a non-linear Phillips curve and also ensures a long-run growth consistent with an external debt service.

The special features of the FRB-US are the non-arbitrage conditions in the financial markets. In the goods sector, the expectational variables are model consistent. The dynamic model is also based in an error-correction approach. VAR expectations are also taken into account to describe transitory shocks. The steady-state part is not a dynamic general equilibrium model (DGE) but na ad-hoc constructed baseline case.

Among the models in the Keynesian paradigm, we can mention the Financial Programming model of the International Monetary Fund, which has the monetarist approach of the balance of payments, which created an entire generation of the IMF programs and are still being applied. The bottom-line is to set a goal to the net domestic assets, as a way to avoid the growth of the monetary supply well above the floor of international reserves. The World Bank already presents a line of models of growth of two gaps, in terms of domestic saving and external saving, called of RMSM-X. In Brazil, we can mention the experience of IPEA in also making a keynesian macroeconomic model, based on national accounts, especially on the balance of payments and the fiscal budgets. A quarterly version of this model has been made recently.

The Central Bank of Chile has built a Keynesian model very similar to the one that is going to be presented in this paper. The major difference is in the way that the steady-state equilibrium is derived. The consumption is divided in durable and non-durable goods, which are a future goal of our model.

Our model main contributions, compared to other macroeconomic models developed in Central Bank of Brazil, are:

- The aggregate demand is calculated by estimating: (1) family consumption; investment in (2) machinery and (3) construction; (4) net exports, (5) government spending, (6) government taxes, (7) changes in inventories;
- The Phillips curve that includes dummies for the structural break in the pass-through coefficient in 1999 and a proxy for the labor productivity (unit labor cost);
- The potential output is estimated by a Cobb-Douglas production function.

- A partial calibrated-estimated Uncovered-Interest-Parity (UIP) equation, together with an equation for the risk premium, which responses for changes in the fiscal and external conditions are added to the model;[1]
- Ad-hoc steady-state conditions for the current account deficit and for the fiscal primary surplus.

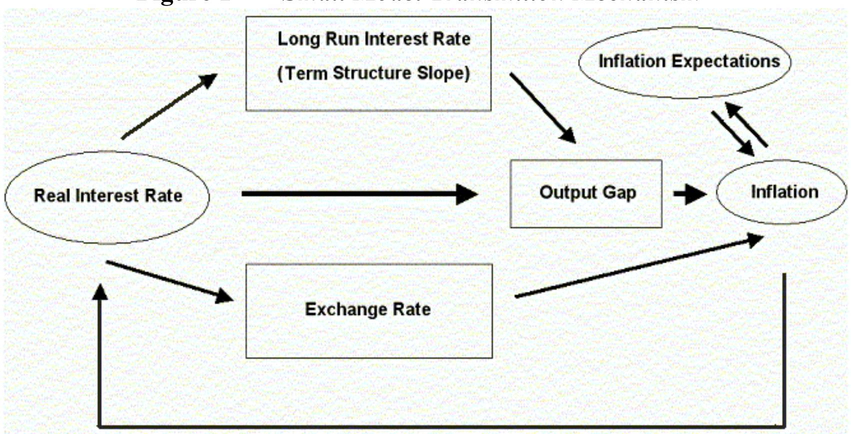
Some simulations for different Taylor rules, impulse responses for a temporary cost-push shock and permanent productivity shocks are presented.

The paper is organized as follows. Section 2 presents the diagrams of the small macroeconomic and the medium size models, showing the transmission mechanisms of the monetary policy and presenting some discussion about the long run equilibrium conditions for the external sector. Section 3 presents estimated and calibrated equations for the demand, supply, external and monetary-fiscal blocks of the model. Section 5 shows some simulation exercises and the last section concludes the paper.

2. Diagrams of The Transmission Mechanisms and Equilibrium Conditions

In order to compare the of monetary transmission mechanisms of the medium-scale and the small-scale models, it is necessary to explain how the mechanisms in the latter model are, as shown in Figure 1. There are two channels, the traditional one, by the output gap and another one by the exchange rate. The IS curve shows that an increase in the real interest rate will affect negatively the output gap, directly and indirectly by the term structure of interest rates. A more negative output gap will decrease inflation by the Phillips curve. By an UIP non-arbitrage condition, an increase in the interest rate causes an appreciation of the exchange rate in the spot market, and, by the Phillips curve, a decrease in the imported prices will generate a smaller inflation.

Figure 1 *Small Model Transmition Mechanism*



The two monetary transmission mechanisms described for the small model also exist in the medium model, shown in Figure 2[2]. But now it is possible to distinguish between supply and demand effects. An increase in the interest rates will affect the household consumption and, through term structure, the investment in construction and machinery, generating a decrease in aggregate demand. A greater interest rate will cause an exchange rate appreciation and a decrease in net exports, decreasing the aggregate demand. In the supply side, the effects of a higher interest rate will take more time to occur, because a lower investment level will cause a decrease in capital growth, affecting the potential output. The decrease in aggregate demand leads to an inflation drop, through a more negative output gap. But this drop would be partially offset by a decrease in potential output described above.

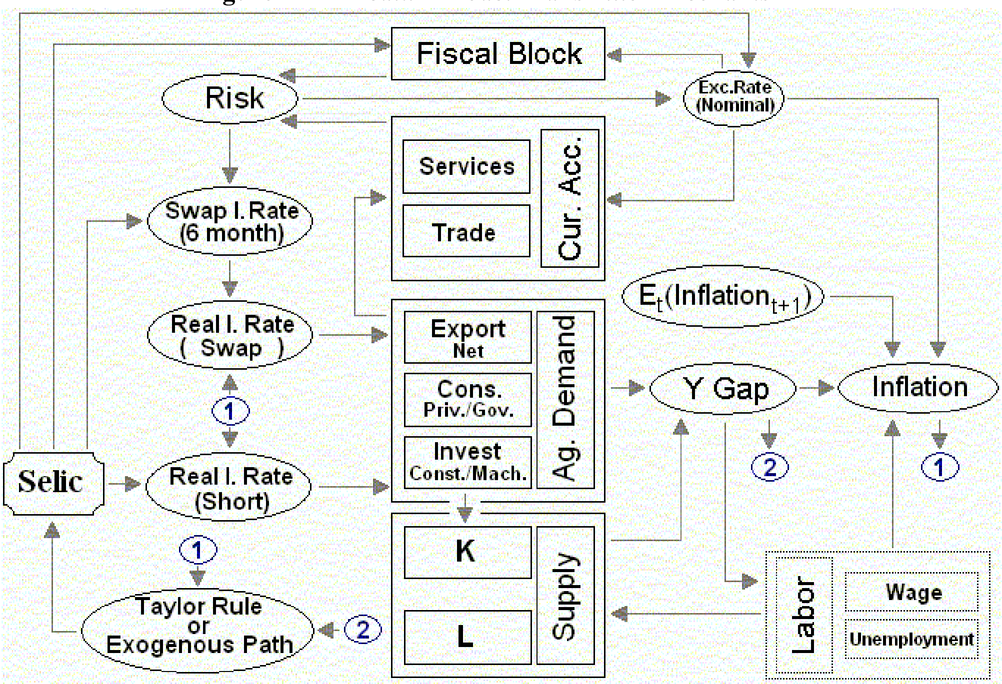
The exchange rate mechanism is still available in the medium model. But now the fiscal and external variables are also part of the model by affecting exchange rate by the risk premium. An increase in the interest rate that worsens the fiscal front will generate an increase in the risk premium and a devaluation of the exchange rate that might offset the aggregate demand channel. The current account

[1] Muinhos, Alves and Riella (2002) already have similar equations for UIP and Risk premium.

[2] Although the main blocks of the medium model are represented in Figure 2, there are some interactions between variables not shown in the figure in order to obtain a clean representation of the model. Nevertheless, the model equations are commented in the text. As this is still a work in progress, our blocks are subject to future improvements.

deficit is also affecting the risk premium and consequently, the exchange rate and inflation. A rapid GDP growth may cause an increase in inflation by the output gap and also by the trade balance worsening.

