

Dynamic Regression

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Consider the following 'dynamic' model

$$Y_t = \beta_1 + \beta_2 X_t + \gamma Y_{t-1} + u_t,$$

where $t = 1, \dots, T$ indicates time-series observations and all other classical assumptions hold, and $|\gamma| < 1$.

- **Dynamic model:** observations in one period are linked to those in some other period.
- For example, this may be a model for cigarettes consumption: consumption (Y_t) today is explicitly linked to past consumption Y_{t-1} , even after controlling for other determinants X_t (current income, for example).

The ADL Model

A very simple dynamic structure that contains many interesting dynamic models as particular examples is the *autorregressive distributed lag (ADL)* model:

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \gamma Y_{t-1} + \epsilon_t, \quad |\gamma| < 1, \quad t = 1, \dots, T$$

where all other classical assumptions hold.

Several interesting specifications arise as particular cases.

The model can be estimated by standard OLS regressing Y_t on X_t , X_{t-1} and Y_{t-1} .

Static regression ($\beta_1 = \gamma = 0$): The simple model introduced in Chapter 2 is an extreme case with no dynamics.

The simple model with AR(1) autocorrelation ($\beta_0 = \beta_1/\gamma$):

It is interesting to see that the simple linear model with AR(1) autocorrelation also arises as a particular case of the ADL model. Let us rewrite it using the following notation:

$$Y_t = b_0 + b_1 X_t + u_t, \quad t = 1, \dots, T \quad (1)$$

$$u_t = \phi u_{t-1} + \epsilon_t, \quad |\phi| < 1 \quad (2)$$

Using a trick from the previous class

$$Y_t = (b_0 - \phi b_0) + b_1 X_t - \phi b_1 + \phi Y_{t-1} + \epsilon_t$$

This can be written as:

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \gamma Y_{t-1} + \epsilon_t$$

where $\alpha \equiv b_0 - \phi b_0$, $\beta_0 \equiv b_1$, $\beta_1 \equiv -\phi b_1$ and $\gamma \equiv \phi$. But according to the definitions, for the specification to hold, we must impose the restriction $\beta_0 = -\beta_1/\gamma$. Hence the linear model with serial correlation actually corresponds to a particular configuration of the ADL model.

Partial adjustment ($\beta_1 = 0$): It is better to understand this specification through an example.

Y_t^* is the desired demand for money in real terms, and consider a money demand function:

$$Y_t^* = b_0 + b_1 X_t + u_t \quad (3)$$

where X_t is real income.

Now assume that due to market imperfections or adjustment costs agents cannot realize their desired holdings immediately, instead, the **observed** demand for money (Y_t) is determined as:

$$Y_t - Y_{t-1} = \delta (Y_t^* - Y_{t-1}), \quad 0 < \delta \leq 1 \quad (4)$$

This is the **partial adjustment** hypothesis.

- $\delta = 1$ means *observed* changes are equal to *desired* changes in money demand: full adjustment.
- $\delta < 1$ agents are able to fulfill they desired change only partially. δ is usually called the adjustment coefficient.

Estimation: We cannot estimate the demand model directly: Y^* is not observable.

Replace Y_t^* and solve for Y_t (we will leave the simple algebraic steps for you to check) to get:

$$Y_t = \delta b_0 + \delta b_1 X_t + (1 - \delta)Y_{t-1} + \delta u_t \quad (5)$$

$$= \alpha + \beta_0 X_t + \gamma Y_{t-1} + \epsilon_t \quad (6)$$

where $\alpha \equiv \delta b_0$, $\beta_0 \equiv \delta b_1$, $\gamma \equiv 1 - \delta$ and $\epsilon_t \equiv \delta u_t$. This is the ADL model with $\beta_1 = 0$. Can be estimated by OLS.

Short-run and long-run effects

Consider the simple partial adjustment model:

$$Y_t = \alpha + \beta_0 X_t + \gamma Y_{t-1} + \epsilon_t$$

suppose β_0 and γ are positive. For example, Y can be cigarettes consumption and X is personal income.

This about the effect of increasing X today in one unit (at time t).

- 1 Today consumption will increase in β_0 (the **short run** effect).
- 2 In period $t + 1$ consumption will continue to increase, since past consumption has a positive effect: $\gamma\beta_0$
- 3 In period $t + 2$ we will have another effect: $\gamma\gamma\beta_0 = \gamma^2\beta_0$
- 4

It can be easily shown that the addition of all these effects, the **long run** effect of altering X

$$\beta_0 + \beta_0\gamma + \beta_0\gamma^2 + \beta_0\gamma^3 + \dots$$

is equal to $\beta_0/(1 - \gamma)$.

Intuition: suppose we increase income. Then there will be an effect today in consumption, but due to adjustment costs, part of the desired increase will be postponed to the next period, to the following next, etc. So there will be a short run effect (the effect today) and a delayed effect in all other periods. The aggregate effect is the *long run* effect.

Empirical example: dynamic demand for cigarettes

- Explained variable: $l\text{sales}$ (log of sales)
- Explanatory variables:

$l\text{t}$ = log of tax rate

$l\text{price}$ = log of net price

$l\text{sales1}$ = $l\text{sales}$ in previous period

$d1, d2, d2$ = seasonal dummies

$l\text{windup}$ = log of industrial wages

$d882, d893, d901$ = dummies for "anomalous" periods.

We fitted a partial adjustment model: addictive behavior

Coefficient	Std. Err	t- stat	p-value	
lt	-0.370	0.059	-6.300	0.000
Lprice	0.026	-2.530	0.015	0.000
Lsales1	0.406	0.065	6.250	0.000
d1	-0.084	0.009	-9.740	0.000
d2	-0.105	0.007	-14.900	0.000
d3	-0.061	0.007	-8.500	0.000
lwindup	0.195	0.038	5.210	0.000
d882	-0.087	0.021	-4.180	0.000
d893	-0.157	0.022	-7.260	0.000
d901	-0.088	0.020	-4.430	0.000
cons	11.043	1.205	9.170	0.000

Comments

- Sales are strongly related to its past (addictive behavior).
- The short run effect of price (short run price elasticity) is -0.066.
- The long-run effect of price (long run price elasticity) is $-0.066/(1-0.406) = 0.111$.
- The short run effect of income (short run price elasticity) is 0.195
- The long-run effect of income (long run price elasticity) is $0.195/(1-0.406) = 0.328$
- The adjustment coefficient of the partial adjustment process is: $1-0.406 = 0.594$