

Current distribution simulation for superconducting multi-layered structures

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Abstract

The software package 3D-MLSI is developed, which allows us to calculate the current distribution and to extract inductances from multi-layered high- T_c and low- T_c superconducting circuits. Both kinetic and magnetic inductances as well as the three-dimensional distribution of the magnetic field are taken into account. We discuss the numerical approach used in 3D-MLSI and some new features such as visualization of sheet currents and analysis of circuits with holes. As an example, we present a simulation of a high- T_c double-layer transformer.

1. Motivation

Modern high- T_c and low- T_c superconducting circuits contain components where the magnetic field has essentially a three-dimensional (3D) structure. In many cases, both kinetic and magnetic parts of the inductance are of the same order and equally important. These make it necessary to develop adequate computer-aided design (CAD) tools which will allow us to extract the circuit parameters (e.g. inductances) from the layout. An up-to-date survey of existing software for inductance estimation in superconducting circuits can be found in [2].

Recently, a new 3D technique [3, 4] and a computer program 3D-MLSI for multi-layered circuits have been proposed. The approach is based on the London equations written for the so-called stream function instead of the electric current. The finite element method has been used for calculations and it has proven that it is fast and precise.

In this paper, we consider further enhancements in the numerical technique and the 3D-MLSI software. The program now allows us to calculate and view sheet currents excited by given terminal currents, by the external magnetic field and by given flux trapped in the holes. Now the program can also automatically calculate the reduced inductance matrix for circuits with holes.

To demonstrate the new features and the program's efficiency, we perform a simulation of a high- T_c double layer YBCO transformer with a hole in the ground plane, which increases the mutual inductance (transformation ratio).

2. Method

The straightforward and numerically stable method to calculate the inductances is to calculate the full energy. For this purpose, it is necessary to solve a set of problems for the electric current with specific excitations and for each excitation to calculate the full energy.

Our method assumes that the thickness of superconducting films is of same order or less than the London penetration depth. Formally, the method is applicable for thicker (3–5 London penetration depth) conductors. But, in this case, the accuracy of the solution is less than in the case of thin conductors. The case of single-layer flat conductors is special because symmetry makes planarity assumptions valid for relatively thick conductors.

The approach for the current solution takes into account the planarity of superconducting films and is based on the potential representation $\psi_m(r)$ of a two-dimensional sheet current \vec{J}_m which is called a *stream function*:

$$J_{m,x}(r) = \frac{\partial \psi_m(r)}{\partial y}, \quad J_{m,y}(r) = -\frac{\partial \psi_m(r)}{\partial x} \quad (1)$$

where $m = 1, \dots, N_c$ is the number of simply-connected conductors. Then, using stationary London and Maxwell equations

$$\lambda^2 \nabla \times \vec{j} + \vec{H} = 0, \quad (2)$$

$$\nabla \times \vec{H} = \vec{j}, \quad (3)$$

where λ is the London penetration depth, it is possible to obtain the following expression in terms of stream functions $\psi_m(r)$:

$$\begin{aligned} -\lambda_m^s \Delta \psi_m(r_0) + \frac{1}{4\pi} \sum_{n=1}^{N_c} \iint_{S_n} (\nabla \psi_n(r), \nabla_{xy} G_{mn}(r, r_0)) ds_r \\ = -H_{z,\text{ext}}(r_0). \end{aligned} \quad (4)$$

Here $\lambda_m^s = \lambda_m^2 / t_m$, t_m is the thickness of the superconducting film m and $H_{z,\text{ext}}(r_0)$ is the external or exciting magnetic field.