Figure 2 ***Medium Model Transmition Mechanism***



We also include the labor market as a channel of monetary transmission. An increase in the labor force may increase the potential output by the Cobb-Douglas production function. Besides that an increase in productivity measured by the Total Factor Productivity will increase GDP and decrease inflation allowing for a loosening in the monetary policy.

The absence of micro-founded behavior equations does not allow us to endogenously find endogenous steady state values for variables, such as interest rate or exchange rate. The UIP equation, for instance, presents a calibrated error correction term that drives the system to the long run equilibrium steady-state exchange rate, defined as the one that leads to an ad-hoc long-run current account/GDP ratio. And this ration, on the other hand, should be consistent with a steady-state external liabilities/GDP ratio. For the interest rate, the use of the Taylor rule assures long run equilibrium compatible with the inflation target and a neutral output gap.

3. Modeling Brazilian Economy

Since the third quarter of 2001, IBGE[3] has been releasing quarterly income components series, whose samples start in 1991:01. This fact made possible to construct and to run more complete and detailed macro models for the Brazilian economy. However, studies using these new quarterly data are very recent and the constructed models are still based on new Keynesian paradigm, using ad hoc relations between the variables rather than micro founded structural relationship. The main reason is that ad hoc relations are easier to run and model, not requiring many computational time nor very complex coding technology. Even considering that our results, in some sense, are subjected to the Lucas critique, it is still worth working on the model, due the fact that this is the first large one developed in the Research Department of Central Bank of Brazil, and it is in line, in some parts, with other models developed using this new Brazilian quarterly data as in Cavalcanti, Kai and Carvalho (2002). The use of micro founded models is our agenda for the next generation of the Central Bank of Brazil structural modeling.

The estimation samples depend upon the availability and behavior of the data. We decide to not homogenize the starting point of the estimations. Because since we are estimating each equation separately if we do not consider the series since as long as possible we are throwing information away. Nevertheless when there are severe structural breaks that cannot be fixed with dummy variables, we decide not to consider the series before the break. Inflation before 1994 is a good example of that

[3] Brazilian Institute of Geography and Statistics.

problem. As we did not estimate the equations in a system, simultaneity bias was avoided using two stages estimations or considering only lagged variables on the right side of the equations.

3.1. The Aggregated Demand Side

The aggregate demand is determined by its definition identity, shown in Equation 1. In this section, we will model each of its components and related variables as taxes, government expenditures and fiscal deficit (primary concept).

$$Y_t = C_t + G_t + I_t + X_t^{net} + \Delta S_t \tag{1}$$

Where:

Y_t	*output;*
C_t	*household consumption;*
G_t	*government consumption;*
I_t	*investment;*
X_t^{net}	*net exports;*
ΔS_t	*inventory investment.*

Although both nominal and real data are released, real income components do not sum as Equation 1[4]. As a solution, we estimated income real components using their nominal income share applied to real income. This method guarantees Equation 1, in real terms, in the whole sample.

3.1.1. Household Consumption

Household consumption is far the most important income component, since it responds for about 70% of output. In this context, we used a very simple specification: a first difference equation, in logarithms, with an error correction process to achieve a constant long run fraction of disposable income. In order to guarantee this restriction, the disposable yield must have a unitary coefficient in logarithm, as shown in Equation 2. As it is an error correction equation type, coefficient α_1 must be negative. As consumption decreases on higher real interest rates, α_2 must be negative, as well. Estimation output is shown in Table 1.

$$\Delta ln(C_t) = \alpha_0 + \alpha_1 \cdot \left[ln(C_{t-1}) - \sum_{j=1}^{3} \frac{ln(Y_{t-j}^d)}{3} \right] + \alpha_2 \cdot r_{t-1} + \sum_{i=1}^{3} \beta_i \cdot Seas_i \tag{2}$$

Where:

C_t	*household consumption;*
Y_t^d	*disposal income:* $Y_t^d = Y_t - T_t^d$;
T_t^d	*direct taxess;*
r_t	*short run real interest rate:* $r_t = \dfrac{Selic_t}{4} - \sum_{i=0}^{3} \dfrac{\pi_{t-i}}{4}$;
$Selic_t$	*annualized Brazilian over night nominal interest rate.*

Table 1 ***Equation 2***

Method: *OLS*			**Sample:** *1991:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-0.266	0.038	-7.071	0.000
α_1	-0.707	0.118	-5.988	0.000
α_2	-0.126	0.034	-3.741	0.001
β_1	-0.022	0.018	-1.220	0.231
β_2	0.027	0.017	1.596	0.120
β_3	0.079	0.016	4.823	0.000
$D_{94:01}$	0.182	0.036	5.095	0.000
R^2 = 0,987			$R^2_{Ajust.}$ = 0,985	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.889 (p =0.420)
Jarque-Bera Normality Test: 0.055 (p =0.973)
White Heteroskedasticity Test: F = 1.170 (p =0.345)

[4] Even when changing base period values in order to guarantee the income identity in some quarters, there are always some periods in which income identity does not fit. Probably it is due the fact that they are not deflated by the same deflator income is.

3.1.2. Investment

As household consumption, investment has an important role on determining the economic activity, since it affects both output, directly, and potential output, indirectly, via capital inventories. We break investment series into its construction and machinery components and modeled each one. These two series are better explained by medium run real interest rates (6 months)[5]. However, available 6-month interest rate series data are very recent, which made us use a short sample in the estimations (from 1994 on). As regressions seemed to have a good explanation power - good fit and good residual statistics – we considered them as acceptable reality representations. Construction[6] and machinery investments are represented in Equations 3 and 4, respectively, and its estimation outputs are shown in Tables 2 and 3, respectively. Equation 3 was estimated with a step dummy indicating a level change after 1996. Total investment is determined by its identity, in Equation 5.

$$\Delta ln(IC_t) = \alpha_0 + \alpha_1 \cdot [ln(IC_{t-1}) - ln(Y_{t-1})] + \alpha_2 \cdot r_t^{Swap6} + \sum_{i=1}^{3} \beta_i \cdot Seas_i \tag{3}$$

$$\Delta ln(IM_t) = A_0 + A_1 \cdot [ln(IM_{t-1}) - ln(Y_{t-1})] + A_2 \cdot r_{t-1}^{Swap6} + \sum_{i=1}^{3} B_i \cdot Seas_i \tag{4}$$

$$I_t = IC_t + IM_t \tag{5}$$

Where:

I_t	*total investment;*	i_t^{Swap6}	*annualized 6 month swap interest rate;*
IC_t	*construction investment;*		
IM_t	*machinery investment;*	r_t^{Swap6}	*medium run real interest rate:* $r_t^{Swap6} = i_t^{Swap6}/4 - \sum_{i=0}^{3} \pi_{t-i}/4$.
Y_t	*real income;*		

Table 2 ***Equation 3***

Method: *OLS*			**Sample:** *1994:4 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-1.622	0.431	-3.762	0.001
α_1	-0.740	0.213	-3.481	0.002
α_2	-0.299	0.105	-2.848	0.009
β_1	0.099	0.026	3.865	0.001
β_2	0.142	0.026	5.469	0.000
β_3	0.122	0.025	4.962	0.000
D_{96}	0.053	0.027	1.954	0.064
	R^2 = 0,724		$R^2_{Ajust.}$ = 0,649	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.159 (p =0.854)
Jarque-Bera Normality Test: 0.563 (p =0.722)
White Heteroskedasticity Test: F = 1.947 (p =0.109)

Table 3 ***Equation 4***

Method: *OLS*			**Sample:** *1995:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
A_0	-1.116	0.283	-3.949	0.001
A_1	-0.369	0.104	-3.563	0.002
A_2	-0.213	0.111	-1.912	0.069
B_1	0.132	0.026	5.026	0.000
B_2	0.135	0.024	5.532	0.000
B_3	0.130	0.024	5.409	0.000
	R^2 = 0,743		$R^2_{Ajust.}$ = 0,684	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.493 (p =0.618)
Jarque-Bera Normality Test: 1.618(p =0.445)
White Heteroskedasticity Test: F = 0.997 (p =0.462)

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[5] Household consumption is, on the other hand, is better explained by short run real interest rate.

