Generalized Uncertainty Principle

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Abstract

Quantum theory brought an *irreducible* lawlessness in physics. This is accompanied by lack of specification of *state* of a system. We can not measure *states* even though they ever existed. We can measure only *transition* from one state into another. We deduce this lack of determination of *state* mathematically, and thus provide formalism for maximum *precision* of determination of *mixed states*. However, the results thus obtained show consistency with Heisenberg's uncertainty relations.

keywords State • Praxism • Ontism • Schrödinger Operators • Probability • Uncertainty • Indeterminism • Entanglement

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Contents

1 Introduction

Our search is for a valid *interpretation* of quantum theory. Quantum theory does not seem to have form of a fundamental theory. For it does not involve "Law of Nature", but only law of the system under study [26, Finkelstein 2005]. It is rather a phenomenological theory [27]. "Quantum Mechanics" seems misnomer for a non-mechanistic theory. Quantum theory (and its semantics) is in contradiction with "mechanistic order" [6, 7, Bohm 1980, p. 173]. The *mechanistic order* conflicts with practical quantum theory in the sense of *non-locality* and *entanglement*. *There is no mechanistic quantum theory.* The basic idea (basis) of Mechanics is **Ontism**. The basis of Quantum is **Praxism** [23, Finkelstein 1996]. In this connection, the nomenclature "Quantum Mechanics" reflects classical ideology; vestige of classical thought, therefore. The irony is that classical ideology is complete, while quantum ideology is incomplete.

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Quantum theory seems complete *phenomenologically*, for it can explain (all) phenomena (and experimental facts) in its domain. It is incomplete on *ideological* ground. This *quantum spontaneity* is subject to the very definition of "Reality" (as it is). Quantum theory is a theory of *actuality*, not *reality*. Actuality (paradox) is a scientific truth that can be proven scientifically, while reality is an "orthodox", illusion however.

There is no room for an "absolute reality" or "absolute state" (of being) in quantum theory; as there is no room for "absolute time" in relativity.

Physicists use a 'state' vector $|\psi\rangle \,\epsilon\, \mathbb{H}$ in order to define 'state of a system' (as if it existed). A state vector actually reflects our *actions* upon the system, not any its properties [24]. What we measure is a Hermitian observable, not state itself [25].

The *wavefunction* ψ can not be constructed for a system. It is main underlying assertion of my approach. If a system existed (in *isolation*), having a probability of existence (w) [18, Dwivedi 2005], its wavefunction can not be constructed by means of properties of the system. Properties of a system (in classical sense) can completely be determined by means of Hamilton-Jacobi function '$\mathcal{A}$ction' $s(q_i,t)$. It absorbs certain constants of integration a system has. It suffices system's complete determinacy. It was a successful formulation of classical ideology. Wavemechanics is not a successful formulation of quantum ideology. Not even self-consistent [25].

We have two (parallel) versions of quantum theory:

— **Praxic theory.** Matrix-Mechanics invented by Heisenberg. The fundamental physical entities are *actions*, *operations* and *processes*.

— **Ontic theory.** Wave-Mechanics invented by Schrödinger. The fundamental physical entities are *states of being*; absolute reality.

The Ontic theory better survived because it is more visualized. The Praxic theory has been overlooked [25]. Although the *two* versions are equivalent methodologically, Praxic theory is more valid than Ontic, ideologically. It is self-consistent too.

The distinction *state* for ψ has been claimed absurd [4, 3, 23, 45, Weizsäcker 2006, p. 263]. In this paper I claim that Ontic theory assumes an *state* that can not be constructed. Quantum theory is a theory about probabilities and predictions [45, Weizsäcker 2006, p. 260]. However, I define system's existence in probabilistic manner. I assign a probability (w) in order to define a system in isolation. For $w = 1$ system is in *pure state* and all its variables are accessible, for $w \,\epsilon\, (0, 1)$ it is in *mixed state* as certain of variables are *hidden* or non-accessible (e.g. in presence of many type of interactions [35]). For $w = 0$ the system is in *forbidden state* and all its variables are hidden and system can be represented by none. I quantize this observable using Schrödinger's quantization rule and obtain $\widehat{w} = -i\hbar\partial/\partial s$. However, the two observables (s and w) are found *incompatible* and we obtain an uncertainty relation $\sigma_s\sigma_w \geq \hbar/2$. As a consequence, for a pure state ($\sigma_w = 0, \sigma_s = \infty$), s is undetermined and ψ can not be constructed. Thus if system existed, it can not be specified by a vector in Hilbert space. The state vector can not be constructed with certainty. After all I provide a maximum precision for construction of state vector for system in *mixed state*.

