FUNDAMENTAL EXAM

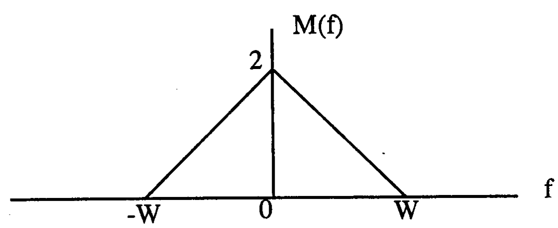
DATE : April 1999

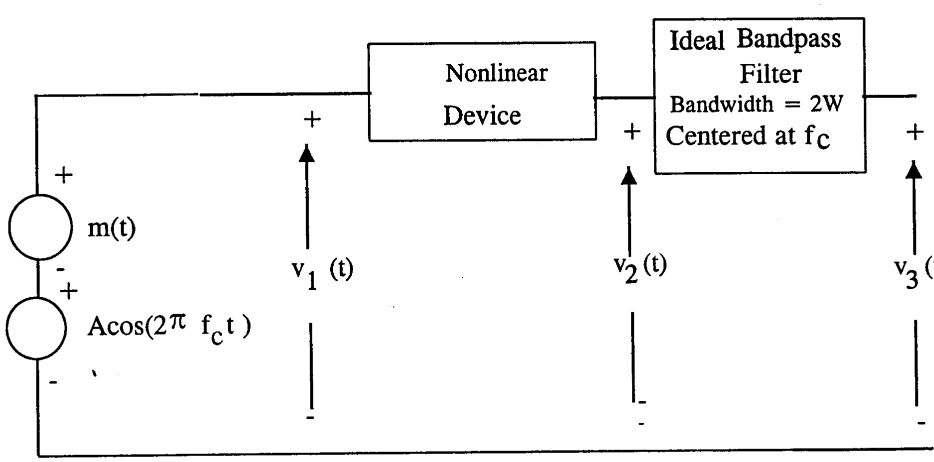
AREA : COMMUNICATIONS

STUDENT CODE :

PROBLEM # : 2

The signal m(t) in the system shown below has a spectrum M(f) also shown below:





The nonlinear device is described by: $v_2(t) = 2v_1(t) + 0.1v_1^2(t)$.

Plot the respective spectrum of $v_1(t)$, $v_2(t)$, and $v_3(t)$

ADVANCED EXAM

Date: April 1999

Area: Communications

Student Code:

Problem # 3

A ternary information source produces an iid, equiprobable sequence of symbols from the alphabet $\{a, b, c\}$. Suppose that the three symbols are encoded into the respective binary passwords 00, 01, 10. Let B_n be the sequence of binary symbols that result from encoding the binary sumbols.

1. Find the joint probability mass function of B_n and B_{n+1}.
2. Is B_n stationary ?
3. Find the mean and covariance function of B_n.
4. Is B_n wide sense stationary ?

Explain all your answers

ADVANCED EXAM

Date: April 1999

Area: Communications

Student Code:

Problem # 4

Suppose that computer A sends messages to computer B over two unreliable telephone lines. Computer B can detect when errors have occurred in either line. Let the probability of message transmission error in line 1 and line 2 be q_1 and q_2, respectively. Computer B requests retransmissions until it receives an error-free message on either line.

1. Find the probability that more than k transmissions are required.

2. Find the probability that in the last transmission, the message on line 2 is received free of errors.

Explain all your answers

ADVANCED EXAM

Area: Communications

Date: November 5, 1999

Student Code:

Problem 2: A communication receiver filters and amplifies the voltage across the terminals of an antenna. The final output of the receiver, sampled at a certain time instant t, is a random variable $\mathbf{X}$. When no signal, but background noise is present at the input of the receiver (hypothesis H_0), the probability density function of $\mathbf{X}$ is

$$f_{\mathbf{X}}(x|H_0) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

When a signal has also arrived (hypothesis H_1), the probability density function of $\mathbf{X}$ is

$$f_{\mathbf{X}}(x|H_1) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{x - m^2}{2\sigma^2}\right)$$

The prior probability that the signal is present is 1/2.

1. Given that a particular value x of the random variable $\mathbf{X}$ has been observed at the output of the receiver, what is the posterior (conditional) probability that the signal is present ? That is calculate $\Pr(\textit{signal present}|\mathbf{X} = x]$.
2. If we wish to min imize the probability of error P_e in deciding whether or not a signal is present, over what range of values x should we decide "signal present" and over what range should we decide "signal absent" ?

