Executive Summary

The purpose of this report is to determine the best *forecasting method* to forecast Edison's quarterly revenues for 2000. These Methods include Naive I, Moving Average, Simple Exponential Smoothing, Holt's Linear Exponential Smoothing, and Winter's Seasonal Linear Exponential Smoothing. After we find the best method, we will use the method to make final forecast for first quarter of 2000.

To choose the best method, we analyze each *forecast error*. Particularly, we compare its *Mean Squared Error* (MSE), perform *p-value* test, and look at *autocorrelation* of its error terms. The best forecasting method must have the following characteristics:

- Least MSE
- p-value greater than 0.05; do not reject null hypothesis (H_0 = errors are random)
- Autocorrelation of the error terms is random:
  - ✓ Autocorrelation plot is inside the control limit
  - ✓ Autocorrelation plot has no pattern
  - ✓ Autocorrelation plot has a fairly equal distribution of +'s and –'s

The first model we look at is *Naive I* method (see Appendix 1). The p-value is way below 0.05 and the autocorrelation plot is outside control limit. Thus there is no doubt that the *error terms are not random*. We should expect this result because Naive I method works best with stationary data, when the data has seasonality pattern with trend.

The next model is *Moving Average* method (see Appendix 2). We use two period of moving average in this instance. The model has lower MSE than Naive I's but the p-value is still way below 0.05. In addition, the autocorrelation plot is still outside control limit. These indicate that the *errors are not random*. This is not surprising when Moving Average method works best with stationary data as well.

We then proceed with *Simple Exponential Smoothing* method (see Appendix 3). The smoothing adjustment factor that we use is 0.2, after some trial and error. The MSE is a bit higher than Moving Average's, the p-value is still below 0.05, and the autocorrelation plot is outside control limit. They all indicate that the *errors are not random*. This is expected since Simple Exponential Smoothing is not a good method for the data with seasonality and trend.

Our next model is *Holt's Linear Exponential Smoothing* method (see Appendix 4). The method requires two parameters: smoothing adjustment factor and trend adjustment factor. Knowing that the data is linear, we should expect to use a low trend adjustment factor. We start with 0.5 for both factors. The error terms are not much different from Exponential Smoothing's, thus not random. After some trial and error, we find the best factors in this case are 0.1. The MSE has improved a bit from other methods, but the rest of *error terms still reject random characteristics*. This is not surprising as well. Although the method is suitable for data with trend, our data has trend with strong seasonality.

The last model we use is *Winter's Seasonal Linear Exponential Smoothing* method (see Appendix 5). The method requires three parameters, in addition to number of seasons that is four: a regular smoothing, trend, and seasonal factors. Since our data has regular seasonal amplitude, we should expect to use a low seasonal factor. Again we start with 0.5 for all factors. The error terms have improved a lot from other methods'. The MSE is much lower, and the p-value is higher than 0.05 to not

reject random hypothesis. But the correlation plot is still outside the control limit, an indication that the errors are still not random. Further trial and error gives us the best factors: 0.6 for regular factor, 0.1 for trend factor, and 0.4 for seasonal factor. The MSE is now the lowest from others', the p-value is higher than 0.05, and the correlation plot has all characteristics mentioned previously to indicate that the *error terms is random*. We should expect this since this is a good method for trend and seasonal data.

We summarize the result of our finding at Appendix 6 for comparison of error terms of different methods. To confirm our finding that the Winter's method with chosen parameters is the best method, we plot the actual against the forecast (see Appendix 7) and list the forecast errors (see Appendix 8). Scatter plot in Appendix 7 does indicate that the forecast fit quite well with the actual data. Forecast errors list in Appendix 8 shows that the errors are quite random indeed. Therefore we can conclude that *Winter's Seasonal Linear Exponential Smoothing is the best forecasting method* to forecast Edison's quarterly revenues for 2000.