[6] Equation 3 was better explained by contemporaneous real interest rate, what could somehow be an indicative of no rigidity in the construction investment decision process. On the other hand, it may only indicate that the rigidity lag is shorter than 3months.

3.1.3. Net Exports

Since nominal exports and imports, in US dollars, are modeled in Sections 3.3.3 and 3.3.4, real exports and imports modeling are quite simple. We only have to transform the currency into Brazilian real and deflate them, as described in Equations 6 and 7. Net exports are determined by its definition, in Equation 8.

$$\Delta ln(X_t) = \Delta ln(Exports_t) - \Delta e_t - \pi_t \tag{6}$$

$$\Delta ln(M_t) = \Delta ln(Im\,ports_t) - \Delta e_t - \pi_t \tag{7}$$

$$X_t^{net} = X_t - M_t \tag{8}$$

Where:

$Exports_t$	*nominal US dollar exports;*
$Imports_t$	*nominal US dollar imports;*
e_t	*exchange rate (logarithm);*
π_t	*IPCA inflation rate;*

3.1.4. Government Consumption

Brazilian fiscal surplus (primary concept) is defined as total taxes revenue minus government expenditures. Those are divided, as usual, in direct and indirect taxes and these are divided into government investment and government consumption.

We model total taxes as a function of lagged total taxes and lagged income, in a simple specification as described in Equation 9. In order to capture the most recent behavior of tax police, estimation sample was very short and the result is shown in Table 4. A step dummy was used to model a level change occurred in from 1999 on and a pulse dummy was used to capture a 1997:04 outlier. Direct taxes, modeled in order to have a disposal income measure $\left(Y_t - T_t^d\right)$, are modeled by a similar specification, described in Equation 10. Its output is shown in Table 5.

Government investment, as a ratio of total taxes, is modeled as a AR(3) process described in Equation 11, with outlier dummies. The result is shown in Table 6. Government consumption is calculated as a residual, in Equation 12, since we determined an exogenous path to fiscal deficit (primary concept) as a GDP ratio (annual average).

$$T_t^T = \alpha_0 + \alpha_1 \cdot T_{t-1}^T + \sum_{i=1}^{3} \beta_i \cdot Y_{t-i} \tag{9}$$

$$T_t^d = \alpha_0 + \alpha_1 \cdot T_{t-1}^d + \sum_{i=1}^{3} \beta_i \cdot Y_{t-i} \tag{10}$$

$$\frac{I_t^G}{T_t^T} = \alpha_0 + \alpha_1 \cdot \frac{I_{t-1}^G}{T_{t-1}^T} + \alpha_2 \cdot \frac{I_{t-2}^G}{T_{t-2}^T} + \alpha_3 \cdot \frac{I_{t-3}^G}{T_{t-3}^T} \tag{11}$$

$$G_t = \sum_{j=0}^{3}\left(T_{t-j}^T\right) - FSPR_t \cdot \sum_{j=0}^{3} Y_{t-j} - \sum_{j=1}^{3} G_{t-j} \tag{12}$$

Where:

T_t^T	*total taxes;*
T_t^d	*direct taxes;*
I_t^G	*government investment;*
$FSPR_t$	*fiscal deficit (primary concept).*

Table 4 *Equation 9*

Method: *OLS*			Sample: *1996:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-25.297	12.177	-2.077	0.053
α_1	-0.391	0.097	-4.050	0.001
β_1	0.464	0.079	5.889	0.000
β_2	-0.290	0.083	-3.475	0.003
β_3	0.371	0.079	4.697	0.000
D_{99}	-4.782	1.004	-4.762	0.000
$D_{97:04}$	-10.449	1.684	-6.205	0.000
R^2 = 0,926			$R^2_{Ajust.}$ = 0,899	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 1.560 (p =0.242)
Jarque-Bera Normality Test: 2.274 (p =0.321)
White Heteroskedasticity Test: F = 0.516 (p =0.850)

Table 5 *Equation 10*

Method: *OLS*			Sample: *1996:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-29.538	11.458	-2.578	0.020
α_1	-0.449	0.104	-4.333	0.000
β_1	0.342	0.075	4.546	0.000
β_2	-0.231	0.083	-2.779	0.013
β_3	0.288	0.078	3.675	0.002
D_{99}	4.224	1.000	4.226	0.001
$D_{97:04}$	-9.587	1.674	-5.726	0.000
R^2 = 0,899			$R^2_{Ajust.}$ = 0,863	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 1.031 (p =0.381)
Jarque-Bera Normality Test: 4.323 (p =0.115)
White Heteroskedasticity Test: F = 0.470 (p =0.881)

Table 6 *Equation 11*

Method: *OLS (White Heteroskedasticity-Consistent Standard Errors & Covariance)*			Sample: *1996:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	0.031	0.012	2.497	0.022
α_1	0.509	0.142	3.584	0.002
α_2	0.447	0.173	2.581	0.019
α_3	-0.320	0.171	-1.871	0.078
$D_{96:02}$	0.032	0.003	9.171	0.000
$D_{97:04}$	0.072	0.002	31.423	0.000
R^2 = 0,859			$R^2_{Ajust.}$ = 0,820	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 2.125 (p =0.152)
Jarque-Bera Normality Test: 0.122 (p =0.941)
White Heteroskedasticity Test: F = 3.084 (p =0.029)

3.1.5. Inventory Investment

We define the inventory dynamics as follows:

$$S_t = (1-\delta)\cdot S_{t-1} + \Delta S_{t-1} \tag{13}$$

Where:

S_t *inventory level;*
ΔS_t *inventory investment;*
δ *depreciation rate, invariant over time.*

The basic hypothesis considered is that firms produce in order to maintain a minimum inventory as a long run time invariant demand ratio $\frac{S}{Z}$, where $Z_t = Y_t - \Delta S_t$. Keeping this assumption in mind and dividing both sides of Equation 13 by Z_t , we can get to the result described in Equation 14, once considering that $Z_{t+1} = Z_t \cdot \left(1 + g_{t+1}^Z\right)$. In the steady state, the ratio $\frac{S}{Z}$ must converge. Hence, by Equation 15, equilibrium ratio $\frac{\Delta S}{S}$ must depend on the depreciation rate plus the quarterly Z growth. Assuming that the inventory dynamics over the last decade (1991/2001) have behaved in average as Equation 3, we can estimate two latent variables related to the inventory formation: the initial inventory (1991:01) and the depreciation rate (δ). Note that if the ratio $\frac{\Delta S}{S}$ is supposed to be constant over the sample, the summation of quadratic deviations from the sample average must be minimum. In this context, we just have to find the values of those two non-observed variables that minimize that summation since g_{t+1}^Z values are known. Doing so, we find $S_{91:01} = 59{,}82$ and $\delta = 8{,}62\%$ *(annual basis).*

$$\frac{S_{t+1}}{Z_{t+1}} - \frac{S_t}{Z_t} = -g_{t+1}^Z \cdot \frac{S_{t+1}}{Z_{t+1}} - \delta \cdot \frac{S_t}{Z_t} + \frac{\Delta S_t}{Z_t} \tag{14}$$

$$\left(\delta + g_{t+1}^Z\right) \cdot \frac{S_t}{Z_t} = \frac{\Delta S_t}{Z_t} \quad \therefore \quad \frac{\Delta S_t}{S_t} = \left(\delta + g_{t+1}^Z\right) \tag{15}$$

Where:

$Z_{t+1} = Z_t \cdot \left(1 + g_t^Z\right)$

g_t^Z *quarterly Z_t growth rate.*

3.1.5.1 Estimating an Inventory Investment Dynamic Model

We model $\frac{\Delta S_t}{S_t}$ as a function of its own lags and of Z_t growth rate, as shown in Equation 16[7]. In order to avoid seasonal variations of this rate, we considered a 1-lagged 4-quarter growth rate average instead of 1-quarter growth rate. We impose that the long-run $\frac{\Delta S_t}{S_t}$ values must follow Equation 15. Table 7 shows the estimation output with outlier dummies.

