Problem 2

Electrons in atoms are known to have energies in the range of 1 eV. Show that the uncertainty principle allows such electrons to be confined in an area the size of an atom (~0.1nm).

Order of magnitude problem: solve in 1D

We have

$$p \sim \Delta p$$

$$\Delta x \, \Delta p \sim \hbar$$

We need an estimate for p. But

$$E = 1eV = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

So

$$\Delta x \sim \frac{\hbar}{\sqrt{2mE}} = \frac{4.136\times10^{-15}\ eV\cdot s/2\pi}{\sqrt{2 \cdot 5.11\times10^{5}\ eV/c^2 \cdot 1eV}}$$

$$= \frac{(3\times10^{8}\ m/s)(4.136\times10^{-15}\ eV\cdot s)/2\pi}{\sqrt{2 \cdot 5.11\times10^{5}\ eV}}$$

$$= 1.94\times10^{-10}\ m \simeq 0.2\ nm$$

Within a factor of 2: close enough