Our final forecast for Edison's quarterly revenue of first quarter 2000, using Winter's method with chosen parameters, is 1831.44 (see Appendix 5 and 8)

Appendix 1: Naive I Forecast Error Terms

10/23/2001_19:23_

FILE: Edison, NO. OF VARIABLES: 1, NO. OF CASES: 60 (MISS. CASES: 0)
LABEL: Quarterly Revenues for Consolidated Edison ($ million), 1985-1999

NAIVE FORECAST OF Qtr_Revenue

Mean Error = 3.10169
Mean % Error = -1.63403
Mean Absolute Error = 278.017
Mean Absolute % Error = 18.0609
Mean Squared Error = **90382.2**
Next Period Forecast = 1624
BOX-PIERCE Q(29) = 719.38, P-VALUE = **8.84613E-133**

ERROR AUTOCORRELATION PLOT

	CORRELATION	STD.ERROR	-1 -.75 -.5 -.25 0 .25 .5 .75 1
Lag -1	-0.871355	0.130189	***********(*****) . .
Lag -2	0.740498	0.206608	. . (********)*******
Lag -3	-0.80791	0.247537	******(**********) .
Lag -4	0.89494	0.288792	. (. ***********)******
Lag -5	-0.805339	0.332491	***(********* .) .
Lag -6	0.697459	0.364055	(. **********)
Lag -7	-0.742503	0.386038	(********** .)
Lag -8	0.820132	0.409529	(. . **********)
Lag -9	-0.752444	0.43648	(********** . .)
Lag -10	0.65675	0.457938	(. . ********** .)
Lag -11	-0.693388	0.473633	(.********** . .)
Lag -12	0.742465	0.490536	. . **********
Lag -13	-0.669193	0.509227	. ********** . .
Lag -14	0.588535	0.52392	. . ********** .
Lag -15	-0.63233	0.535009	. ********** . .
Lag -16	0.678195	0.547529	. . **********.
Lag -17	-0.596378	0.561587	. ********** . .
Lag -18	0.512066	0.57222	. . ********** .
Lag -19	-0.564394	0.579935	. ********** . .
Lag -20	0.616162	0.589171	. . ********** .
Lag -21	-0.542119	0.599994	. ********** . .
Lag -22	0.463157	0.608239	. . **********. .
Lag -23	-0.500991	0.614188	. ********** . .
Lag -24	0.546207	0.621075	. . ********** .
Lag -25	-0.480142	0.629164	. ********** . .
Lag -26	0.402596	0.635345	. . ********* .
Lag -27	-0.440681	0.639654	. .********** . .
Lag -28	0.492071	0.644779	. . ********** .
Lag -29	-0.431759	0.651113	. .********** . .

