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the book's suggested procedure, while physically correct, is needlessly complex. My apologies: I had not realized as much when I assigned the problem.

Instead, apply the Lorentz transformations for a distance  $\Delta x$  & a ~~time interval~~  $\Delta t$ , and in the rest frame, and  $\Delta x'$ ,  $\Delta t'$  in the moving frame.

The Lorentz transforms are

$$\Delta x' = \frac{\Delta x - u \Delta t}{\sqrt{1 - u^2/c^2}} \quad \text{or, equivalently} \quad \Delta x = \frac{\Delta x' + u \Delta t'}{\sqrt{1 - u^2/c^2}}$$

(interchange roles)

$$\Delta t' = \frac{\Delta t - \frac{u}{c^2} \Delta x}{\sqrt{1 - u^2/c^2}} \quad \Delta t = \frac{\Delta t' + \frac{u}{c^2} \Delta x'}{\sqrt{1 - u^2/c^2}}$$

$$\begin{aligned} \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta t \rightarrow 0}} \frac{\Delta x}{\Delta t} &\equiv V = \lim_{\substack{\Delta x' \rightarrow 0 \\ \Delta t' \rightarrow 0}} \frac{\Delta x' + u \Delta t'}{\Delta t' + \frac{u}{c^2} \Delta x'} \\ &= \lim_{\substack{\Delta x' \rightarrow 0 \\ \Delta t' \rightarrow 0}} \frac{\Delta t' \left( \frac{\Delta x'}{\Delta t'} + u \right)}{\Delta t' \left( 1 + \frac{u}{c^2} \frac{\Delta x'}{\Delta t'} \right)} \\ &= \frac{V' + u}{1 + \frac{uV'}{c^2}} \end{aligned}$$