Module 6

Time Series II

1. Introduction

The Unit Root Test can be undertaken using Microfit in two ways. In earlier DOS versions of Microfit, the researcher had to type ADF X(k), when k is the order of the test, in the Command window. This gives the t value of the 'unit root' for the Dicky-Fuller equation, the Augmented Dicky-Fuller Equations (up to the given order) and the critical value of t from Dicky-Fuller tables. The results are given for two models – with only intercept (but no time trend) and another model with both intercept and time trend. This option has been retained in the Microfit for Windows version.

The other way to test for Unit Root is to run the regression: X on intercept and time trend. From the Post Regression Menu proceed to Hypothesis Testing, and choose Unit Root Tests. You are asked to specify the order of the test. This gives the t value of the 'unit root' for the Dicky-Fuller equation, the Augmented Dicky-Fuller Equations (up to the given order) and the critical value of t from Dicky-Fuller tables.

If we compare the two results we will find that they are dissimilar. Even the conclusions regarding stationarity may differ. The Microfit Help Menu only states that the two results may be different depending upon the number of I(1) variables in the latter model – excluding the intercept and time trend – and whether there is a time trend. However, a close look at the underlying models will explain their difference.

2. Model Underlying ADF Command

Run the command: ADF X(3).[1] Note the t-value for ADF(3) for the model with both Intercept and Time.

Estimate the regression: X INT TIME. Save the residuals under the name RES. Create difference of RES: DRES = RES – RES(-1).

Estimate the regression:

DRES INT TIME RES(-1) DRES(-1) DRES(-2) DRES(-3).

Look at the t-value of RES(-1). This is the same as the earlier t-value obtained running the ADF command.

3. Unit Root Test Under Hypothesis Menu

Estimate the regression:

DRES RES(-1) DRES(-1) DRES(-2) DRES(-3).

Note down the tabulated value of RES(-1).

Now run the equation: X INT TIME. Your earlier steps (taking lags of DRES) will have reduced the sample period. **So you must change the sample period to cover your entire data set**. In the Post regression Menu opt for Hypothesis Testing and then Unit Root Test. Specify the order as 3. The t-value corresponding to ADF(3) will be the same as the t-value obtained from the OLS regression on residuals.

4. Which Test to Perform?

When the researcher wants to undertake the Unit Root Test using Microfit then s/he should use the ADF command. If, however, the two step Engle-Granger Test is to be

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[1] We have taken the order to be 3 for convenience. The discussion is true for any order.

used to test whether two variables Y_t and X_t are cointegrated then we should use the Unit Root option from the Hypothesis Testing Menu. We will discuss cointegration later on. Meanwhile let us consider why we use the ADF command to undertake the Unit Root Test.

5. Underlying Logic of the Microfit Process

Econometric theory tells us that we should regress:

$$\Delta X_t = \alpha + \beta T + (\rho - 1) X_{t-1} + u_t \qquad [1]$$

or, $$X_t = \alpha + \beta T + \rho X_{t-1} + u_t \qquad [2]$$

This is not what Microfit does. It regresses:

$$X_t = \alpha_1 + \beta_1 T + u_t \qquad [3]$$

Saves the residuals and then again regresses:

$$\Delta u_t = \alpha_2 + \beta_2 T + \gamma u_{t-1} + e_t \qquad [4]$$

This may appear confusing. Actually, however, the test $\rho = 0$ using [4] implies that $(1 - \rho) = 0$ in [1]. So whether we undertake the Unit Root Test on the variable or its residuals does not matter. The logic is clear from the following simplification.

Start from [4]. We have:

$$\Delta u_t = \alpha_2 + \beta_2 T + \gamma u_{t-1} + e_t$$

or, $$u_t - u_{t-1} = \alpha_2 + \beta_2 T + \gamma u_{t-1} + e_t$$

or, from [3] : $[X_t - \alpha_1 + \beta_1 T] – [X_{t-1} - \alpha_1 + \beta_1(T-1)] = \alpha_2 + \beta_2 T + \gamma[X_{t-1} - \alpha_1 + \beta_1(T-1)] + e_t$

or, $$X_t - \alpha_1 - \beta_1 T - X_{t-1} + \alpha_1 + \beta_1 T + \beta_1 = \alpha_2 + \beta_2 T + \gamma X_{t-1} - \gamma\alpha_1 + \gamma \beta_1 T - \gamma \beta_1 + e_t$$

or, $$X_t - X_{t-1} + \beta_1 = \alpha_2 + \beta_2 T + \gamma X_{t-1} - \gamma\alpha_1 + \gamma \beta_1 T - \gamma \beta_1 + e_t$$

or, $$X_t = [\alpha_2 - \beta_1 - \gamma\alpha_1 - \gamma \beta_1] + [\beta_2 + \gamma \beta_1] T + [\gamma + 1] X_{t-1} + e_t \qquad [5]$$

This has the same structure as equation [2]:

$$\alpha = [\alpha_2 - \beta_1 - \gamma\alpha_1 - \gamma \beta_1]$$

$$\beta = [\beta_2 + \gamma \beta_1] T$$

$\rho = [\gamma + 1] X_{t-1} +$

$u_t = e_t$

So if we test for $[\gamma + 1] = 0$ this is equivalent to testing whether $\rho = 1$, i.e. whether X_t has a unit root or not.