2 Probability Eigenvalue Formalism

We have a general form of Schrödinger's wavefunction[1] belonging to system's Hilbert space $\mathbb{H}$, in generalized perspective [5]

$$\psi(R(q_i,t), s(q_i,t)) := R(q_i,t) \exp\left(\frac{i}{\hbar} s(q_i,t)\right), \qquad i = 1, 2, 3, \ldots, f\,, \tag{2.1}$$

which is orthonormalizable

$$\langle\psi_\alpha|\psi_\beta\rangle = \int_{-\infty}^{+\infty} \psi_\alpha^*(R(q_i,t), s(q_i,t))\psi_\beta(R(q_i,t), s(q_i,t))\, d\tau = \delta_{\alpha\beta}\,, \tag{2.2}$$

[1] It is notable that ψ is function of q_i and t implicitly as well as function of R and s explicitly. Although $\mathcal{A}$ction $s[\ldots]$ is not function of q_i and t necessarily, instead it is often functional of the path. It has been taken function of q_i and t here for mere convention, that does not hurt assertion. Nevertheless, the result holds intact if one prefers $\psi(R, s) := R \exp\left(\frac{i}{\hbar} s\right)$ over (2.1).

where $d\tau$ $(= \prod_{i=1}^{f} h_i dq_i$, h being scale factor and f is degrees of freedom) is generalized volume element of the *configuration space*. [The system has all these variables, except ψ (and tacitly its space $\mathbb{H}$) in Praxic perspective]. Differentiate (2.1) partially *w.r.t.* $\mathcal{A}$ction $s(q_i, t)$ to obtain

$$\frac{\partial \psi(R(q_i,t), s(q_i,t))}{\partial s(q_i,t)} = \frac{i}{\hbar}\psi(R(q_i,t), s(q_i,t))\,. \tag{2.3}$$

I entail a unit (zero-order differential) operator that satisfies for an ordinary function f as well as for wavefunction (See Appendix A)

$$\mathcal{I} f = f\,; \qquad \mathcal{I}\psi(R(q_i,t), s(q_i,t)) = \psi(R(q_i,t), s(q_i,t))\,. \tag{2.4}$$

Following deduction (2.4) for (2.3), we obtain

$$\mathcal{I}\psi(R(q_i,t), s(q_i,t)) + i\hbar\frac{\partial \psi(R(q_i,t), s(q_i,t))}{\partial s(q_i,t)} = 0\,, \tag{2.5}$$

which is in the form of eigenvalue equation. We deduce *Schrödinger unit operator* $\widehat{\mathcal{I}}$ [in the sense of Schrödinger's quantization rule] satisfying *unit eigenoperator equation* [17, Dwivedi 2005]

$$\widehat{\mathcal{I}}|\psi\rangle = \mathcal{I}|\psi\rangle\,; \qquad \widehat{\mathcal{I}} = -i\hbar\frac{\partial}{\partial s}\,. \tag{2.6}$$

Its expectation value is given by inner-product

$$\begin{aligned}\langle\widehat{\mathcal{I}}\rangle = \langle\psi|\widehat{\mathcal{I}}|\psi\rangle &= \int_{-\infty}^{+\infty} \psi^*(R(q_i,t), s(q_i,t)) \left(-i\hbar\frac{\partial \psi(R(q_i,t), s(q_i,t))}{\partial s(q_i,t)}\right) d\tau \\ &= \int_{-\infty}^{+\infty} |\psi(R(q_i,t), s(q_i,t))|^2\, d\tau = Prob.\,(-\infty,+\infty)\,.\end{aligned} \tag{2.7}$$

[it could also be obtained alternatively using (2.4) and (2.6) in inner-product (2.7).] The operator $\widehat{\mathcal{I}}$, having *trace* $Prob.\,(-\infty,+\infty)$, entails properties of our probability operator $\widehat{w}$. For a system in isolation:

$$\begin{cases} Prob.\,(-\infty,+\infty) = w_{pure} = 1 & \text{for pure state;} \\ Prob.\,(-\infty,+\infty) = w_{mixed}\ \epsilon\ (0,1) & \text{for mixed state;} \\ Prob.\,(-\infty,+\infty) = w_{forbidden} = 0 & \text{for forbidden state.} \end{cases} \tag{2.8}$$

