

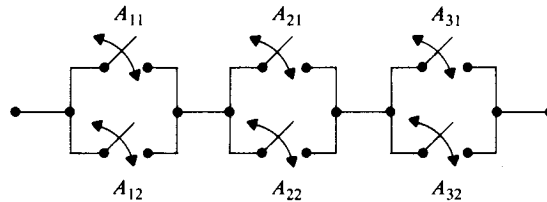
# EEL5542 – Fall 2001

## HOMEWORK #1

Due on Thursday, Sep 6<sup>th</sup> 2001

14. A system consists of three key subsystems that are duplicated for redundancy. The system is “up” if at least one unit in each of the subsystems is functioning.
- Let  $A_{jk}$  correspond to the event “unit  $k$  in subsystem  $j$  is functioning,” for  $j = 1, 2, 3$  and  $k = 1, 2$ . Explain why the above problem is equivalent to the problem of having a connection in the network of switches shown in Fig. P2.1.
  - Find a network that corresponds to a system that is “up” if both units in the first subsystem are functioning and at least one of the units in each of the other subsystems is functioning.

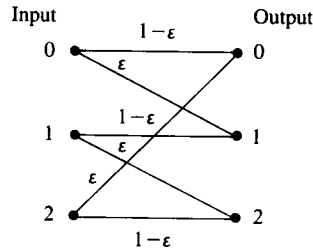
FIGURE P2.1



(Use 25 to solve 26; do not solve 25)

25. Use Corollary 7 to prove the following:
- $P[A \cup B \cup C] \leq P[A] + P[B] + P[C]$ .
  - $P\left[\bigcup_{k=1}^n A_k\right] \leq \sum_{k=1}^n P[A_k]$ .
- The second expression is called the **union bound**.
26. A document consisting of  $n$  characters is typed into a computer. The probability that a single character is typed incorrectly is  $p$ . Use the union bound to obtain an upper bound to the probability of there being any errors in the typed document.
30. Consider an experiment for which the sample space is the real line. A probability law assigns probabilities to subsets of the form  $(-\infty, r]$ .
- Show that we must have  $P[(-\infty, r]] \leq P[(-\infty, s]]$  when  $r < s$ .
  - Find an expression for  $P[(r, s]]$  in terms of  $P[(-\infty, r]]$  and  $P[(-\infty, s]]$ .
52. A number  $x$  is selected at random in the interval  $[-1, 1]$ . Let  $B$  be the event  $\{|x - 1/2| < 1\}$  and let  $C$  be the event  $\{x > 3/4\}$ . Find  $P[B | C]$  and  $P[C | B]$ .
60. A ternary communication channel is shown in Fig. P2.3. Suppose that the input symbols 0, 1, and 2 occur with probability  $1/2$ ,  $1/4$ , and  $1/4$ , respectively.
- Find the probabilities of the output symbols.
  - Suppose that a 1 is observed as an output. What is the probability that the input was 0? 1? 2?

FIGURE P2.3



68. A random experiment is repeated a large number of times and the occurrence of events  $A$  and  $B$  is noted. How would you test whether events  $A$  and  $B$  are independent?
70. In the binary communication system in Example 2.23, find the value of  $\epsilon$  for which the input of the channel is independent of the output of the channel. Can such a channel be used to transmit information?
71. A block of 100 bits is transmitted over a binary communication channel with probability of bit error  $p = 10^{-3}$ . Find the probability that the block contains three or more errors.
78. A runlength coder segments a binary information sequence into strings that consist of either a “run” of  $k$  “zeros” punctuated by a “one”, for  $k = 0, \dots, m - 1$ , or a string of  $m$  “zeros.” The  $m = 3$  case is:

<i>string</i>	<i>runlength k</i>
1	0
01	1
001	2
000	3

Suppose that the information sequence is produced by a sequence of independent Bernoulli trials with  $P(1)=P(\text{Success})=p$

- a) Find the probability of runlength  $k$  in the  $m=3$  case.  
 b) Find the probability of runlength  $k$  for general  $m$ .
81. An urn initially contains two black balls and two white balls. The following experiment is repeated indefinitely: A ball is drawn from the urn; if the ball is white it is put back in the urn, otherwise it is left out.
- a. Draw the trellis diagram for this experiment and label the branches by the transition probabilities. *Hint:* The urn can only be in one of three states.  
 b. Find the probabilities for the sequences  $www$ ,  $bww$ ,  $bbw$ , and  $bbwww$ .  
 c. Find the probability that the urn contains no black balls after three draws.  
 d. Find the probability that the urn contains two black balls after  $n$  trials.