Appendix 2: Moving Average Forecast Error Terms

```
_____________________________________________________________________10/23/2001_19:35_

  FILE: Edison,  NO. OF VARIABLES: 1,  NO. OF CASES: 60     (MISS. CASES: 0)
 LABEL:  Quarterly Revenues for Consolidated Edison ($ million), 1985-1999
_____________________________________________________________________________

                2 PERIOD SIMPLE MOVING AVERAGE OF Qtr_Revenue

Mean Error            =     12.4914
Mean % Error          =     -0.300181
Mean Absolute Error   =    148.56
Mean Absolute % Error =      9.4974
Mean Squared Error    =  33119.6
Next Period Forecast  =   1842
 BOX-PIERCE Q(29) = 380.548,  P-VALUE = 6.38378E-63

                          ERROR AUTOCORRELATION PLOT

         CORRELATION STD.ERROR  -1 -.75 -.5  -.25   0   .25  .5   .75   1
         ----------- ---------  |----|----|----|----|----|----|----|----|
 Lag  -1   -0.60541   0.131306       .  *******(*****    )    .    .
 Lag  -2    0.282402  0.172858       .    .  (      *******)  .    .
 Lag  -3   -0.564439  0.180638       .   ****(*******      )  .    .
 Lag  -4    0.837516  0.208844       .    . (       ********)*********
 Lag  -5   -0.599519  0.260391       .  **(**********         )    .
 Lag  -6    0.296914  0.283191       .   (.         *******   .)   .
 Lag  -7   -0.503784  0.288509       .  ( ***********         . )  .
 Lag  -8    0.75505   0.303297       .  ( .         ************)***
 Lag  -9   -0.53603   0.334137       . ( ************         .  ) .
 Lag -10    0.292481  0.348648       .(   .         *******   .   ).
 Lag -11   -0.501251  0.352853       .(   ***********         .   ).
 Lag -12    0.677295  0.364923       (    .         ***************)
 Lag -13   -0.460489  0.385989       (    .**********         .    )
 Lag -14    0.249599  0.395347      (.    .         ******    .    .)
 Lag -15   -0.454081  0.398055      (.    .**********         .    .)
 Lag -16    0.656119  0.406888      (.    .         ************** .)
 Lag -17   -0.405021  0.424738     ( .    . *********         .    . )
 Lag -18    0.197758  0.431345     ( .    .         *****     .    . )
 Lag -19   -0.405997  0.432906     ( .    . *********         .    . )
 Lag -20    0.584455  0.439422    (  .    .         *************  .  )
 Lag -21   -0.375359  0.452626    (  .    . *********         .    .  )
 Lag -22    0.181625  0.457961    (  .    .         *****     .    .  )
 Lag -23   -0.365963  0.459202    (  .    .  ********         .    .  )
 Lag -24    0.527837  0.464203   (   .    .         ************   .   )
 Lag -25   -0.327401  0.474438   (   .    .  ********         .    .   )
 Lag -26    0.130918  0.478318       .    .         ****      .    .
 Lag -27   -0.316483  0.478935       .    .   *******         .    .
 Lag -28    0.479324  0.482527       .    .         ***********    .
 Lag -29   -0.281537  0.490668       .    .   *******         .    .
```

Appendix 3: Simple Exponential Smoothing Forecast Error Terms

```
_____________________________________________________________________10/23/2001_19:46_

  FILE: Edison,  NO. OF VARIABLES: 1,  NO. OF CASES: 60     (MISS. CASES: 0)
 LABEL:  Quarterly Revenues for Consolidated Edison ($ million), 1985-1999
_____________________________________________________________________________

                  SIMPLE EXPONENTIAL SMOOTHING OF Qtr_Revenue

Smoothing Adjustment Factor = 0.2
Mean Error            =     25.7293
Mean % Error          =      0.339927
Mean Absolute Error   =    158.395
Mean Absolute % Error =     10.2363
Mean Squared Error    =  34615.1
Next Period Forecast  =   1744.61
 BOX-PIERCE Q(29) = 501.186,  P-VALUE = 1.64236E-87

                           ERROR AUTOCORRELATION PLOT

         CORRELATION STD.ERROR  -1 -.75 -.5  -.25   0   .25  .5   .75   1
         ----------- ---------  |----|----|----|----|----|----|----|----|
 Lag  -1   -0.64803   0.130189       . ********(*****    )    .    .
 Lag  -2    0.614213  0.176591       .    .  (      *******)*****  .
 Lag  -3   -0.610411  0.209697       .  ****(********       ) .    .
 Lag  -4    0.850511  0.237915       .    (         **********)*******
 Lag  -5   -0.640314  0.284824       . **(***********          .)   .
 Lag  -6    0.559548  0.308258       .  ( .          ************)  .
 Lag  -7   -0.579921  0.325017       . (*************          .  ) .
 Lag  -8    0.749925  0.342106       .(   .          ***************)*
 Lag  -9   -0.624523  0.368918       (  *************          .    )
 Lag -10    0.504857  0.386422       (    .          ***********    )
 Lag -11   -0.574015  0.397444      (.   ************          .    .)
 Lag -12    0.66178   0.411255      (.    .          ************** .)
 Lag -13   -0.564349  0.428925     ( .   ************          .    . )
 Lag -14    0.439048  0.441331    (  .    .          **********.    .  )
 Lag -15   -0.53025   0.448673    (  .   ************          .    .  )
 Lag -16    0.609806  0.459172    (  .    .          *************  .  )
 Lag -17   -0.507766  0.472699   (   .    ***********          .    .   )
 Lag -18    0.370109  0.481855       .    .          ********  .    .
 Lag -19   -0.476792  0.486649       .    ***********          .    .
 Lag -20    0.550842  0.494503       .    .          ************   .
 Lag -21   -0.465144  0.504796       .    .**********          .    .
 Lag -22    0.336537  0.512009       .    .          ********  .    .
 Lag -23   -0.426999  0.515745       .    .**********          .    .
 Lag -24    0.483913  0.521702       .    .          ***********    .
 Lag -25   -0.418779  0.529255       .    . *********          .    .
 Lag -26    0.288875  0.534842       .    .          *******   .    .
 Lag -27   -0.369071  0.53748        .    .  ********          .    .
 Lag -28    0.439493  0.541759       .    .          **********.    .
 Lag -29   -0.378416  0.547768       .    . *********          .    .
```