6. An Alternate Unit Root Test

Alternately we can undertake the Unit Root Test 'manually'. This consists of running the following models and testing for $\rho = 0$ using the appropriate t-statistic.

Model Form	**t-statistic**
$X_t = \rho X_{t-1} + u_t$	t_1
$X_t = \alpha + \rho X_{t-1} + u_t$	t_2
$X_t = \alpha + \beta T + \rho X_{t-1} + u_t$	t_3

7. Augmented Dicky-Fuller Equation

The above method assumes that the error terms u_t 's are not serially correlated. In that case we add sufficient lags of ΔX_t (ΔX_{t-1} , ΔX_{t-2} , ΔX_{t-3} , …) until serial correlation in the residuals is eliminated. The relevant reference is the article by Holden and Perman in Bhaskara Rao ed. Cointegration (1995). Once again we have to use the Dicky Fuller tables. However, the exact nature of the series is clearly understood if we adopt this step wise procedure.

This method starts with the ADF equation

$$\Delta X_t = \alpha + \beta T + (\rho - 1) X_{t-1} + \sum_{i=1}^{n} \gamma_i \Delta X_{t-i}$$

For convenience we will ignore the ADF part.

Step 1: Estimate the ADF equation. Go to Post Estimation Menu\Hypothesis Testing Menu\Variable Deletion Test and delete X_{t-1} and T. Compare F = ϕ_3 with F1. If F > F_1 then proceed to Step 4.

Step 2: If, however, F < F1 then the null hypothesis (β= 0, ρ =1) is accepted. Use the t-statistic of (ρ -1) from the above equation and verify that ρ - 1= 0, so that X has a unit root. The critical value should be t_3. Note that the RW does not have a trend.

Step 3: To ascertain whether a drift exists, we rerun the ADF equation dropping T. Then undertake the variable deletion test dropping intercept and X_{t-1}. Compare F = ϕ_2 with F_2 in the Tables. If F > F_2 then we have a random walk with a drift but no trend. The specification of our model is: $x_t = \alpha + x_{t-1} + u_t$. On the other hand if F < F_2 we have a RW without either drift or trend. The model is: $x_t = x_{t-1} + u_t$.

Step 4: If F > F_1 in Step 1 then we have three possibilities: (β = 0, ρ ≠1), (β ≠ 0, ρ =1) or (β ≠ 0, ρ ≠ 1). Use the t-statistic for ρ - 1 to test whether ρ = 0. Compare t with the normal t table. If t < t_0 then ρ - 1 = 0 and X_t has a unit root. This implies that (β ≠ 0, ρ =1) holds. So we have a random walk with a trend.

Step 5: On the other hand if t > t_0 then ρ - 1 ≠ 0 and X_t does not have a unit root (stationary). In that case either (β = 0, ρ ≠1) or (β ≠ 0, ρ ≠ 1) holds and we have to undertake the standard t-test to ascertain whether β ≠ 0 (stationary with trend) or β = 0 (stationary without trend).

Step 6: Finally, we ascertain, again using the standard t-test, whether α ≠ 0 (stationary with drift) or α = 0 (stationary without drift).

8. Critical Values of t and F

N	t_1		t_2		t_3		F_1		F_2	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
25	-2.66	-1.95	-3.75	-3.00	-4.38	-3.60	10.61	7.24	8.21	5.68
50	-2.62	-1.95	-3.58	-2.93	-4.15	-3.50	9.31	6.73	7.02	5.13
100	-2.60	-1.95	-3.51	-2.89	-4.04	-3.45	8.73	6.49	6.50	4.88
250	-2.58	-1.95	-3.46	-2.88	-3.99	-3.43	8.43	6.34	6.22	4.75
500	-2.58	-1.95	-3.44	-2.87	-3.98	-3.42	8.34	6.30	6.15	4.71
α	-2.58	-1.95	-3.43	-2.86	-3.96	-3.41	8.27	6.25	6.09	4.68

t_1 [t-value when there is no constant]: $x_t = \rho\, x_{t-1}$

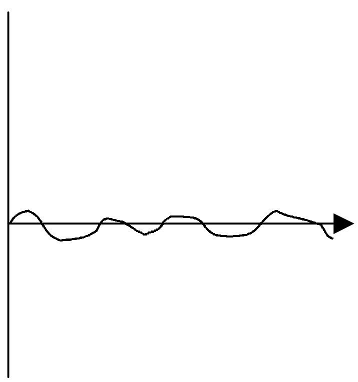
t_2 [t-value when there is a constant]: $x_t = \alpha + \rho\, x_{t-1}$

t_1 [t-value when there is both a constant and an intercept]: $x_t = \alpha + \beta T + \rho\, x_{t-1}$

