Polynomial and Rational Functions

NAME _____ DATE _____ SCORE _____

1. Use synthetic division to divide $4x^3 + 2x^2 + 3x - 5$ by $x - 1$.

2. Use the Remainder Theorem to evaluate $P(x) = -4x^3 + x^2 - 2x + 7$, for $x = -2$.

3. Use the Factor Theorem to determine whether $x - 1$ is a factor of $P(x) = x^4 - 4x^3 + 7x^2 - 5x + 3$.

4. Examine the leading term of $P(x) = -2x^4 + 3x^2 - 4x + 1$ and determine the far-left and far-right behavior of the graph of $P(x)$.

5. Factor to find all the real zeros of $P(x) = 2x^3 + 5x^2 - 12x$.

6. Use the Zero Location Theorem to verify that $P(x) = 3x^3 - 2x^2 - 3x + 1$ has a zero between $x = 1$ and $x = 2$. Show your work.

7. Determine the *x*-intercepts of $P(x) = (2x - 7)^2 (x - 1)^3$ and state whether the graph of *P* crosses the *x*-axis or intersects but does not cross the *x*-axis.

8. Use the Rational Zero Theorem to list the possible rational zeros of $P(x) = 3x^3 - 2x^2 + x - 8$.

9. Use Descartes' Rule of Signs to state the number of possible *positive* real zeros and the number of *negative* real zeros of

$$P(x) = x^4 - 6x^3 - 2x^2 - 5x - 4$$

10. Use a graphing utility to approximate (to the nearest 0.01) the zero of $P(x) = x^3 - 5x + 3$ that lies in the interval $(-3, -2)$.

11. Use the Upper- and Lower-Bound Theorem to find the smallest positive integer that is an upper bound of the real solutions of $P(x) = x^3 - x^2 - 3x + 1$.

12. Verify that $2i$ is a zero of

$$P(x) = x^4 - 2x^3 - 11x^2 - 8x - 60$$

Then proceed to use the Conjugate Pair Theorem to find the remaining zeros of P.

13. Find a polynomial of lowest degree that has real coefficients, a leading coefficient of 1, and with $1 - i$ and 4 as two of its zeros.

14. Use the Theorem on Vertical Asymptotes to find all vertical asymptotes of

$$F(x) = \frac{x^2 - 4}{x^2 + 4x + 3}$$

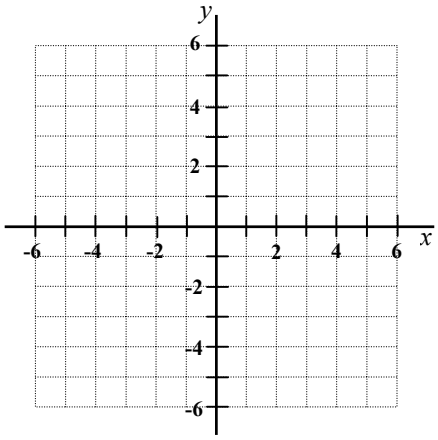
15. Determine the equation of the slant asymptote for the graph of

$$F(x)=\frac{3x^2+2x-5}{x+3}$$

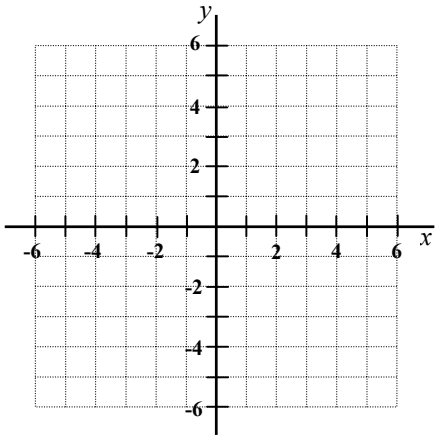
16. Determine the equation of the horizontal asymptote for the graph of

$$F(x)=\frac{2x^2-4x+1}{3x^2+5}$$

17. Graph: $F(x)=\dfrac{3}{x-2}$



18. Graph: $F(x)=\dfrac{x^2}{x^2-9}$



19. Does the graph of $F(x)=\dfrac{x^2+x-6}{x^2+2x-8}$ have a vertical asymptote at $x=2$? Explain.

20. The cost C, in thousands of dollars, to remove p% of the pollutants in a lake is given by $C(p)=\dfrac{50p}{100-p}$, $0\le p<100$. Find the cost of removing 20% of the pollutants.