Appendix 4: Holt's Linear Exponential Smoothing Forecast Error Terms

```
                                                                  10/23/2001_20:0_
  FILE: Edison,  NO. OF VARIABLES: 1,  NO. OF CASES: 60     (MISS. CASES: 0)
 LABEL:  Quarterly Revenues for Consolidated Edison ($ million), 1985-1999
```

LINEAR EXPONENTIAL SMOOTHING OF Qtr_Revenue

Smoothing Adjustment Factor = 0.5
Trend Adjustment Factor = 0.5

```
Mean Error            =     -4.26076
Mean % Error          =     -1.74596
Mean Absolute Error   =    203.464
Mean Absolute % Error =     13.2396
Mean Squared Error    =  54263.7
Next Period Forecast  =   1731.98
 BOX-PIERCE Q(14) = 366.136,  P-VALUE = 1.68803E-69
```

ERROR AUTOCORRELATION PLOT

```
         CORRELATION STD.ERROR  -1 -.75 -.5  -.25   0   .25  .5   .75   1
         ----------- ---------  |----|----|----|----|----|----|----|----|
 Lag  -1   -0.759213  0.130189       **********(*****    )    .    .
 Lag  -2    0.57314   0.191019       .    . (       ********)***   .
 Lag  -3   -0.718075  0.218228       .*****(*********        ).    .
 Lag  -4    0.858079  0.255152       .    (         **********)*******
 Lag  -5   -0.728493  0.300103       ***(************         . )  .
 Lag  -6    0.546968  0.328712       . (  .         ************ ) .
 Lag  -7   -0.648082  0.343793       .(***************         .   ).
 Lag  -8    0.790259  0.363911       (    .         ***************)*
 Lag  -9   -0.671964  0.391919      (. **************         .    .)
 Lag -10    0.519731  0.410983      (.    .         ***********    .)
```

Smoothing Adjustment Factor = 0.1
Trend Adjustment Factor = 0.1

```
Mean Error            =     -4.9714
Mean % Error          =     -1.54621
Mean Absolute Error   =    153.204
Mean Absolute % Error =     10.0759
Mean Squared Error    =  30970.1
Next Period Forecast  =   1833.44
 BOX-PIERCE Q(14) = 309.513,  P-VALUE = 1.22368E-57
```