Let $I_{h,k}$ be a full current circulating around the hole k , then the boundary conditions for equation (4) are

$$\psi_m(r) = I_{h,k}, \quad r \in \partial S_{h,k}, \quad (5)$$

$$\psi_m(r) = F_m(r), \quad r \in \partial S_{\text{ext},m}, \quad m = 1, \dots, N_c \quad (6)$$

where $S_{h,k}$ are hole boundaries and $S_{\text{ext},m}$ are superconductor sheet boundaries. Functions $F_m(r)$ are completely defined by the terminal current distribution. Moreover, the kernels $G_{mn}(r, r_0)$ have singularity $|r - r_0|^{-1}$. In the case of infinitely thin current sheets, the kernel has the form

$$G_{mn}(r, r_0) = 1/\sqrt{|r - r_0|^2 + (h_m - h_n)^2}, \quad (7)$$

where h_k is the height of the current sheet k . For the expression for superconductors of finite thickness, as well as for more details concerning the derivation of the equations, see [3].

The main advantage of equations (4), (5) and (6) is the very clear problem definition ideally suitable for the finite element method [6] solution. The stream function problem given by equations (4), (5) and (6) is practically similar to the well-known Poisson equation and boundary problems for this equation. We also want to attract attention to the fact that excitation currents including terminal and hole circulating currents are taken into account in this approach as simple boundary conditions.

To obtain the solution, we solve equations (4), (5) and (6) numerically using the finite element method on a triangular mesh. The solution can be used for inductance calculation or presented as colour map of current density and streamlines.

3. Full energy and inductance matrix

The functional of the full energy of the system has the form [1, 5]:

$$\begin{aligned} E &= \frac{1}{2} \sum_{n=1}^{N_c} \iint_{S_n} (\mu_0 \lambda_s^n J^2 + \vec{J} \cdot \vec{A}_s) ds_n \\ &= \frac{\mu_0}{2} \sum_{n=1}^{N_c} \iint_{S_n} \lambda_s^n (\nabla \psi_n)^2 ds_n \\ &\quad + \frac{\mu_0}{8\pi} \sum_{n=1}^{N_c} \sum_{m=1}^{N_c} \iint_{S_n} \iint_{S_m} (\nabla \psi_n, \nabla \psi_m) G_{mn} ds_m. \end{aligned} \quad (8)$$

The finite element method we use can be considered as a procedure for the minimization of the full energy.

Let $N = N_h + N_t$, $\vec{I} = (I_{h,1}, \dots, I_{h,N_h}, I_{t,1}, \dots, I_{t,N_t})$, where $I_{t,i}$ are the full currents through corresponding terminals, $i = 1, \dots, N_t$, and $I_{h,i}$ are the full currents circulating around the i th hole, $i = 1, \dots, N_h$. Since the equations (4), (5) and (6) are linear, expression (8) is a positive

quadratic form with respect to \vec{I} . This means that there is an $N \times N$ symmetric positive-definite matrix L

$$E = \frac{1}{2} (L\vec{I}, \vec{I}). \quad (9)$$

Matrix L is the matrix of mutual inductances and self-inductances. Fluxoids $\vec{\Phi} = (\Phi_1, \dots, \Phi_N)$ can be calculated as

$$\vec{\Phi} = L\vec{I}. \quad (10)$$

A simple proof of equation (10) can be found in [5].

For $i = 1, \dots, N_h$ values Φ_i in equation (10) are fluxoids trapped in the holes in the conductors. For $i = N_h + 1, \dots, N_h + N_t$ each value Φ_i is the sum of fluxoids for the given sequence of terminals on the conductors. The fluxoids for terminal currents are discussed in [5]. In our program, it is not necessary to introduce contours to calculate these values.

In the geometries with holes, certain coefficients of the full inductance matrix can be excluded as they are sometimes useless for practical purposes. Those are the coefficients which describe the mutual inductances and self-inductances corresponding to the currents flowing around the holes. We are interested only in mutual inductances and self-inductances associated with the externally applied currents. The currents circulating around the holes are not excited independently from terminal currents. Therefore, we can exclude the mutual inductances and self-inductances associated with the currents circulating around the holes (due to the trapped flux) from the inductance matrix and reduce its size. Let $\vec{\Phi} = (\vec{\Phi}^{(1)}, \vec{\Phi}^{(2)})$ where $\vec{\Phi}^{(1)} = (\Phi_1, \dots, \Phi_h)$ is the flux trapped in the holes and $\vec{\Phi}^{(2)} = (\Phi_{h+1}, \dots, \Phi_N)$ is the flux associated with the external excitation currents. We consider the same decomposition of the vector of full current I on sub-vectors $I^{(1)}$ of length h and $I^{(2)}$ of length $l = N - h$. Let us calculate the reduced inductance matrix \tilde{L} of dimension $l \times l$ by excluding parts related to the hole currents $I^{(1)}$ from L . For this purpose, we present the full matrix L in the block form

