S-2 $\psi_0 = A\cos\left(\frac{2\pi x}{\lambda} + \pi/6\right)$ $\quad \psi_1 = \alpha\cos\left(\frac{4\pi x}{\lambda} + \phi\right)$

Find α, ϕ

at $x=0$, Continuity of $\psi \Rightarrow$ $\quad -A\cos\pi/6 = \alpha\cos\phi$

at $x=0$, Continuity of $\psi' \Rightarrow$ $\quad \psi_0'(0) = \psi_1'(0)$ so

$$\frac{2\pi}{\lambda} A \sin\left(\frac{2\pi x}{\lambda} + \pi/6\right)\Big|_{x=0} = \frac{-4\pi}{\lambda}\alpha\sin\left(\frac{4\pi x}{\lambda} + \phi\right)\Big|_{x=0}$$

$$\frac{2\pi}{\lambda} A \sin\pi/6 = -\frac{4\pi}{\lambda}\alpha\sin\phi$$

$$\alpha = -\frac{A}{2}\frac{\sin\pi/6}{\sin\phi}$$

then using continuity of ψ

$$-A\cos\pi/6 = -\frac{A}{2}\frac{\sin\pi/6}{\sin\phi}\cos\phi$$

$$\cot\pi/6 = \frac{1}{2}\cot\phi, \text{ or } \tan\pi/6 = 2\tan\phi$$

$$\phi = \tan^{-1}\left(\tfrac{1}{2}\tan\pi/6\right) = 0.28 \text{ rad}$$

$$\alpha = 0.904A$$

Equivalently, we assume $\psi_1 = C_1\cos\frac{4\pi x}{\lambda} + C_2\sin\frac{4\pi x}{\lambda}$. Then

$$-A\cos\pi/6 = C_1$$

$$\frac{2\pi}{\lambda} A\sin\pi/6 = \frac{4\pi}{\lambda} C_2, \quad C_2 = \frac{A}{2}\sin\pi/6$$