$$\frac{\Delta S_t}{S_t} = \frac{\delta}{\left(1 - \alpha_1 - \alpha_2\right)} + \alpha_1 \cdot \frac{\Delta S_{t-1}}{S_{t-1}} + \alpha_2 \cdot \frac{\Delta S_{t-3}}{S_{t-3}} + \left(1 - \alpha_1 - \alpha_2\right) \cdot \frac{\ln\left(Z_{t-1}\right)}{4 \cdot \ln\left(Z_{t-5}\right)} \tag{16}$$

Table 7 ***Equation 16***

Method: *OLS*			**Sample:** *1992:2 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_1	-0.204	0.086	-2.371	0.024
α_2	-0.363	0.109	-3.331	0.002
$\alpha_{94:01}$	-0.143	0.003	-47.576	0.000
$\alpha_{94:04}$	-0.108	0.017	-6.515	0.000
$\alpha_{97:04}$	0.116	0.007	17.673	0.000
$R^2 = 0{,}638$			$R^2_{Ajust.} = 0{,}596$	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.858 (p =0.434)
Jarque-Bera Normality Test: 1.58 (p =0.453)
White Heteroskedasticity Test: F = 1.010 (p =0.478)

3.2. The Supply Side

The supply side was modeled considering a traditional Cobb-Douglas production function approach and a Phillips type curve.

[7] The $\Delta S_t / S_t$ correlogram indicated that just lagged correlation 1 and 3 were significant.

3.2.1. Cobb-Douglas Production Function

A Cobb-Douglas production function, with capital inventory and labor, was modeled as described in Equation 17.

$$Y_t = A_t \cdot (K_t \cdot uci_t)^{\alpha_t} \cdot (L_t)^{1-\alpha_t} \quad (17)$$

Where:

$L_t = PEA_t \cdot (1 - u_t)$

$K_t = (1-\delta) \cdot K_{t-1} + \sum_{i=2}^{4} \beta_i \cdot I_{t-i}$

$\sum_{i=2}^{4} \beta_i = 1$

PEA *labor force (Age ≥ 15 years)*[8]*;*
u *unemployment rate (%)*[9]*;*
uci *installed industrial capacity used (%)*[10]*;*
δ *depreciation rate*[11]*.*

The total factor productivity (TPF) series is extracted as a residual of Equation 17, since the series of GDP, PEA, u, uci and α_t are known. It is important to emphasize here that the TPF series (A_t) depend only on the latent values of β_i and on the initial capital inventory (K_0). Knowing the TPF series, we can define the potential output as in Equation 18. The output gap[12], written in a logarithmic form $h_t = \ln\left(\frac{Y_t}{\overline{Y}_t}\right)$, can be easily derived as described in Equation 19[13].

$$\overline{Y}_t = A_t \cdot (K_t \cdot uci_{fe})^{\alpha_t} \cdot (L_t^{fe})^{1-\alpha_t} \quad (18)$$

$$h_t = \alpha_t \cdot [\ln(uci_t) - \ln(uci_{pe})] + (1-\alpha_t) \cdot [\ln(1-u_t) - \ln(1-\overline{u})] \quad (19)$$

Where:

$L_t^{fe} = PEA_t \cdot (1 - \overline{u})$

$\overline{u}$ *natural unemployment rate (%);*
fe *full employment index.*

Note that each quarter investment contributes to the capital inventory formation up to 4 quarters ahead. The intuition behind this modeling is that certain investment percentage may be converted into capital inventory faster than others. However, we restricted the investment percentage lag to be between 2 and 4 quarters.

In this model, we considered the dynamic behavior of the α, the capital share yield. Actually, this series had a structural break in 1994 as shown on Graph 1, where annual values are plotted, since that is the frequency released by IBGE[14]. In order to obtain α quarterly values, we considered an alternative that assured the smoothness pattern and annual quarterly values average restriction, as described and justified in Appendix 1. A visual comparison between annual and quarterly estimated values is presented in Graph

[8] Released on a monthly basis by the *Brazilian Institute of Geography and Statistics* (IBGE), the *População Economicamente Ativa* (PEA) is the potential labor force in the economy, accounting for employed people and unemployed who looked for a job in the last 30 days, both older than 15 y.

[9] Released on a monthly basis by the *Brazilian Institute of Geography and Statistics* (IBGE).

[10] Released on a quarterly basis by the *Brazilian Fundação Getúlio Vargas* (FGV).

[11] We considered, for simplification sake, that capital inventory depreciates by the same rate estimated in the inventory investment section.

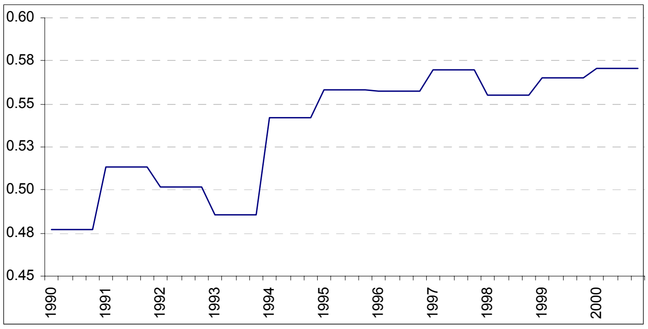
[12] "Hiato" in Portuguese.

[13] In the simulations, we did not model u_t and uci_t. Hence, the simulations output gap is obtained from the aggregated demand output and from the potential output described here.

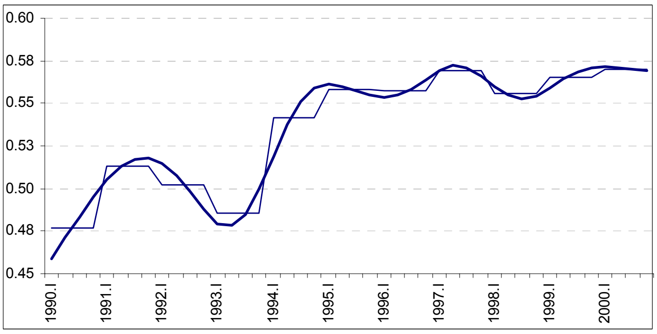
[14] Released on a annual basis by the *Brazilian Institute of Geography and Statistics* (IBGE) on "Table 4 - Composição do Produto Interno Bruto sob as três óticas - 1996-2000", Contas Nacionais do Brasil.

2. As IBGE tables only included, by the time this paper was made, information up to 2000, we used α forecasts made by Equation 26 in order to estimate the output gap and potential output in 2001.

Graph 1 *Structural break of capital share yield*



Graph 2 *Quarterly α_t series*



Following, we will present some results based on Solow's growth model, considering the effect of a not constant α path. First, we rewrite the potential output equation considering the labor efficiency (E_t), as in Equation 21. Equation 22 shows the last equation transformed into a logarithm first difference. It is a known result that, in stead state, k_t^{fe} and E_t must grow by the same rate. Accepting that the first difference in natural logarithm equals the growth rate, we may expect that in the steady state or in a large enough sample, Equation 22 reduces to Equation 23[15].

$$\bar{y}_t = \left(k_t^{fe}\right)^{\alpha_t} \cdot \left(E_t\right)^{1-\alpha_t} \tag{21}$$

$$\Delta\ln(\bar{y}_t) = \alpha_t \cdot \Delta\ln\left(k_t^{fe}\right) + \ln\left(k_{t-1}^{fe}\right)\cdot\Delta(\alpha_t) + (1-\alpha_t)\cdot\Delta\ln(E_t) - \ln(E_{t-1})\cdot\Delta(\alpha_t) \tag{22}$$

$$\Delta\ln(\bar{y}_t) = \Delta\ln\left(k_t^{fe}\right) + \left[\ln\left(k_{t-1}^{fe}\right) - \ln(E_{t-1})\right]\cdot\Delta(\alpha_t) \tag{23}$$

Where:

$$E_t = (A_t)^{\frac{1}{1-\alpha_t}} \qquad \bar{y}_t = \frac{\bar{Y}_t}{L_t^{fe}} \qquad k_t^{fe} = \frac{K_t \cdot uci_{fe}}{L_t^{fe}}$$

Regarding Equation 23, whose both sides are functions of non-observed variables: K_0, β_i, uci_{fe} and $\bar{u}$ [16], its empirical validity depends much on the estimates – or calibrations - of those latent variables. Hence, we decided that the estimation of such variables should include an optimization process aiming to reduce the deviations between both sides, over a sample in which we could validate the main hypothesis of this result, that k_t^{fe} and E_t grow by the approximately the same rate. However, there was another restriction that should be considered. Actually, it is a restriction present in the Phillips curve modeled in the next section. After modeling this curve, we will present the latent variables estimation process.