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \quad (11)$$

where L_{11}, L_{12}, L_{21} and L_{22} are $h \times h$, $h \times l$, $l \times h$ and $l \times l$ sub-matrices of matrix L . Then, after excluding $I^{(1)}$ from equation (10) and simple calculations, we obtain

$$\vec{\Phi}^{(2)} = \tilde{L}I^{(2)} + L_{21}L_{11}^{-1}\vec{\Phi}^{(1)}, \quad (12)$$

where

$$\tilde{L} = L_{22} - L_{21}L_{11}^{-1}L_{12}. \quad (13)$$

In equation (12) the first term contains the new reduced inductance matrix \tilde{L} , while the second term just denotes that the fluxes in the holes affect the fluxes created by the external excitation current. From equations (12) and (13) we can see that the trapped flux $\vec{\Phi}^{(1)}$ does not enter the expression for \tilde{L} and, therefore, for the sake of simplicity, can be taken to be equal to zero. Thus, 3D-MLSI may calculate the reduced inductance matrix assuming zero trapped flux in each hole.

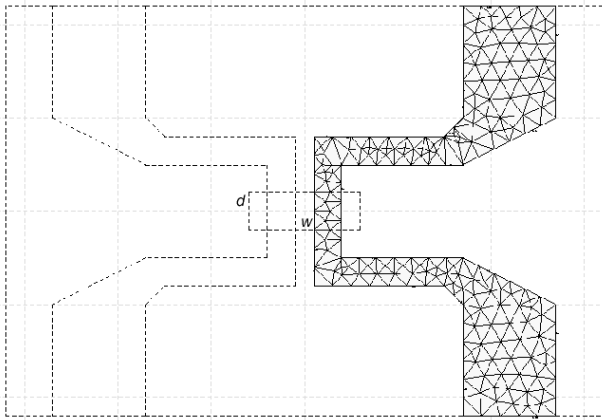


Figure 1. Transformer layout with a finite element mesh on one of the conductors. The coordinate cell size is $10 \mu\text{m}$.

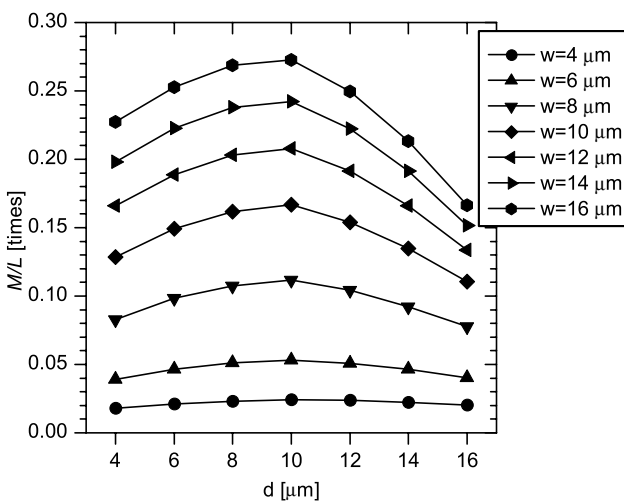


Figure 2. Transformation ratio as a function of w and d .

