Two-fold stacks of long Josephson junctions with different parameters

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The properties of the two-layer magnetically coupled long Josephson junction stacks are investigated theoretically. An expression for the Swihart velocities $\bar{c}_+$ and $\bar{c}_-$ is given for the case of different junction parameters. It is predicted that for asymmetric stack the first critical field H_{c1} is represented by two values $H_{c1}^{\downarrow}$ and $H_{c1}^{\uparrow}$ different from the fields H_{c1}^A and H_{c1}^B that characterize separate junctions. A calculation of the first critical field $H_{c1}^{\downarrow}$ as a function of the junction parameters is made for the case of small difference in effective magnetic field penetration depths of the junctions.

1. INTRODUCTION

Stacked Josephson junctions have recently received much attention, because they posses a variety of new physical phenomena as compared with a single Josephson junction and have a potential for applications. Such structures can be made as artificially prepared stacks of low-T_c Josephson junctions. The examples of the characteristic behavior of the stacked Josephson junctions is the splitting of the Swihart velocity, due to inductive coupling in the stack [1, 2], and a more sophisticated mechanism of magnetic field penetration into the junctions.

Up to now, only symmetric stacks of long Josephson junctions (JJ's) were investigated theoretically and expressions for new characteristic velocities $\bar{c}_\pm$ and the first critical field H_{c1} were obtained [3, 4, 5]. However, in experiment one has to deal with stacks with different parameters of JJ's. Due to technological limitations, one usually has different critical currents and thicknesses of the electrodes. In this paper, a two-fold stack consisting of different JJ's is studied. We give the expression for the characteristic velocities $\bar{c}_\pm$ as a function of Swihart velocities of individual JJ's $\bar{c}_A$ and $\bar{c}_B$ ($\bar{c}_A \neq \bar{c}_B$) and discuss the penetration of magnetic field into the JJ's.

2. RESULTS

The simplest Josephson stack consists of only two JJ's with arbitrary electrode thicknesses d^A, d^m, d^B. Individual Swihart velocities of uncoupled JJ's is given by

$$\bar{c}_{\rm Sw}^{A,B} = c\sqrt{\frac{d_I^{A,B}}{\varepsilon d'^{A,B}}} \qquad (1)$$

where $d'^{A|B} = \lambda\left(\coth d^{A|B}/\lambda + \coth d^m/\lambda\right)$ is the so called magnetic thickness of JJ, λ is the London penetration depth, $d_I^{A,B}$ is the thickness of the barrier and ε is the dielectric constant of the barrier. In the symmetric case the splitting of $\bar{c}$ is given by [4]:

$$\bar{c}_\pm = \bar{c}_0/\sqrt{1 \pm S}. \qquad (2)$$

Here $-1 < S < 0$ is the dimensionless coupling parameter defining the strength of the inductive interaction between the two JJ's [3] and given by

$$S = s_m/d', \qquad (3)$$

where $s_m = -\lambda/\sinh(d^m/\lambda)$.

One can see that the condition $d^A \neq d^B$ results in $\bar{c}^A \neq \bar{c}^B$ and $\bar{c}_\pm$ have to be calculated using more general formula than Eq. (2). To derive it we use the approach of Ref. [4],which yields the following expression for $\bar{c}_\pm$ as a function of $\bar{c}_A$ and $\bar{c}_B$

$$\bar{c}_\pm = \frac{\bar{c}_A\bar{c}_B\sqrt{2}}{\sqrt{\bar{c}_A^2 + \bar{c}_B^2 \pm \sqrt{(\bar{c}_A^2 - \bar{c}_B^2)^2 + 4\bar{c}_A^2\bar{c}_B^2 S^2}}}, \qquad (4)$$

where coupling parameter S is now defined in a more general way as

$$S = \frac{s_m}{\sqrt{d'^A d'^B}}. \qquad (5)$$

In the case $d^A = d^B$ i.e. $d'^A = d'^B$, $\bar{c}_A = \bar{c}_B$ and the Eq. (5) coincides with (3) and Eq. (4) becomes (2).

To calculate H_{c1} of a double stack of JJ's we use the static version of equations derived by Sakai et al. [3]. For equal parameters of JJ's ($\lambda_J^A = \lambda_J^B$ and $\Lambda^A = \Lambda^B$) it can be written as

$$\begin{cases} \frac{1}{1-S^2}\varphi_{\tilde{x}\tilde{x}} = & \sin\varphi + \quad \frac{S}{1-S^2}\psi_{\tilde{x}\tilde{x}} \\ \frac{1}{1-S^2}\psi_{\tilde{x}\tilde{x}} = & \sin\psi + \quad \frac{S}{1-S^2}\varphi_{\tilde{x}\tilde{x}} \end{cases}. \tag{6}$$

In this case the external magnetic field penetrates into both JJ's in the same way and we can assume $\varphi = \psi$ in (6) which yields

$$\varphi(x) = \psi(x) = 4\arctan\exp\frac{x - x_0}{\lambda_J^+}, \tag{7}$$

where $\lambda_J^+ = \lambda_J/\sqrt{1+S}$ and $\lambda_J^+ > \lambda_J$.

