UNIVERSITY OF CALIFORNIA, BERKELEY
Spring Semester 2001

Department of Civil and Environmental Engineering
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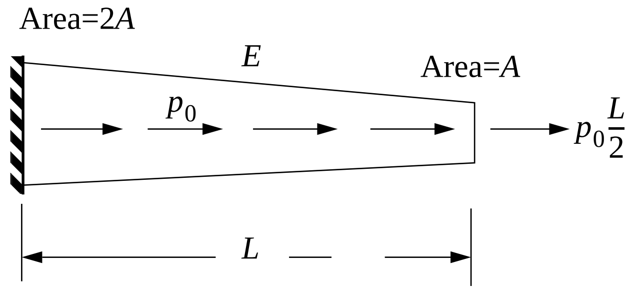
CE 222— FINITE ELEMENT METHODS

PROBLEM SET #2

Problem 1

Use the Rayleigh–Ritz procedure with three (3) trial functions to compute the displacement and axial force for the non–prismatic rod (the cross sectional area varies linearly). The load is uniformly distributed and there is a concentrated load at the free end.

Compare the approximate solution with the exact solution (from solving the differential equation).



Problem 2

For a prismatic rod with linearly varying area, such as in Problem 1:

a) Form the element stiffness matrix and load vector for uniformly distributed load for a two node element.

b) Use the element formulation in (a) to analyze the prismatic rod in Problem 1 with three elements. Plot the approximate and exact displacements.

c) Use the solution from (b) compute the axial force.

d) Determine the axial force at the nodes from equilibrium. How to they compare with the “consistent” axial forces from (c).

e) Repeat calculation with six elements (you should automate assembly and solution for N-elements). Comment on the convergence of the approximate axial force to the exact solution as the number of elements increases.

f) Compute the total potential for each analysis in part (e): three and six element solutions. Compare the two values.

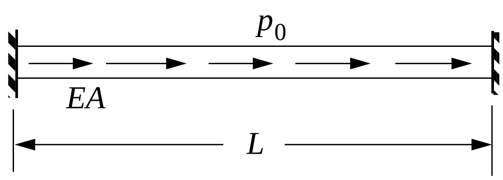
Problem 3

a) For the rod problem, a higher order element uses a quadratic displacement approximation. Using three nodes for an element, define three quadratic shape functions for the approximate displacement over an element.

b) Form the element stiffness matrix and load vector with a uniformly distributed load for the three node element with prismatic cross–section (*EA*).

c) Analyze the rod on the next page with a single three node element.

d) Repeat the analysis of the rod using two 2–node elements. The solutions in (c) and (d) have both have one DOF. How do they compare in accuracy.



Problem 4

A prismatic rod has an elastic support with stiffness $k = \alpha \cdot \frac{EA}{L^2}$ to model a friction pile, where *EA* is the axial rigidity and *L* is the length of the rod.

a) Write the differential equation of equilibrium (strong form). Construct the weak form for the differential equation including a concentrated load at one end. How does the relate to the principle of virtual displacements for this problem?

b) Determine the element stiffness matrix for a two node element.

c) Determine the element stiffness matrix for a three node element using quadratic shape functions defined in Problem 3.