4. Double-layer transformer simulation

As an example of a real device, a two-layer YBCO circuit—a superconducting transformer—is calculated, see figure 1. Both superconducting layers are $0.2 \mu\text{m}$ thick. The first layer is a ground plane and contains a $12 \times 4 \mu\text{m}$ hole to increase the mutual inductance. The second layer contains superconducting wiring. The narrow part of the wires is $3 \mu\text{m}$ wide. The thickness of the insulator between the layers is $0.19 \mu\text{m}$. The London penetration depth $\lambda = 0.18 \mu\text{m}$. The circuit is symmetric (see figure 1).

We assume that the ground plane carries all return currents. It is possible to calculate a full 3×3 inductance matrix, for two symmetrical current loops via top layer wires and the current around the loop in the ground plane. For the circuit analysis, this matrix is unsuitable because the hole circulating currents is not excited independently of the terminal currents. It is known that, for any currents flowing in the circuit, the hole maintains constant trapped flux, which can be taken to be equal to zero as, in any case, it does not affect the reduced inductance matrix [1]. 3D-MLSI calculates the 2×2 inductance matrix with self-inductances L and mutual inductances $-M$. For

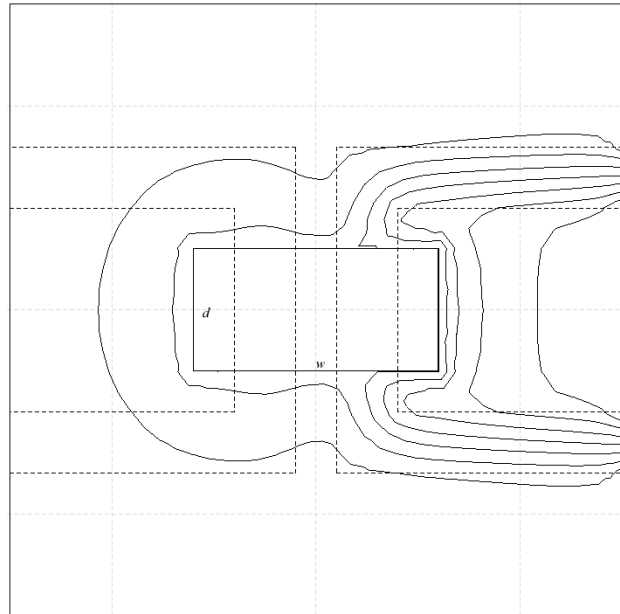


Figure 3. Current flow lines for the ground plane with the hole size $w \times d = 12 \times 6 \mu\text{m}$.

a basic configuration with a rectangular hole $w \times d = 12 \times 4 \mu\text{m}$, the self-inductance is calculated to be $L = 13.61 \text{ pH}$, $M = 1.651 \text{ pH}$ (transformation ratio 0.12). For the same structure without a hole, $L = 10.65 \text{ pH}$, $M = 0.04 \text{ pH}$ (transformation ratio 0.0039). We have also varied the hole dimensions w and d to study their influence on the transformation ratio. For these calculations, we have used a $30 \times 30 \mu\text{m}$ reduced central part of the layout on figure 3. The results are shown in figure 2. The plot is non-monotonic due to the curved form of wires. Designing a real circuit, we can now use an optimal size of the hole which provides maximum transformation ratio. An example of current flow lines for $w = 12$ and $d = 6$ is presented in figure 3.

This task results in 1165 simultaneous linear equations and a total CPU time of 27 s for a 1700+ Athlon computer.

5. Conclusions

The program 3D-MLSI has proven to be a fast and reliable tool for complicated inductance calculations and for studying current distribution induced by given terminal currents, external magnetic field and magnetic flux trapped in the holes.

3D-MLSI effectively solves the problems with the large ground planes. In some modern circuits, very narrow strips (in comparison with the circuit size) are used. 3D-MLSI shows good calculation speed and accuracy for such cases while other programs either lose precision or need ill-affordable time for calculations.

Presently, the program has some inessential limitations related to the shape of the terminals and does not allow us to simulate non-planar circuits. Fortunately, it is clear how to solve these problems in the future. Another improvement which we plan to implement is the phase terminals (as opposed to the current terminals used now), i.e. terminals which have a constant superconducting phase along them.

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