ERROR AUTOCORRELATION PLOT

```
         CORRELATION STD.ERROR  -1 -.75 -.5  -.25   0   .25  .5   .75   1
         ----------- ---------  |----|----|----|----|----|----|----|----|
 Lag  -1   -0.622955  0.130189       .  *******(*****    )    .    .
 Lag  -2    0.608479  0.173506       .    .  (      *******)*****  .
 Lag  -3   -0.615647  0.206531       .  ****(********       ) .    .
 Lag  -4    0.815452  0.235591       .    .(        *********)*******
 Lag  -5   -0.658473  0.279364       . **(***********         .)   .
 Lag  -6    0.519656  0.304536       .  ( .         ***********  )  .
 Lag  -7   -0.604452  0.319212       . (*************         .  ) .
 Lag  -8    0.713931  0.338055       .(   .         **************).
 Lag  -9   -0.632168  0.362711       ( **************         .    )
 Lag -10    0.467887  0.380928       (    .         **********.    )
```

Appendix 5: Winter's Seasonal Linear Exponential Smoothing Forecast Error Terms

```
                                                                  10/23/2001_21:3_
  FILE: Edison,  NO. OF VARIABLES: 1,  NO. OF CASES: 60     (MISS. CASES: 0)
 LABEL:  Quarterly Revenues for Consolidated Edison ($ million), 1985-1999
```

SEASONAL LINEAR EXPONENTIAL SMOOTHING OF Qtr_Revenue

Smoothing Adjustment Factor = 0.5
Trend Adjustment Factor = 0.5
Seasonal Adjustment Factor = 0.5
Number Of Seasons = 4
Mean Error = 2.2358
Mean % Error = 0.0472813
Mean Absolute Error = 53.8712
Mean Absolute % Error = 3.51919
Mean Squared Error = **4468.21**
Next Period Forecast = 1828.06
BOX-PIERCE Q(13) = 12.4539, P-VALUE = **0.490823**

ERROR AUTOCORRELATION PLOT

	CORRELATION	STD.ERROR	-1 -.75 -.5 -.25 0 .25 .5 .75 1
Lag -1	0.088724	0.13484	. . (***) . .
Lag -2	-0.292598	0.135897	. . *(*****) . .
Lag -3	-0.193598	0.146906	. . (*****) . .
Lag -4	-0.0842498	0.151473	. . (***) . .
Lag -5	-0.0963522	0.152323	. . (***) . .
Lag -6	0.100053	0.153427	. . (***) . .
Lag -7	0.116493	0.154609	. . (***) . .
Lag -8	-0.0884938	0.156197	. . (***) . .
Lag -9	-0.0218801	0.157106	. . (*) . .

Smoothing Adjustment Factor = 0.6
Trend Adjustment Factor = 0.1
Seasonal Adjustment Factor = 0.4
Number Of Seasons = 4
Mean Error = 6.07367
Mean % Error = 0.312164
Mean Absolute Error = 48.1767
Mean Absolute % Error = 3.14826
Mean Squared Error = **3583.2**
Next Period Forecast = 1831.44
BOX-PIERCE Q(13) = 7.35038, P-VALUE = **0.883204**

