

7-17 2s $R(r) = \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$

so

$$P(r) = r^2 |R|^2 = \frac{r^2}{8a_0^3} \left(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\right) e^{-r/a_0}$$

$$\begin{aligned} \langle r \rangle &= \int_0^\infty dr r P(r) = \int_0^\infty dr \frac{r^3}{8a_0^3} \left(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\right) e^{-r/a_0} \\ &= \frac{1}{8a_0^3} \int_0^\infty dr r^3 \left(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\right) e^{-r/a_0} \end{aligned}$$

Use $\int_0^\infty x^n e^{-ax} = \frac{n!}{a^{n+1}}$ where $a = \frac{1}{a_0}$

$$\begin{aligned} \langle r \rangle &= \frac{1}{8a_0^3} \left[4 \cdot 3! \cdot a_0^4 - 4 \cdot 4! a_0^4 + 5! a_0^4 \right] \\ &= a_0 \left[\frac{4 \cdot 3! - 4 \cdot 4! + 5!}{8} \right] \\ &= a_0 \left[\frac{4 \cdot 6 - 4 \cdot 24 + 120}{8} \right] = 6a_0 \end{aligned}$$

2p $R(r) = \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$

$$\Rightarrow P(r) = \frac{1}{24a_0^3} \frac{r^4}{a_0^2} e^{-r/a_0}$$

$$\langle r \rangle = \frac{1}{24a_0^5} \int_0^\infty dr r^5 e^{-r/a_0}$$

$$= \frac{1}{24a_0^5} \cdot 5! a_0^6 = \frac{5!}{24} a_0 = 5a_0$$