3.2.2. Phillips Curve Modeling

A Phillips curve, as usual, should consider an inflation rate expectation, a measure of the activity level as the output gap, for instance, and also a pass-through component, to capture import prices variation. Concerning this last component, we model a structural break on the pass-through coefficient after the exchange rate regime has changed into floating in January 1999. We assume that, under the new regime, movements in exchange rates will not be perceived as permanent as they were in the crawling peg regime. Thus, expecting this coefficient to be smaller under the floating exchange rate regime, we introduced a step dummy in a non-linear pass-throw coefficient in order to capture the structural break.

Related to the inflation expectations, we should make some regards concerning the government prices. In Brasil, government prices have a high weight in the IPCA[17] basket, averaging around 30%. And the forecasting of these prices is made with certain accuracy for one year ahead, mainly because of price

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[15] A particular case is when α_t is time invariant, the second term on the right side vanishes and we obtain the known result that k_t^{fe} and $\bar{y}_t$ grow by the same rate as well as E_t.

[16] Remember that δ is already estimated.

[17] Measured and released on a monthly basis by the Brazilian Institute of Geography and Statistics (IBGE), IPCA is the consumer price index chosen for the purpose of gauging yearly inflation targets in the Inflation Target system.

readjusting contract clauses. In this context, our Phillips curve models just the "free prices"[18], which should respond to monetary policy[19]. But free inflation expectations must be a function of full inflation, with backward and forward components. A final feature of the specification is a verticality long run restriction: there must be no intercept coefficient and backward and forward coefficients and the pass-through coefficient must sum 1. The absence of an intercept coefficient is the final restriction to be used on the latent variables estimation mentioned earlier. This restriction should be considered in the estimation described in the next section. Attempting to capture all those features, the Phillips curve specification should be described as in Equation 24[20].

$$\pi_t^{free} = \alpha_1 \cdot \pi_{t-1} + \left(1-\alpha_1-\alpha_3-\alpha_4 \cdot D_{fl}\right)\cdot E_t\pi_{t+1} + \alpha_2 \cdot h_{t-1} + \left(\alpha_3+\alpha_4 \cdot D_{fl}\right)\cdot\left(\Delta e_t + \pi_t^*\right) \tag{24}$$

Where all variables are specified in logarithms:

π_t^{free} *free inflation rate, considering the IPCA;*

π_t *full inflation rate, considering the IPCA;*

π_t^* *foreign inflation rate, considering the US PPI;*

h_t *output gap (hiato in Portuguese);*

e_t *exchange rate;*

D_{fl} *step dummy: 0 before exchange rate regime changing and 1 after.*

3.2.3. Non-Observed Variables Estimation

As commented in Boone, Juillard, Laxton and N'Diaye (2002), NAIRU estimation processes that doe not exploit information about inflation may result in inefficient historical measures of the NAIRU, biased parameter estimates, as well as inefficient forecasts of the NAIRU. In this work, rather than taking into account the NAIRU path, we assumed a constant natural rate of unemployment. Nevertheless, as this last critique adverts, we must use inflation information to estimate the latent variables. In this sense, we considered a Phillips curve restriction described as follows.

As previously mentioned, the latent variables estimation process must approximate both sides of Equation 23, over a sample in which we could validate the main hypothesis of this result, that k_t^{fe} has converged. On the other hand, Equation 24 must have no intercept, what is justifiable when we cannot reject the null hypothesis that, on a specification containing an intercept term, its coefficient equals zero. We assume here that the only reason why the Phillips curve should have a significant intercept is from a misestimating output gap, since it is the only non-observed variable whose estimation includes estimated non-observed variables. Hence, in each interaction of the estimation process, there are two phases: the first consists on a optimization process in which we estimate the latent variables in order to minimize the quadratic sum of the difference between both sides of Equation 23, as described in System 25. The sample used in the optimization process is from 1995:1 to 2001:4, because in this period k_t^{fe} seemed to be very stable.

$$\begin{aligned} &\min.\ \sum_t \left\{\Delta\ln(\bar{y}_t) - \left[\Delta\ln\left(k_t^{fe}\right) + \left[\ln\left(k_{t-1}^{fe}\right) - \ln(E_{t-1})\right]\cdot\Delta(\alpha_t)\right]\right\}^2 \\ &\textit{Subject to:} \\ &0 \le \beta_i,\ uci_{fe}\ \textit{and}\ \bar{u} \le 1 \end{aligned} \tag{25}$$

The second phase of each interaction consists on estimating Equation 24 with an intercept. Assuming that the only misestimated variable is the output gap generated in the first phase, and assuming that this error should be related to the uci_{ef}, the intercept value should equal $\alpha_2 \cdot \varepsilon_h \cdot \bar{\alpha}$, where $\varepsilon_h \cdot \bar{\alpha}$ is the uci_{ef} error multiplied by average α_t. Therefore, we could estimate the uci_{ef} error, and a new measure of

[18] Purging government prices.

[19] In our simulations, we considered the government inflation forecasts up to one year ahead based on the contracts. But, for higher forecasting horizons, we assume that government prices should move together with free prices.

[20] Due to Central Bank of Brazil's internal policy, we are not allowed to release our Phillips curve estimation output.

uci_{ef}. With the new uci_{ef} estimated, we could do phase one again but with the restriction that uci_{ef} should be the estimated one.

The process generates another value for the natural rate of unemployment that should be consistent with the uci_{ef} imposed. With new estimates, we could do phase 2 again restarting the cycle of interactions. It is important to note that, in each second phase, the intercept looses more and more significancy. With the estimated parameters, we could estimate[21] Equation 24 without an intercept term. In convergence, the process estimated:

$$K_0 = 904.25 \quad uci_{fe} = 84.93\% \quad \overline{u} = 5.29\% \quad \beta_4 = 0 \quad \beta_3 = 0 \quad \beta_2 = 1$$

3.2.4. Modeling the Components of Cobb-Douglas Production Function

As there was no a priori information about the future dynamics of α_t , we simply estimated an ARIMA(3;1;0) model with no intercept in order to avoid a non-justified trend, as shown in Equation 26. Estimation output is described in Table 9. The total factor productivity (A_t) was modeled, in logarithm, by a seasonal ARIMA(2;1;0), as shown in Equation 27 and its estimation output is described in Table 10. PEA was modeled in logarithms with an autoregressive component, linear trend and seasonality. Trying to account for a level change occurred after 1994:3, we introduced a step dummy. The specification is shown in Equation 28 and the estimation output is described in Table 11. And, finally, the capital inventory, obtained by the estimated parameters into its definition in Equation 5, is shown in Equation 29.