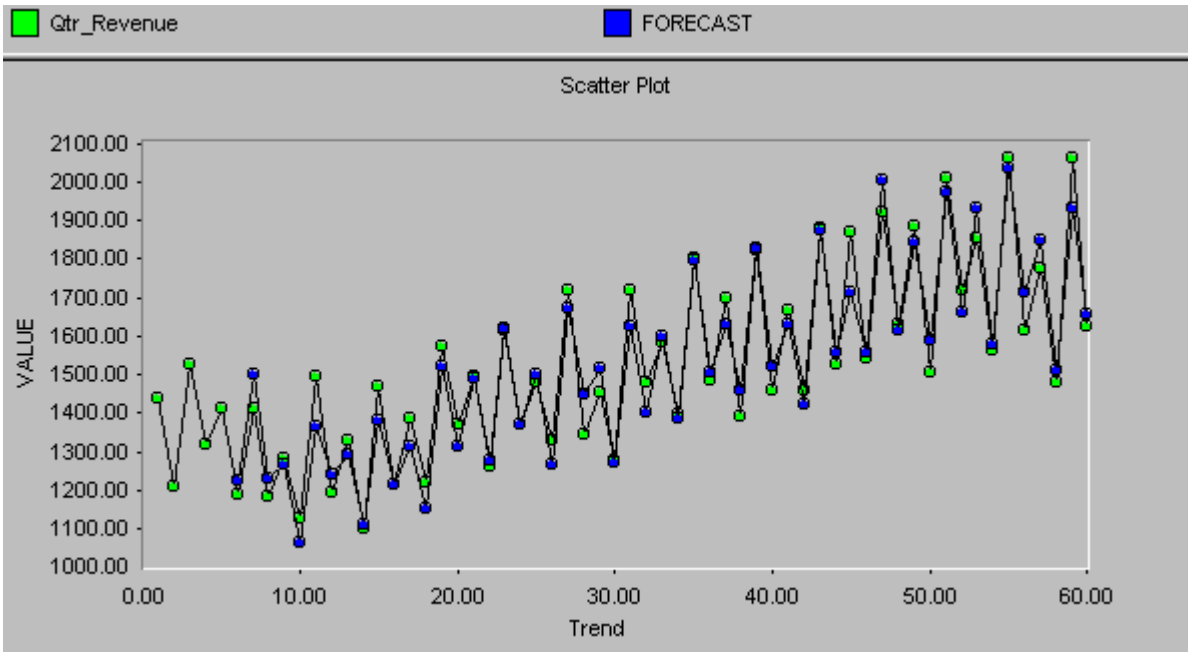
ERROR AUTOCORRELATION PLOT

	CORRELATION	STD.ERROR	-1 -.75 -.5 -.25 0 .25 .5 .75 1
Lag -1	0.0133486	0.13484	. . (*) . .
Lag -2	-0.158646	0.134864	. . (****) . .
Lag -3	-0.0418131	0.138215	. . (**) . .
Lag -4	0.0760028	0.138445	. . (***) . .
Lag -5	-0.150755	0.139202	. . (****) . .
Lag -6	0.0971591	0.142139	. . (***) . .
Lag -7	0.0870768	0.143342	. . (***) . .
Lag -8	-0.0771644	0.1443	. . (***) . .
Lag -9	-0.0941861	0.145049	. . (***) . .

Appendix 6: Error Terms Comparison Table

Forecasting Method	Mean Square Error (MSE)	AutoCorrelation of Error Terms Inside Control Limit?	AutoCorrelation of Error Terms Have no Pattern?	p-value is greater than 0.05?	Actual Error Terms have no Pattern?
Naive I	90,382.20	No	No	No	No
Moving Average (2 periods)	33,119.60	No	No	No	No
Simple Exponential Smoothing (0.2 factor)	34,615.10	No	No	No	No
Holt's Linear Exponential Smoothing (0.1 smoothing factor, 0.1 trend factor)	30,970.10	No	No	No	No
Winter's Seasonal Linear Exponential Smoothing (0.6 smoothing factor, 0.1 trend factor, 4 seasons)	3,583.20	Yes	Yes	Yes	Yes
Time Series Regression (for trend & seasonal)	5,111.56	No	No	No	Yes

Appendix 7: Scatter Plot of Actual vs. Forecast Data (Winter's Seasonal Linear Exponential Smoothing Forecast)



Appendix 8: List of Actual, Forecast, and Error (Winter's Seasonal Linear Exponential Smoothing Forecast)