F_1: Both α and ρ are zero simultaneously

F_2: All three terms α, β and ρ are zero simultaneously

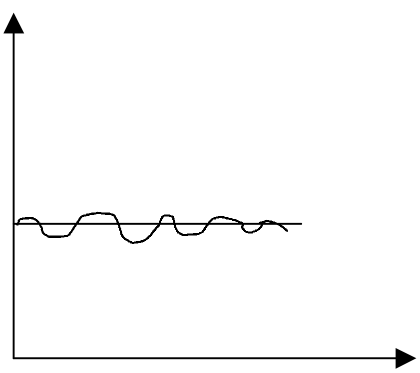
9. Trend Stationary and Difference Stationary Models



Consider the models in Step 5 and 6. The series is stationary. However the nature of each model is different. The three models can be described in terms of the basic model given below:

$$X_t = \alpha + \beta T + \gamma X_{t-1} + u_t$$

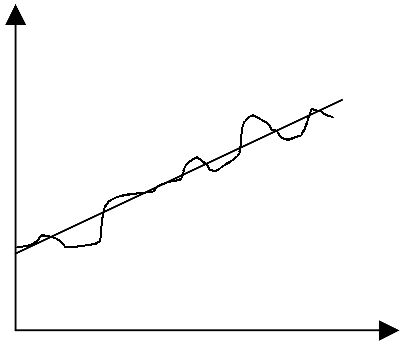
Suppose that the Unit Rot test reveals that γ is NOT equal to 0 so that X is stationary. The deviations can be around zero – zero stationary. The plot will be something like in the adjacent Figure so that the model is $X_t = \gamma X_{t-1} + u_t$.



On the other hand, the deviation can be around a mean value (α). In such cases we have a mean stationary series given by $X_t = \alpha + \gamma X_{t-1} + u_t$. The series has a drift.

Finally, we consider the **Trend Stationary Process** (TSP model). This model has both a drift and trend and is given by

$X_t = \alpha + \beta T + \gamma X_{t-1} + u_t$.



Note that we should check for the appropriate level of stationary. Thus, if we test for Mean Stationarity, when the series is actually trend stationary, we will get the wrong signal to remove the non-stationarity.

If, however, the series really has a Unit Root then X_t is non-stationary. Such variables can be made stationary by **differencing.** The model then becomes:

$$\Delta X_t = \beta + \gamma X_{t-1} + \Delta u_t$$

This is called a **Difference Stationary Model.** Many researchers in macroeconomics arbitrarily use the DSP models. This leads to a loss in valuable information and should be undertaken only if the series is non-stationary. So the correct procedure is to estimate:

$$\Delta X_t = \alpha + \beta T + \gamma X_{t-1} + u_t$$

and check whether it has a Unit Root. Nelson and Plosser suggests that we estimate the above regression, test whether it has a unit root, and then undertake the variable deletion test for ($\beta = 0$, $\gamma = 0$) using F_2.Pnly if the null hypothesis is accepted we should use the DSP model. On the other hand if the series is stationary then we should **detrend** the series by using the appropriate TSP model. Greene (pp. 780-781) provides a concise treatment of the difference between DSP and TSP models.

10. Regressing Random Walks on Another Random Walk

What happens when two series are both non-stationary? Regressing one Random Walk (RW) against another can lead to spurious results and the conventional tests will tend to indicate a relationship when in fact none exists. This is one reason why it is important to test for RW. Differencing, although acceptable, may result in a loss of information about the long-run relationship. Is it possible to run regression between two variables even though both variables are RWs?

In steady-state equilibrium, economic variables take the same values from period to period, so $X_t = X_{t-1} = X_{t-2} = ... = X^*$, until the system is disrupted. Furthermore, in equilibrium there are relationships between economic variables like consumption and income that economists would like to understand.

11. Cointegration

Co-integration, developed by Engle & Grange, is important for reasons that go beyond its use as a diagnostic for linear regression. In many cases theory tells us that two variables should be co-integrated, and a test for co-integration is a test of the theory.

$$E_t = Y_t - \beta X_t$$

" Aggregate consumption and disposable income behave as RWs, we would expect these two variables to move together over the long-run, so that a linear combination of the two should be stationary."

" Although dividends and stock prices both follow RWs, the two series should be cointegrated, with the co-integrating parameter (β) equal to the discount rate used by investors to calculate the present value of earnings."