$$\Delta\alpha_t = \beta_1 \cdot \Delta\alpha_{t-1} + \beta_2 \cdot \Delta\alpha_{t-2} + \beta_3 \cdot \Delta\alpha_{t-3} \tag{26}$$

$$\Delta \ln(A_t) = \beta_0 + \beta_1 \cdot \Delta \ln(A_{t-1}) + \beta_2 \cdot \Delta \ln(A_{t-2}) + \sum_{i=1}^{3} \alpha_i \cdot Seas_i \tag{27}$$

$$\ln(PEA_t) = \beta_0 + \beta_1 \cdot \ln(PEA_{t-1}) + \beta_2 \cdot Trend_{91:01} + \sum_{i=1}^{3} \alpha_i \cdot Seas_i + \beta_3 \cdot D_{94:03} \tag{28}$$

$$K_t = 0.98 \cdot K_{t-1} + I_{t-2} \tag{29}$$

Table 9 ***Equation 26***

Method: *OLS*			Sample: *1995:1 to 2000:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
β_1	2.200	0.117	18.784	0.000
β_2	-1.926	0.176	-10.930	0.000
β_3	0.619	0.090	6.851	0.000
R^2 = 0,985			$R^2_{Ajust.}$ = 0,983	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.281 (p =0.758)
Jarque-Bera Normality Test: 0.529(p =0.337)
White Heteroskedasticity Test: F = 0.905 (p =0.768)

Table 10 ***Equation 27***

Method: *OLS*			Sample: *1995:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
β_0	-0.052	0.012	-4.294	0.000
β_1	0.745	0.182	4.082	0.000
β_2	-0.380	0.148	-2.568	0.017
α_1	0.083	0.020	4.160	0.000
α_2	0.080	0.017	4.626	0.000
α_3	0.056	0.017	3.316	0.003
R^2 = 0,673			$R^2_{Ajust.}$ = 0,602	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.166 (p =0.852)
Jarque-Bera Normality Test: 0.886 (p =0.642)
White Heteroskedasticity Test: F = 0.536 (p =0.797)

[21] This equation was estimated using Two-Stage Least Squares with lagged inflation and inflation forecasts made by an univariated model, as instrumental variables for the forward component. The results are robust and we could confirm that pass-through coefficient reduced significantly after changing the exchange rate regime.

Table 11 *Equation 28*

Method: *OLS*			**Sample:** *1991:1 to 2001:4*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
β_0	6.2527	1.805	3.464	0.001
β_1	0.6225	0.109	5.710	0.000
β_2	0.0014	0.000	2.947	0.006
α_1	-0.0058	0.003	-2.017	0.051
α_2	0.0041	0.003	1.434	0.160
α_3	0.0027	0.003	0.979	0.334
β_3	0.0090	0.004	2.470	0.018
	$R^2 = 0{,}987$		$R^2_{Ajust.} = 0{,}985$	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.889 (p =0.420)
Jarque-Bera Normality Test: 0.055 (p =0.973)
White Heteroskedasticity Test: F = 1.170 (p =0.345)

3.3. External Block

3.3.1 Exchange Rate

We modeled exchange rate by an equation loosely based on UIP non-arbitrage condition. It is completely detailed in Muinhos, Alves and Riella (2002) and is a first difference logarithm equation, since we could not reject the null hypothesis that exchange rate has a unit root in the used sample. The exchange rate expectatation was modeled as a lagged exchange rate plus the expected inflation differential, in order to maintain the real exchange rate. The risk premium was modeled as a linear function of the C-Bond spread over treasury. And, instead of imposing a unitary interest rate differential coefficient, with negative signal, we relaxed this assumption and let this coefficient to be estimated[22]. The specification is described in Equation 30 and its output estimation is shown in Table 12.

$$\Delta e_t = \alpha_1 \cdot \Delta e_{t-1} + \alpha_2 \cdot \Delta(Selic_t - FFunds_t) + \alpha_3 \cdot \Delta SCBond_t + (1-\alpha_1)\cdot(\pi_{t-1} - \pi^*_{t-1}) \qquad (30)$$

Table 12 Equation 30

Method: *TSLS*			**Sample:** *May1999 to Dec2001*	
Instruments: Δe_{t-1}, $\Delta(Selic_{t-1} - FFunds_t)$, $(\pi_{t-1} - \pi^f_{t-1})$, $\Delta(\text{FiscalDebt/GDP}_t)$, CurAc/GDP_t and Res/GDP_{t-1}				
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_1	0.513	0.116	4.423	0.000
α_2	-9.238	3.554	-2.599	0.015
α_3	49.938	8.991	5.554	0.000
	$R^2 = 0.672$		$R^2_{Ajust.} = 0.649$	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 2.212 (p =0.129)
Jarque-Bera Normality Test: 0.418 (p =0.811)
White Heteroskedasticity Test: F = 1.170 (p =0.406)

Completing this specification, we added an ad hoc error correction component, in order to achieve an equilibrium real exchange rate determined as the one necessary to make the current account path end on an ad hoc value. The error correction coefficient was calibrated to correspond to a 3 years half-life correction process.

3.3.2 Risk Premium

As completly described in Muinhos, Alves and Riella (2002), we modeled C-Bond spread over treasury, used in Equations 30 and 31, in order to capture sovereign risk perceptions generated by fiscal variables, external commerce and solvency/liquidity variables. Using a parsimonious criterion, we focused on relevant variables avoiding, on the other hand, *over fitting* estimations. In the best-fit estimation, foreign reserves (%GDP), public debt (%GDP) and current account balance (%GDP) coefficients were significant and representative of fiscal variables, external commerce and

[22] This was a previous work. In the most recent version of the model we use a UIP condition to model the real exchange rate.

solvency/liquidity indicators [23]. The specification is described in Equation 31 and its output estimation, by TSLS, is shown in Table 13.

$$SCBond_t = \alpha_0 + \alpha_1 \cdot SCBond_{t-1} + \alpha_2 \cdot \Delta Res/GDP_t + \alpha_3 \cdot \Delta PD/GDP_t + \alpha_4 \cdot CurAc/GDP_t \qquad (31)$$

Where:
Res/GDP_t foreign reserves (%$GDP^{12\ month}$);
PD/GDP_t public debt (%$GDP^{12\ month}$);
$CurAc/GDP_t$ 12 month accumulated current account balance (%$GDP^{12\ month}$).

Table 13 ***Equation 31***

Method: *TSLS*		**Sample:** *Jan1996 to Dec/2001*		
Instruments: $SCBond_{t-1}$, Res/PIB_{t-1} , Res/PIB_{t-2} , $\Delta DLT/PIB_t$, $TCor/PIB_t$ e Δe_{t-1}				
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-0.0004	0.0044	-0.0821	0.9348
α_1	0.8597	0.0472	18.2134	0.0000
α_2	-0.8396	0.4274	-1.9646	0.0536
α_3	0.1360	0.0788	1.7259	0.0890
α_4	-0.2536	0.1074	-2.3608	0.0212
$R^2 = 0.874$			$R^2_{Ajust.} = 0.867$	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 1.586 (p =0.212)
Jarque-Bera Normality Test: 19.412 (p =0.000)
White Heteroskedasticity Test: F = 0.910 (p =0.543)

3.3.3 Exports

In this section and in the next, we present our nominal net export modeling in US dollars. For simplification sake, we modeled exports quantum and imports quantum. Prices are modeled as ARMA models, as described in Muinhos, Alves and Riella (2002). Equation 32 presents the quarterly estimation for the quantitative index of exports. The sample starts at 1988 and the coefficients and the t statistics are in Table 14.

$$\exp_t = \alpha_0 + \alpha_1 \cdot \exp_{t-1} + \alpha_2 \cdot y_t^* + \alpha_3 \cdot \theta_{t-1} + \alpha_4 \cdot lpxt_t + \sum_{j=1}^{3} \beta_j \cdot Seas_j + \alpha_5 \cdot D_{91:03} \qquad (32)$$

where:
exp_t quantitative index of exports for period t;
y_t^* world GDP for period t, measuared as the log of the world imports;
θ_t real exchange rate for period t;
$lpxt_t$ price index of exports for period t;
$Seas_j$ seasonal dummies;
$D_{91:03}$ outlier dummy for 1991:03.