Explain all your answers

ADVANCED EXAM

Area: Communications

Date: November 5, 1999

Student Code:

Problem 3: Let **X** be a random variable with mean of 2 and variance of 9. Let **Y** be a random variable with mean of 2 and variance of 4. Given that the covariance of X, Y is equal to 2, calculate the variance of the random variable **Z**, where $\mathbf{Z} = 2\mathbf{X} + 3\mathbf{Y}$. Show all your work carefully.

Explain all your answers

ADVANCED EXAM

Area: Communications

Date: November 5, 1999

Student Code:

Problem 4: If **X** and **Y** are Gaussian random variables with zero mean, unit variance and correlation coefficient of 1/6, calculate the conditional mean $\mathbf{E}[X|Y = 3]$.

Explain all your answers

ADVANCED EXAM

Area: Communications

Date: April 7, 00

Student Code:

Problem 1:

Let X be an input to a communication channel. X takes on the values ± 1 with equal probability. Suppose that the output of the channel is $Y = X + N$, where N is a Laplacian random variable with probability density function

$$f_N(z) = a/2 \cdot \exp\{-a|z|\} \quad ; \quad -\infty < z < \infty$$

1. Find $\Pr[X = k, Y \leq y]$ for $k = \pm 1$.
2. Find the marginal probability density function of Y.
3. Suppose we are given that $Y > 0$. Which is more likely, $X = 1$ or $X = -1$.

Explain all your answers.

ADVANCED EXAM

Area: Communications

Date: April 7, 00

Student Code:

Problem 2:

Consider a random variable S which is the sum of a random number N of independent, identically distributed random variables X. That is

$$S = \sum_{k=1}^{N} X_i$$

where N is a random integer and the X_i's are the identically distributed random variables. Find the characteristic function of S in terms of the characteristic function in terms of the characteristic function of X and the moment generating function of N.

Explain all your answers.

ADVANCED EXAM

DATE: April 7, 2000

AREA: Communication

STUDENT CODE:

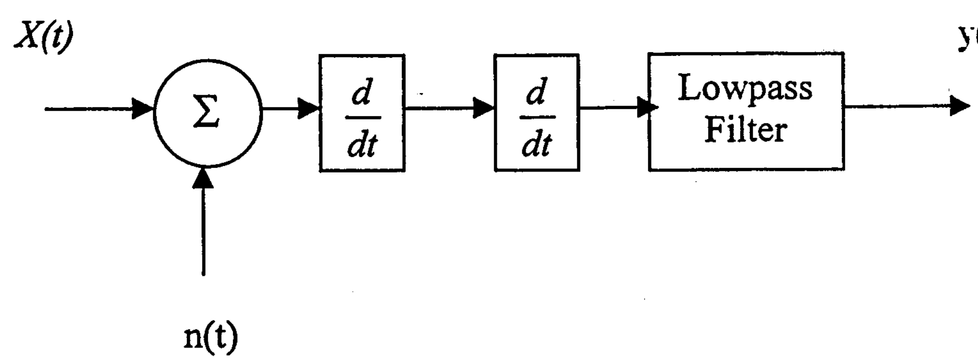
PROBLEM #: 3

Instructions:
Show all of your work very neatly and clearly state what is given.

2. Suppose we have the input signal given by $x(t)$,

$$x(t) = A \cos (2\pi f_C t)$$

The lowpass filter has a gain of unity in the passband and a bandwidth of W, where $f_C < W$. the noise n(t) is whithe with two-sided power spectral density of $No/2$. The signal component of $y(t)$ is defined to be the component at frequency f_C. Determine the SNR of $y(t)$.



ADVANCED EXAM

DATE: April 7, 2000

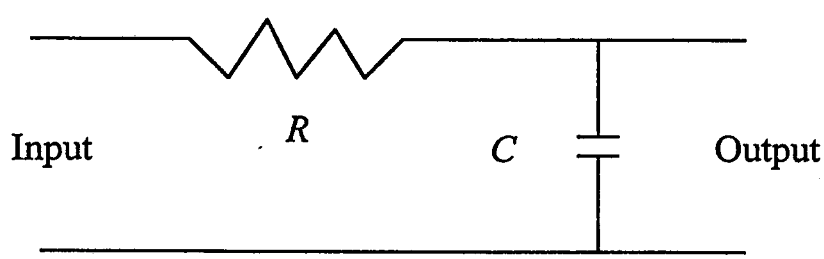
AREA: Communication

STUDENT CODE:

PROBLEM #: 4

Instructions:
Show all of your work very neatly and clearly state what is given.