A pair of series X_t and Y_t are said to be co-integrated if they are each I(1) but there exists a linear combination of them $E_t = Y_t - \alpha - \beta X_t$, that is I(0) (that is, stationary).

Suppose that the long-run equilibrium relationship is defined by $\mathbf{Y_t = \alpha + \beta X_t + e_t}$. In this case e_t would represent how far Y_t and X_t **were away from equilibrium and could be called 'equilibrium error'**. If Y_t and X_t are co-integrated and the error et is stationary with mean zero then:

$$Y_t = \alpha + \beta X_t + E_t$$

Since $e_t \sim(0, \sigma^2)$ is stationary, the variables If Y_t and X_t follow a stable, long-run relationship. In this case, X_t and Y_t are co-integrated, least squares estimation of the equation provides in large samples an excellent (super consistent) estimator of β which describes the long-run steady state equilibrium relationship between Yt and Xt. Therefore, the series Y_t and X_t is cointegrated if their error term is stationary.

12. Engle Granger 2-Step Test

The Engle-Granger test consists of checking whether the two series are non-stationary. If both variables are RWs then we have to test whether the residual of Y_t on X_t is stationary.

To undertake this test regress Y_t on X_t. Then go to Post Estimation Menu\Hypothesis Testing\Unit Root and enter the appropriate order. This regresses – as seen before in section 3 – the model:

DRES = Lag of RES + lagged values of DRES.

The researcher should take as many lagged values as necessary to eliminate serial correlation in the residuals.

13. Removing Serial Correlation

Run the following regression

$$\Delta u_t = \gamma\, u_{t-1} + v_t$$

or in MFIT format: DRES RES(-1)

The LM statistic suggests auto-correlation in the residuals.

Adding Δu_{t-1} yields the ADF(1) equation

$$\Delta u_t = \gamma\, u_{t-1} + \Delta u_{t-1} + v_t$$

Add as many lags, as you require removing serial correlation. Once you removed serial correlation you have the optimum lag length for the unit root test.

14. Testing For Co-Integration - CRDW

An alternative and quicker method of testing for cointegration is described in Gujrati (pp. 824) – th Cointegrating Regression Durbin Watson test. This test uses the DW statistic. However the null hypothesis is now that $d \approx 0$ rather than $d \approx 2$. So the critical values of d will change. At 1%, 5% and 10% levels the critical values for n = 100 are 0.511, 0.386 and 0.322 respectively. If d is less than d_0 than we reject the null hypothesis of cointegration.

This test, however, has limited applicability. It can be used only if the disequilibrium errors are generated by a first order AR process. An easier test is described below.

In order to assess whether the relationship is co-integrated or not we run the above regression:

$$Y_t = \alpha + \beta\, X_t$$

and save the residuals. Close the results screen, by pressing Esc once. From the post regression menu choose option 3. At the next screen,

Option 0: Takes you back to the post regression menu.

Option 1: Displays the residuals and fitted values.

Option 2: Plots actual and fitted values

Option 3: Plots residuals. The bands represent +/-2SE, where SE is the estimated standard error of the regression.

Option 4: Displays graphs of the ACF and the standardised spectral density function of the residuals.

Option 5: Displays histogram of residuals.

6: Allows you to save the residuals and forecast errors (if any) in a variable for use in subsequent analysis.

7: Allows you to save fitted and forecast values (if any) in a variable for use in subsequent analysis.

Choose Option 6. Microfit asks you to supply a name for it, say RES. Return to the Estimation Window and regress:

$$Y_t = \alpha + \beta X_t + \gamma RES_{t-1}$$

If the t-ratio on RES(-1) is significant then we conclude that the relationship co-integrated.

15. Error Correcting Mechanism

If the two series are both non-stationary we can still run a regression model if they are cointegrated. Now cointegration implies that there is a long run equilibrium relationship between the two variables. The error term then becomes an equilibrium error. This suggests that the error term can be used to tie the short run behaviour of Y_t to its long run value. The ECM then becomes:

$$\Delta Y_t = \alpha + \beta \Delta X_t + \theta u_{t-1} + \varepsilon_t \text{ when } u_t = Y_t - \alpha - \beta X_t$$

In short, regress: Y INT X. Save residuals (RES), create lagged values of RES and differences of Y and X (DY and DX) and regress DY INT DX RES(-1) for the ECM. Note that if θ is not significant then this implies that Y adjusts to changes in X in the same period.

16. Alternative Approaches to Cointegration

In recent times Johansen has suggested an alternative method of approaching the problem of Cointegration. In a paper the Autoregressive Lagged Distribution (ARDL) model has also been suggested by Pesharan & Shin (1999). Both these approaches can be undertaken using Microfit.

References:

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