Table 14 ***Equation 32***

Method: *OLS*			**Sample:** 1988:01 to 2001:02	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-0.572	0.804	-0.711	0.481
α_1	0.559	0.081	6.865	0.000
α_2	0.445	0.085	5.249	0.000
α_3	0.139	0.054	2.561	0.014
α_4	-0.257	0.137	-1.874	0.067
β_1	-0.090	0.024	-3.729	0.001
β_2	0.151	0.026	5.822	0.000
β_3	0.098	0.022	4.554	0.000
α_5	-0.239	0.056	-4.307	0.000
R^2 = 0,954			$R^2_{Ajust.}$ = 0,946	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 1226 (p =0.303)
Jarque-Bera Normality Test: 2.17 (p =0.95)
White Heteroskedasticity Test: F = 1.156 (p =0.36)

[23] Even against intuition, exchange rate volatility were not a significant risk premium explainer.

In the literature there are some papers that also estimate the price (real exchange rate) and income (world GDP) elasticities for export. Pastore e Pinotti (1999) e Gonzaga e Bevilacqua (1997) found similar coefficients for the income elasticity. However the price elasticity of 0.14 is smaller than found for those papers. Patore e Pinnoti (1999) for example estimated in 0.24 for the price elasticity and 0.81 for the income elasticity.

3.3.3 Imports

Equation 33 present the estimated equation for the quantitative index for imports, being the value of the coefficients and t statistics in Table 15. The quantitative index for imports presents a structural break in the first quarter of nineties, being necessary to introduce a level dummy in order to avoid a unit root process.

$$imp_t = \alpha_0 + \alpha_1 \cdot imp_{t-1} + \alpha_2 \cdot y_t + \alpha_3 \cdot \theta_{t-1} + \sum_{j=1}^{3} \beta_j \cdot Seas_j + \alpha_4 \cdot D_{imp} + \sum_{aa:tt} \beta_{aa:tt} \cdot D_{aa:tt} \tag{33}$$

where:

imp_t	*quantitative index for imports in period t;*
y_t	*domestic GDP in period t;*
θ_t	*real exchange rate in period t;*
$Seas_j$	*seasonal dummies for the period j;*
Dimp	*step dummy which is 0 until 1993:4 and 1 after 1994:3 being 0.5 in between ;*
Daa:tt	*outlier dummies for 95:03, 97:01 and 99:01.*

Table 15 ***Equation 33***

	Method: *OLS*		**Sample:** *1988:01 to 2001:02*	
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-3.077	1.610	-1.911	0.063
α_1	0.568	0.081	7.039	0.000
α_2	1.170	0.413	2.831	0.007
α_3	-0.191	0.082	-2.337	0.024
β_1	-0.102	0.038	-2.675	0.011
β_2	-0.016	0.036	-0.442	0.661
β_3	0.024	0.040	0.595	0.555
D_{imp}	0.332	0.099	3.367	0.002
$D_{95:03}$	-0.189	0.094	-2.004	0.051
$D_{97:01}$	-0.305	0.090	-3.382	0.002
$D_{99:01}$	-0.206	0.089	-2.302	0.026
	$R^2 = 0,985$		$R^2_{Ajust.} = 0,982$	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 2.516 (p =0.09)
Jarque-Bera Normality Test: 0.67 (p =0.71)
White Heteroskedasticity Test: F = 1.37 (p =0.21)

Our coefficient for the real exchange rate is smaller that usually seen in the literature. However the income-elasticity is closer with other estimations. On the other hand, income-elasticity of exports quantum index is closer to the ones obtained in similar works. Pastore e Pinotti (1999) found the price-elasticity of (-0,96) and their income elasticity is 1,02 (take into account the industrial production). Even considering a level dummy after 1993, it looks like that the income elasticity still presents a structural break after that year. When we decrease the sample, this coefficient almost doubles.

3.3.4 Foreign Direct Investment

Equation 34 presents the estimated equation for Foreign Direct Investment, and its output is shown in Table 16. The appearance of profit and the first difference of the risk premium in the FDI equation is important, not only in terms of significance or but also the results are as we were expecting indeed. An increase in the risk premium is a leading indicator of decrease in FDI, and also if the profit sending abroad is increase so is the FDI.

$$FDI_t = \alpha_0 + \alpha_1 \cdot FDI_{t-1} + \alpha_2 \cdot \Delta(SCBond_{t-1}) + \alpha_3 \cdot y_{t-1} + \alpha_4 \cdot lucro_{t-1} \tag{34}$$

Where:

FDI_t	*Foreign Direct Investment in period t , in 2000 US$;*
$\Delta(SCBond_t)$	*first difference in the spread of C-Bond in period t;*
y_t	*GDP in period t;*
$lucro_t$	*net profit in the Balance of Payment in period t, is 2000 US$.*

Table 16 ***Equation 34***

Method: *OLS*		Sample: *1994:03 to 2001:01*		
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	-18.840	8.663	-2.175	0.041
α_1	0.616	0.095	6.478	0.000
α_2	-14.936	4.015	-3.720	0.001
α_3	3.942	1.906	2.068	0.051
α_4	0.454	0.109	4.164	0.000
	R^2 = 0,904		$R^2_{Ajust.}$ = 0,887	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 0.296 (p =0.747)
Jarque-Bera Normality Test: 0.342 (p =0.843)
White Heteroskedasticity Test: F = 0.9180 (p =0.565)

3.3.1. Monetary and Fiscal Block

For simulation purpose, interest rate follows a usual Taylor rule described in Equation 35, where γ_1 is weight in the persistence of interest rate, γ_2 is the weight on inflation and γ_3 is the weight in output gap. The variable i_t^{Eq} is the long run equilibrium of the interest rate, and it was set to be around 6%. In the baseline scenario, the values were chosen in an ad-hoc manner and γ_1 is 0.8, γ_2 is 1.3 and γ_3 is 0.8.

$$i_t = \gamma_1 \cdot i_{t-1} + (1-\gamma_1) \cdot \left\{ \gamma_2 \cdot \left[\sum_{i=1}^{4} \left(\pi_{t-i} - \pi_{t-i}^{Target} \right) \right] + \gamma_3 \cdot h_{t-1} + i_t^{Eq.} \right\} \tag{35}$$

Although the traditional way to forecast 6-month rates is extracting information from term structure, empirical results suggest that, due to low liquidity on future contracts, term curve information is not a good forecaster to future 6-month rate. Hence, we modeled 6-month interest rate as a function of contemporaneous Selic rate and contemporaneous and lagged risk premium values, as described in Equation 36. Outlier dummies were also used. Estimation output is shown in Table 17. Fiscal debt can be broken in three components: external fiscal debt, internal debt denominated in exchange rate variation plus a risk premium and internal debt denominated in Selic rate. Thus, we modeled these fiscal debt components, subtracting fiscal surplus, as in Equation 37.

$$Swap6_t = \alpha_0 + \alpha_1 \cdot Swap6_{t-1} + \alpha_2 \cdot Selic_t + \alpha_3 \cdot Selic_{t-1} + \alpha_4 \cdot \Delta SCBond_t \tag{36}$$

Table 17 ***Equation 36***

Method: *OLS*		Sample: 1988:01 to 2001:02		
Coefficient	**Estim. Value**	**St. Deviation**	**t**	**P-Value**
α_0	0.029	0.008	3.599	0.001
α_1	0.731	0.122	5.999	0.000
α_2	0.416	0.151	2.764	0.008
α_3	-0.293	0.078	-3.763	0.000
α_4	2.054	0.487	4.215	0.000
$D_{98:08}$	-0.038	0.019	-2.030	0.047
$D_{99:01}$	0.043	0.013	3.230	0.002
$D_{99:02}$	0.072	0.014	5.270	0.000
	R^2 = 0,955		$R^2_{Ajust.}$ = 0,950	

Breusch-Godfrey Serial Correlation LM Test (2 lags): F = 104.846 (p =0.000)
Jarque-Bera Normality Test: 1.191 (p =0.551)
White Heteroskedasticity Test: F = 0.562 (p =0.849)

$$
\begin{aligned}
&D_t^{Ext} = D_{t-1}^{Ext} \cdot \left(1 + i_t^f\right) \cdot \left(1 + Risk_t\right) \\
&D_t^{Int} = D_{t-1}^{Int\ (No\ \text{exchange rate variation})} \cdot \left(1 + Selic_t\right) + \\
&\qquad + D_{t-1}^{Int\ (With\ \text{exchange rate variation})} \cdot \left(1 + Risk_t\right) \cdot \left(1 + \Delta E_t^{\%}\right) \\
&D_t = D_t^{Ext} + D_t^{Int} - FS_t
\end{aligned}
\tag{37}
$$

4. Simulations

The model is simulated in a Matlab/Simulink environment. Our closure rule is an ad-hoc end-point for the ratio current account/GDP, which brings us a long-run equilibrium value for the real exchange rate. The current account/GDP deficit was set to 2.7% at the last period of simulation (2020:4). This value corresponds to a steady-state value of the external liabilities. The fiscal primary surplus follows an exogenous vanishing path to the long run. Due to internal policy of Central Bank of Brazil, level forecasts of the simulated variables are not allowed to be shown, what made us construct some graphics without a scale.

