Quantum mechanical equations for many particle systems

Saurav Dwivedi

I have formulated the fundamental equations of quantum mechanics in their generalized form. It includes the basic aspects of classical theoretical physics.

KEY WORDS: Generalized coordinates; System of particles; Internal and external potential energy; Continuity equation.

"The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."

(Eugene P. Wigner).

Schrödinger equation for many particle systems in generalized form

We consider the quantum mechanical system of particles in generalized form. The Hamiltonian may be defined as,

$$H(q_1, q_2, \dots q_S, p_1, p_2, \dots p_S, t) \tag{1}$$

where the subscript s is the degrees of freedom of the system.

And considering the classical theoretical physics;

$$H(q_i, p_i, t) = \sum_{\alpha=1}^{\alpha} \sum_{i=1}^{N} \frac{p_{\alpha i}^2}{2m_i} + \sum_{i=1}^{S} V_i(q_i, t) + \sum_{i \neq j=1}^{S} V_{ij}(q_i, q_j, t) \tag{2}$$

where $V(q_i, q_j, t) = \sum_{i \neq j=1}^{S} V_{ij}(q_i, q_j, t)$ is the internal potential energy of the system.

And considering the eigenvalue equation,

$$\hat{\mathcal{A}}\psi = \alpha\psi \tag{3}$$

and in the consideration of eq (1) eq (2) and eq (3) and the operator values of Hamiltonian and momentum, I get

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t}+\frac{\hbar^2}{2}\sum_{\alpha=1}^{\alpha}\sum_{i=1}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-\sum_{i=1}^{S}V_i(q_i,t)\,\psi(q_i,t)-\sum_{i\neq j=1}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t)=0 \tag{4}$$

which is the Schrödinger equation for many particle systems in generalized form.

Saurav equation for many particle systems in generalized form

Now considering the generalized treatise for many particle systems for my equation (Saurav equation) (See my recent paper "A theoretical Study of ψ-waves" eq (15) and eq (16)), I get

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t}+\hbar^2\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-i\hbar\frac{d\psi(q_i,t)}{dt}=0 \tag{5}$$

which is the Saurav equation for many particle systems in its generalized form.
and canceling for $i\hbar$ in eq (5), I get another Saurav equation for many particle system in generalized form,

$$\frac{d\psi(q_i,t)}{dt}+i\hbar\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-\frac{\partial\psi(q_i,t)}{\partial t}=0 \tag{6}$$

which is my equation for many particle systems in generalized form.

Continuity equation for many particle systems in generalized form

Now treating the Schrödinger equation for many particle systems in generalized form (4), I get

$$\begin{aligned}&i\hbar\,\psi^*(q_i,t)\frac{\partial\psi(q_i,t)}{\partial t}+\frac{\hbar^2}{2}\psi^*(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_{i=1}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)\\&-\psi^*(q_i,t)\sum_{i=1}^{S}\hat{V}_i(q_i,t)\,\psi(q_i,t)-\psi^*(q_i,t)\sum_{i\neq j=1}^{S}\hat{V}_{ij}(q_i,q_j,t)\,\psi(q_i,t)=0\end{aligned} \tag{7}$$

and getting its complex conjugate, I get

$$-i\hbar\,\psi(q_i,t)\frac{\partial\psi^*(q_i,t)}{\partial t}+\frac{\hbar^2}{2}\psi(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_{i=1}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi^*(q_i,t)$$
$$-\psi(q_i,t)\sum_{i=1}^{S}\hat{V}_i{}^*(q_i,t)\,\psi^*(q_i,t)-\psi(q_i,t)\sum_{i\neq j=1}^{S}\hat{V}_{ij}{}^*(q_i,q_j,t)\,\psi^*(q_i,t)=0 \tag{8}$$

Now we consider the hermitian properties of external and internal potential energy;

$$\psi\hat{V}^*(q_i,t)\,\psi^*-\psi^*\hat{V}(q_i,t)\,\psi=\beta(q_i,t)(\psi\psi^*-\psi^*\psi)=0 \tag{9}$$

and

$$\psi\hat{V}^*(q_i,q_j,t)\,\psi^*-\psi^*\hat{V}(q_i,q_j,t)\psi=\beta(q_i,q_j,t)(\psi\psi^*-\psi^*\psi)=0 \tag{10}$$

where β is the potential energy eigenvalue.
and in the consideration of eq (7) and eq (8) and considering the hermitian properties of internal and external potential energy (9) and (10), I get

$$i\hbar\frac{\partial\{\psi^*(q_i,t)\,\psi(q_i,t)\}}{\partial t}+\frac{\hbar^2}{2}\left\{\psi^*(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_{i=1}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-\psi(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_{i=1}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi^*(q_i,t)\right\}=0 \tag{11}$$

Now considering from mathematical analysis;

$$\psi(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi^*(q_i,t)-\psi^*(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)$$
$$=\mathrm{div}\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{1}{m_i}\{\psi(q_i,t)\nabla_{\alpha i}\psi^*(q_i,t)-\psi^*(q_i,t)\nabla_{\alpha i}\psi(q_i,t)\} \tag{12}$$

and considering the probability density and current density for many particle system in generalized form,

$$\rho(q_i,t)=\psi(q_i,t)\,\psi^*(q_i,t) \tag{13}$$

and we here consider the additivity of current density,

$$\mathbf{j}(q_i,t)=\sum_i^S \mathbf{j}_i(q_i,t)=\frac{i\hbar}{2}\sum_{\alpha=1}^{\alpha}\sum_i^N \frac{1}{m_i}\{\psi(q_i,t)\nabla_{\alpha i}\psi^*(q_i,t)-\psi^*(q_i,t)\nabla_{\alpha i}\psi(q_i,t)\} \tag{14}$$

and in the consideration of eq (11), eq (12), eq (13) and eq (14), I get

$$\frac{\partial\rho(q_i,t)}{\partial t}+\operatorname{div}\mathbf{j}(q_i,t)=0 \tag{15}$$

which is the equation of continuity for many particle systems in generalized form.

