Problem 4

Show that the radial probability density of the 1s level of hydrogen has a maximum value at the Bohr radius. You may use

$$R(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$$

$$P(r) = r^2 |R(r)|^2$$

$$= \frac{4r^2}{a_0^3} e^{-2r/a_0}$$

then

$$0 = \frac{dP(r)}{dr} = \frac{4}{a_0^3} \frac{d}{dr}\left[r^2 e^{-2r/a_0}\right]$$

$$2re^{-2r/a_0} - \frac{2r^2}{a_0} e^{-2r/a_0} = 0$$

$$r - \frac{r^2}{a_0} = 0$$

$$1 - r/a_0 = 0$$

$$a_0 = r$$