		Qtr_Revenue	FORECAST	**ERROR**	
Qtr1_1985	1	1441	*******	**********	
Qtr2_1985	2	1209	*******	**********	
Qtr3_1985	3	1526	*******	**********	
Qtr4_1985	4	1321	*******	**********	
Qtr1_1986	5	1414	*******	**********	
Qtr2_1986	6	1187	1225.8	**-38.7996**	
Qtr3_1986	7	1411	1500.98	**-89.9784**	
Qtr4_1986	8	1185	1233.35	**-48.352**	
Qtr1_1987	9	1284	1265.41	**18.5862**	
Qtr2_1987	10	1125	1066.63	**58.3669**	
Qtr3_1987	11	1493	1364.07	**128.931**	
Qtr4_1987	12	1192	1241.8	**-49.7975**	
Qtr1_1988	13	1327	1294.17	**32.8262**	
Qtr2_1988	14	1102	1113.72	**-11.7224**	
Qtr3_1988	15	1469	1383.75	**85.2465**	
Qtr4_1988	16	1213	1212.97	**0.0320329**	
Qtr1_1989	17	1387	1315.11	**71.888**	
Qtr2_1989	18	1218	1152.14	**65.8608**	
Qtr3_1989	19	1575	1522.76	**52.2428**	
Qtr4_1989	20	1371	1312.8	**58.2028**	
Qtr1_1990	21	1494	1490.31	**3.69275**	
Qtr2_1990	22	1263	1276.77	**-13.7675**	
Qtr3_1990	23	1613	1621.88	**-8.88214**	
Qtr4_1990	24	1369	1369.91	**-0.909749**	
Qtr1_1991	25	1479	1503.08	**-24.0815**	
Qtr2_1991	26	1330	1268.22	**61.7768**	
Qtr3_1991	27	1720	1672.7	**47.3039**	
Qtr4_1991	28	1344	1447.38	**-103.383**	
Qtr1_1992	29	1456	1515.55	**-59.55**	
Qtr2_1992	30	1280	1270.68	**9.3222**	
Qtr3_1992	31	1717	1623.01	**93.9891**	
Qtr4_1992	32	1480	1401.71	**78.2888**	
Qtr1_1993	33	1586	1600.79	**-14.7858**	
Qtr2_1993	34	1396	1386.02	**9.97838**	
Qtr3_1993	35	1800	1794.98	**5.02021**	
Qtr4_1993	36	1483	1504.66	**-21.6636**	
Qtr1_1994	37	1697	1630.14	**66.8601**	
Qtr2_1994	38	1392	1460.67	**-68.6721**	
Qtr3_1994	39	1822	1826.78	**-4.78353**	
Qtr4_1994	40	1461	1519.76	**-58.761**	
Qtr1_1995	41	1669	1631.67	**37.334**	
Qtr2_1995	42	1460	1420.69	**39.3144**	
Qtr3_1995	43	1880	1873.11	**6.88822**	
Qtr4_1995	44	1528	1556.06	**-28.0598**	
Qtr1_1996	45	1867	1712.02	**154.976**	
Qtr2_1996	46	1540	1559.94	**-19.9363**	
Qtr3_1996	47	1920	2006.32	**-86.3221**	
Qtr4_1996	48	1632	1615.23	**16.7678**	
Qtr1_1997	49	1886	1841.76	**44.2447**	
Qtr2_1997	50	1504	1587.11	**-83.1084**	
Qtr3_1997	51	2011	1975.07	**35.9291**	
Qtr4_1997	52	1720	1664.07	**55.9317**	
Qtr1_1998	53	1853	1929.75	**-76.7544**	
Qtr2_1998	54	1561	1576.34	**-15.3448**	
Qtr3_1998	55	2062	2036.66	**25.3366**	
Qtr4_1998	56	1617	1713.45	**-96.4483**	
Qtr1_1999	57	1777	1849.65	**-72.6519**	
Qtr2_1999	58	1479	1510.47	**-31.4709**	
Qtr3_1999	59	2060	1933.25	**126.748**	
Qtr4_1999	60	1624	1653.85	**-29.848**	
Qtr1_2000	**61**	**n/a**	**1831.44**	**n/a**	**<- see Appendix 5**