In the first simulation, whose graphics are on Appendix (see Simulation 1), we set different weights for the Taylor rule. The baseline one presents $\gamma1$, $\gamma2$ and $\gamma3$ as 0.8, 1.3 and 0.8, respectively. A more aggressive rule against inflation places $\gamma1$, $\gamma2$ and $\gamma3$ as 0.6, 1.5 and 0.8, respectively, meaning less persistence in the interest rates and also less weight in the output gap. An opposite rule with higher weight on the output $\gamma1$, $\gamma2$ and $\gamma3$ as 0.8, 1.1 and 0.9, respectively. The results show a good convergence of the model. Inflation goes toward the target, GDP grows close to the potential, and external liabilities and fiscal debt are decreasing in the medium run. The comparisons between the three Taylor rules show more output volatility associated with a more aggressive rule against inflation and also a worsening of the fiscal stance. The second simulation (see Simulation 2) is presented in impulse response terms. A temporary shock of 100 basis points, in 2002:2, plus 50 base points in 2002:3, in C-Bond spread brings volatility to inflation, growth, interest rates and risk, but all those variables are back in track after a while. A permanent shock to the productivity, meaning a increase in the total factor productivity (TFP) growth, causes persistent decrease in inflation and increase in GDP growth.

5. Conclusions and Next Steps

The main objective of this paper was to present the main features of the Keynesian macroeconomic model in development in Central Bank of Brazil. As this paper is still a work in progress, we have many more steps to accomplish and close conclusions. The model with a disaggregated demand and a potential output with production function shows a good convergence. We still can detect problems with the import and consumption equations. The simulations brought about consistent paths for output, inflation, current account and fiscal balance simultaneously. However the long-run equilibrium of some variables are dependent of the end-points for interest rate and exchange rate.

Another problem that we have with this kind of model is that they are not robust to the Lucas critique. Some of the parameters may vary through the sample period due to policy changes. Aware of that we still consider that Keynesian models are useful to identify the transmission mechanics of the monetary policy. The simulation have been consider with caution especially for Brazilian economy, full of cases of structural breaks and policies swings.

As future goals we can point out:

- A consumption disaggregated in durable and non-durable goods;
- A rational expectations forward looking term for inflation in the Phillips curve and for exchange rate in the UIP equation;
- More equations for the wage sector, using the Phillips curve with the unit labor cost on it;
- A more detailed external sector modeling;
- A more structured fiscal block;
- A production function with more than one kind of capital.

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Appendix 1 Obtaining α Quarterly Values For The Production Function

In order to obtain α quarterly values, three alternatives were highlighted. The first is to maintain the annual values in each quarter. However, as the resulting quarterly series present a step shaped pattern, with abruptly level changes on every first quarter of each year, this alternative was discarded because we expect a smoother behavior rather than step shaped. A natural choice, as a second alternative, is to consider a filtered series, obtained by a HP filter, for instance, instead of the original one. Again, there was another undesired behavior: the average quarterly values of each year should equal the original annual values, which was not the case when using the regular filtering process. Hence, we considered a third alternative that assured these two features: the smoothness and the annual quarterly values average restriction. It was based on the quarterly data generating process, based on an annual frequency data, presented in Alves (2001) and is described in System 38.

$$\begin{cases} \alpha^A \equiv \text{Annual capital share yield series, with n observations} \\ \alpha_t^A \equiv \text{ Particular value for } \alpha^A \text{ in year t}: \ \ t \in [1, n] \end{cases}$$

One wishes to estimate the quarterly capital share yield series α^Q such as :

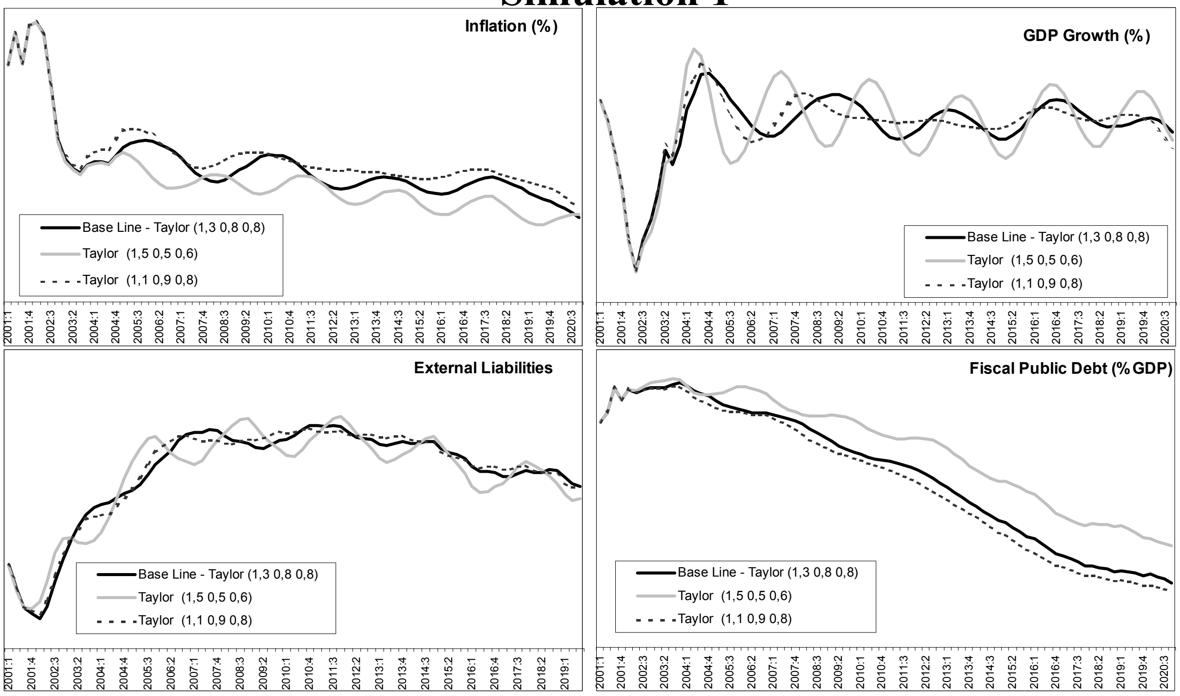
$$\begin{cases} \alpha^Q \ \equiv \text{Quarterly capital share yield series, with 4n observations} \\ \alpha_{t,Q}^Q \equiv \text{ Particular value for } \alpha^Q \text{ in quarter Q of year t}: \ \ t \in [1, n], \ \ Q \in [1, 4] \end{cases} \tag{38}$$

$\alpha_{t,Q}^Q$ series should ensure that :

$$\begin{cases} \text{Minimize} \qquad \mathrm{L} = \sum_{t=1}^{n} \sum_{Q=1}^{4} \left(\Delta^2 \alpha_{t,Q}^Q \right)^2 \\ \text{Subject to} \qquad \sum_{Q=1}^{4} \alpha_{t,Q}^Q = 4 \cdot \alpha_t^Q \ , \quad \forall \ \ t \in [1, n] \end{cases}$$

Appendix 2 Simulation Graphics

Simulation 1



Simulation 2