1. The input to the filter shown below is $A\cos(2\pi f_C t)$ plus white noise with double-sided power spectral density of $N_o/2$. Compute the SNR at the filter output in terms of N_o, A, R, C, and f_C.



ADVANCED EXAM

DATE: November 2000

AREA: COMMUNICATIONS

STUDENT CODE:

PROBLEM 2:

Let the number of message transmissions by a computer in 1 hour be a binomial random variable with parameters n and p. Suppose that the probability of a message transmission error is ε. Let S be the number of transmission errors in 1-hour period.

- ✓ Find the mean and the variance of S.
- ✓ Find $E[z^S]$.
- ✓ What is the utility of $E[z^S]$?

Explain all your answers.

ADVANCED EXAM

DATE: November 3, 2000

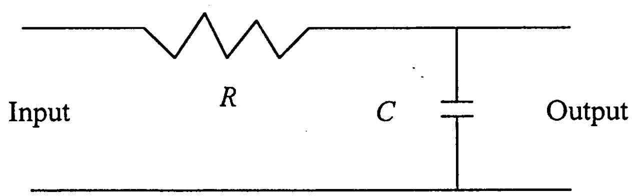
AREA: **Communications**

STUDENT CODE:

PROBLEM #: 3

Instructions:
Show all of your work very neatly and clearly state what is given.

1. The input to the filter shown below is $A\sin(2\pi f_C t)$ plus white noise with spectral density of N_o. Compute the SNR at the filter output in terms of N_o, A, R, C, and f_C.



ADVANCED EXAM

DATE: November 3, 2000

AREA: **Communications**

STUDENT CODE:

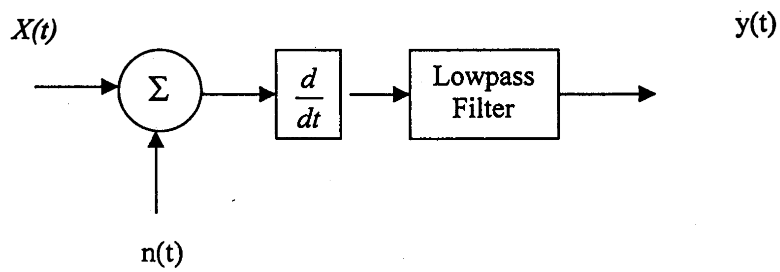
PROBLEM #: 4

Instructions:
Show all of your work very neatly and clearly state what is given.

2. Suppose we have the input signal given by $x(t)$,

$$x(t) = A\cos(2\pi f_C t)$$

The lowpass filter has a gain of unity in the passband and a bandwidth of W, where $f_C < W$. The additive Gaussian noise n(t) is white with power spectral density of No. The signal component of $y(t)$ is defined to be the component at frequency f_C. Determine the SNR of $y(t)$.



ADVANCED EXAM

Date: April 6, 2001

Area: Communications

Student Code:

Problem 1:

Consider an experiment for which the sample space is the real line. A probability law assigns probabilities to subsets of the form $(-\infty, r]$.

a. Show that we must have $\Pr[(-\infty, r] \leq \Pr[(-\infty, s]$ when $r < s$.
b. Find an expression for $\Pr[(r, s]]$ in terms of $\Pr[(-\infty, r]]$ and $\Pr[(-\infty, s]]$.

Explain all your answers.

ADVANCED EXAM

Date: April 6, 2001

Area: Communications

Student Code:

Problem 2:

Suppose that computer A sends a message to computer B over two unreliable transmission lines. Computer B can detect when errors have occurred in either line. Let the probability of message transmission error in line 1 and line 2 be q_1 and q_2, respectively. Computer B requests retransmissions until it receives an error- free message on either line.

a. Find the probability that more than *k* transmissions are needed before computer B receives an error-free message on either line.
b. Find the probability that in the last transmission, the message on line 2 is received free of errors.

Explain all your answers.

ADVANCED EXAM

Date: April 6, 2001

Area: Communications

Student Code:

Problem 4:

A binary communication system transmits signals $s_i(t) (i = 1,2)$. The receiver test statistic $z(T) = a_i + n_0$, where the signal component a_i is either +1 or −1, and the noise component n_0 is uniformly distribute, yielding the conditional density $p(z \mid s_i)$ given by $p(z \mid s_1) = \frac{1}{2}$ for $-0.2 \le z \le 1.8$, and zero otherwise, while $p(z \mid s_2) = \frac{1}{2}$ for $-1.8 \le z \le 0.2$, and zero otherwise.

Find the probability of error, P_B, for the case of equally likely signaling and the use of an optimum threshold.

Explain all your answers.