And now treating my equation in generalized form (6), I get

$$\psi^*(q_i,t)\frac{d\psi(q_i,t)}{dt}+i\hbar\,\psi^*(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_i^N\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-\psi^*(q_i,t)\frac{\partial\psi(q_i,t)}{\partial t}=0 \tag{16}$$

and getting its complex conjugate, I get

$$\psi(q_i,t)\frac{d\psi^*(q_i,t)}{dt}-i\hbar\,\psi(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_i^N\frac{\nabla_{\alpha i}^2}{m_i}\psi^*(q_i,t)-\psi(q_i,t)\frac{\partial\psi^*(q_i,t)}{\partial t}=0 \tag{17}$$

and in the consideration of eq (16) and eq (17), I get

$$\begin{aligned}\frac{d\{\psi(q_i,t)\,\psi^*(q_i,t)\}}{dt}&=\frac{\partial\{\psi(q_i,t)\,\psi^*(q_i,t)\}}{\partial t}\\&+i\hbar\left\{\psi(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_i^N\frac{\nabla_{\alpha i}^2}{m_i}\psi^*(q_i,t)-\psi^*(q_i,t)\sum_{\alpha=1}^{\alpha}\sum_i^N\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)\right\}\end{aligned} \tag{18}$$

and in the consideration of eq (12), eq (13), eq (14) and eq (18), I get

$$\frac{d\rho(q_i,t)}{dt}=\frac{\partial\rho(q_i,t)}{\partial t}+2\operatorname{div}\mathbf{j}(q_i,t) \tag{19}$$

and in the consideration of eq (15) and eq (19), I get

$$\frac{d\rho(q_i,t)}{dt}-\operatorname{div}\mathbf{j}(q_i,t)=0 \tag{20}$$

and

$$\frac{d\rho(q_i,t)}{dt}+\frac{\partial\rho(q_i,t)}{\partial t}=0 \tag{21}$$

Fundamental equations of quantum mechanics for many particle systems in generalized form

Now we consider the fundamental equations of quantum mechanics for many particle systems in generalized form. Let we consider the generalized treatise for the known fundamental equations of quantum mechanics (See my first paper, "A theoretical study of ψ-waves" eq (122) to eq (137)), I get

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t}+\hbar^2\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)+L(q_i,\dot{q}_i,t)\,\psi(q_i,t)=0 \tag{22}$$

$$i\hbar\frac{d\psi(q_i,t)}{dt}-\frac{\hbar^2}{2}\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-\sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t)-\sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t)=0 \tag{23}$$

$$i\hbar\frac{d\psi(q_i,t)}{dt}-\hbar^2\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-E(q_i,p_i)\,\psi(q_i,t)=0 \tag{24}$$

$$E(q_i,p_i)\,\psi(q_i,t)+\frac{\hbar^2}{2}\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)-\sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t)-\sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t)=0 \tag{25}$$

$$E(q_i,p_i)\,\psi(q_i,t)+\hbar^2\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)+L(q_i,\dot{q}_i,t)\,\psi(q_i,t)=0 \tag{26}$$

$$L(q_i,\dot{q}_i,t)\,\psi(q_i,t)+\frac{\hbar^2}{2}\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{\nabla_{\alpha i}^2}{m_i}\psi(q_i,t)+\sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t)+\sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t)=0 \tag{27}$$

$$E(q_i,p_i)\,\psi(q_i,t)-\sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{2m_i}\psi(q_i,t)-\sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t)-\sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t)=0 \tag{28}$$

$$E(q_i, p_i)\,\psi(q_i,t) - \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{m_i}\psi(q_i,t) + L(q_i,\dot{q}_i,t)\,\psi(q_i,t) = 0 \qquad (29)$$

$$L(q_i,\dot{q}_i,t)\,\psi(q_i,t) - \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{2m_i}\psi(q_i,t) + \sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t) + \sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t) = 0$$
(30)

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t} - \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{2m_i}\psi(q_i,t) - \sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t) - \sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t) = 0$$
(31)

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t} - \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{m_i}\psi(q_i,t) + L(q_i,\dot{q}_i,t)\,\psi(q_i,t) = 0 \qquad (32)$$

$$i\hbar\frac{d\psi(q_i,t)}{dt} + \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{2m_i}\psi(q_i,t) - \sum_{i}^{S}V_i(q_i,t)\,\psi(q_i,t) - \sum_{i\neq j}^{S}V_{ij}(q_i,q_j,t)\,\psi(q_i,t) = 0$$
(33)

$$i\hbar\frac{d\psi(q_i,t)}{dt} + \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{m_i}\psi(q_i,t) - E(q_i,p_i)\,\psi(q_i,t) = 0 \qquad (34)$$

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t} - \sum_{\alpha=1}^{\alpha}\sum_{i}^{N}\frac{p_{\alpha i}^2}{m_i}\psi(q_i,t) - i\hbar\frac{d\psi(q_i,t)}{dt} = 0 \qquad (35)$$

which are the fundamental equations of quantum mechanics for many particle systems in generalized form. These equations are clever enough to describe the quantum phenomena's.

References

[1] Saurav Dwivedi. (2005). *A Theoretical Study of* $\psi-$ *waves* (submitted for publication)

[2] Saurav Dwivedi. (2005). *A Fundamental Treatise on Continuity Equation* (submitted for publication)