H_{c1} is the field at which fluxons penetrate into the junction from its edge. To calculate H_{c1} one may consider single-soliton solution (7) of Eq. (6), such that the coordinate x_0 is outside the junction and only fluxon tail penetrates inside. The value of x_0 has to be chosen to satisfy the boundary conditions $\left.\frac{d\varphi}{dx}\right|_{x=0} = \left.\frac{d\psi}{dx}\right|_{x=0} = H\Lambda$. If one increases the magnetic field, x_0 becomes closer to zero. At $H = H_{\rm core}$ one gets $x_0 = 0$ and half of the fluxon penetrates into the JJ. This field is the first critical field of the symmetric stack [5]

$$H_{c1}^+ = \frac{\Phi_0}{\pi\Lambda\lambda_J^+} = H_{c1}\sqrt{1+S} < H_{c1}, \tag{8}$$

where $H_{c1} = H_{c1}^A = H_{c1}^B = \Phi_0/\pi\Lambda\lambda_J$. Now, let us consider the case of slightly different $\Lambda^{A,B}$: $|\Lambda^A - \Lambda^B| \ll \min(\Lambda^A, \Lambda^B)$. To make it definite we assume first that $\Lambda^A > \Lambda^B$. In this case, the boundary conditions for fluxons penetrating in JJ^A and JJ^B are different and fluxons shift relative to each other. To understand this in more detail, one may consider the symmetric solution (7) as a sum of uncoupled solution and the "image" $\xi(x - x_0)$ where

$$\xi(x) = 4\arctan\exp\frac{x}{\lambda_J^+} - 4\arctan\exp\frac{x}{\lambda_J}. \tag{9}$$

For $\Lambda^A \neq \Lambda^B$ fluxons are shifted and have the shape

$$\varphi(x) = 4\arctan\exp\frac{x - x_0}{\lambda_J} + \xi(x - x_1); \tag{10}$$

$$\psi(x) = 4\arctan\exp\frac{x - x_1}{\lambda_J} + \xi(x - x_0). \tag{11}$$

Here we assume that S is small and the images remains unchanged in the first approximation. The equations for x_0 and x_1 can be obtained from the boundary condition

$$d\varphi/dx|_{x=0} = H_{c1}^{\downarrow}\Lambda^A \tag{12}$$

In addition, we assume that at $H = H_{c1}^{\downarrow}$ half of the fluxon penetrates into JJ^B so that

$$d^2\psi/dx^2\big|_{x=0} = 0 \tag{13}$$

Due to the fact that x_0 and x_1 are small, one can Taylor expand Eqs. (10,11) with respect to x_0 and x_1. In this case Eqs. (12,13) can be solved analytically. The field $H_{c1}^{\downarrow}$ can be obtained from

$$d\psi/dx|_{x=0} = H_{c1}^{\downarrow}\Lambda^B \tag{14}$$

Making this calculations one gets

$$H_{c1}^{\downarrow} \approx H_{c1}^A\left[\sqrt{1+S} + \left(\frac{\Lambda^A}{\Lambda^B} - 1\right)(1+2S)\right]. \tag{15}$$

Thus, $H_{c1}^{\downarrow}$ decreases linearly with $|S|$ and increases with the difference in Λ starting from the initial value H_{c1}^+. For $\Lambda^A = \Lambda^B$ Eq. (15) coincides with (8). In the case $S = 0$ (15) gives $H_{c1}^{\downarrow} = H_{c1}^B$.

In the case $\Lambda^A < \Lambda^B$ one has to write Eq. (13) for φ which yields

$$H_{c1}^{\downarrow} \approx H_{c1}^A\left[\sqrt{1+S} + \frac{3}{2}S\left(\frac{\Lambda^B}{\Lambda^A} - 1\right)\right]. \tag{16}$$

At $S = 0$ the last Eq. (16) gives $H_{c1}^{\downarrow} = H_{c1}^A$.

In general, for each junction H_{c1}^+ splits into two critical fields: $H_{c1}^{\downarrow}$ and $H_{c1}^{\uparrow}$. $H_{c1}^{\downarrow}$ corresponds to the penetration of fluxons into the JJ^B while JJ^A remains in the Meissner state. $H_{c1}^{\uparrow}$ corresponds to the penetration of fluxons into JJ^A while the fluxon chain is already present in JJ^B ($H_{c1}^{\uparrow} > H_{c1}^{\downarrow}$). The calculation of $H_{c1}^{\uparrow}$ is a more difficult task and is beyond the scope of this contribution.

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