Thus $\widehat{\mathcal{I}}$ is essentially $\widehat{w}$ that satisfies *probability eigenvalue equation*

$$\widehat{w}|\psi_w\rangle = w|\psi_w\rangle\,; \qquad \widehat{w} = -i\hbar\frac{\partial}{\partial s}\,. \tag{2.9}$$

Or

$$w\psi_w(R(q_i,t), s(q_i,t)) + i\hbar\frac{\partial \psi_w(R(q_i,t), s(q_i,t))}{\partial s(q_i,t)} = 0\,, \tag{2.10}$$

having solution

$$\psi_w(R(q_i,t), s(q_i,t)) = A\exp\left(\frac{i}{\hbar}ws(q_i,t)\right)\,. \tag{2.11}$$

For now we will treat ψ as function of s solely, for mere convention. For *orthonormalization* we have the inner-product,

$$\begin{aligned}\langle\psi_{w'}|\psi_w\rangle &= \int_{-\infty}^{+\infty} \psi^*_{w'}(s)\psi_w(s)\,ds \\ &= |A|^2\int_{-\infty}^{+\infty} \exp\left(\frac{i}{\hbar}(w-w')s\right)\,ds = |A|^2 2\pi\hbar\delta(w-w')\,.\end{aligned} \tag{2.12}$$

For $A = 1/\sqrt{2\pi\hbar}$, we have

$$\psi_w(s) = \frac{1}{\sqrt{2\pi\hbar}}\exp\left(\frac{i}{\hbar}ws\right) \tag{2.13}$$

that follows *Dirac orthonormality*

$$\langle\psi_{w'}|\psi_w\rangle = \delta(w-w')\,. \tag{2.14}$$

However, these eigenfunctions form **complete** set $(\psi = \sum_w c_w\psi_w)$. For (square-integrable) function $\psi(s)$,[2]

$$\psi(s) = \int_0^1 c(w)\psi_w(s)\,dw = \frac{1}{\sqrt{2\pi\hbar}}\int_0^1 c(w)exp\left(\frac{i}{\hbar}ws\right)\,dw\,. \tag{2.15}$$

The expansion coefficient is obtained by *Fourier's trick*

$$\langle\psi_{w'}|\psi\rangle = \int_0^1 c(w)\langle\psi_{w'}|\psi_w\rangle\,dw = \int_0^1 c(w)\delta(w-w')\,dw = c(w')\,, \tag{2.16}$$

or

$$c(w) = \langle\psi_w|\psi\rangle\,. \tag{2.17}$$

Exploiting completeness (2.15), the amplitude R in (2.1) is obtained

$$R = \frac{1}{\sqrt{2\pi\hbar}}\int_0^1 c(w)\exp\left(\frac{i}{\hbar}s(w-1)\right)\,dw\,. \tag{2.18}$$

3 Quantum Dynamical Equations

Dynamics is a law relating physical quantities in course of *time* (or some *internal observables* [28]). In Praxic theory $\mathcal{A}ction$ is a fundamental physical entity [23]. However, it could often be customary to deduce dynamics in course of $\mathcal{A}ction$. Let differentiate the inner-product,

$$\langle\widehat{\mathcal{A}}\rangle = \langle\psi|\widehat{\mathcal{A}}|\psi\rangle = \int_{-\infty}^{+\infty}\psi^*\widehat{\mathcal{A}}\psi\,d\tau\,, \tag{3.1}$$

exactly *w.r.t.* $\mathcal{A}$ction with differential-integral rule

$$\widehat{f}g(\kappa) = \widehat{f}\int_{-\infty}^{+\infty}\phi(\tau)\mathcal{K}(\kappa,\tau)\,d\tau = \int_{-\infty}^{+\infty}\widehat{f}\left\{\phi(\tau)\mathcal{K}(\kappa,\tau)\right\}\,d\tau\,, \tag{3.2}$$

we obtain (using chain rule for $\widehat{f} := \frac{\partial}{\partial s}$)

$$\frac{\partial}{\partial s}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{\partial\psi}{\partial s}|\widehat{\mathcal{A}}|\psi\rangle + \langle\psi|\frac{\partial\widehat{\mathcal{A}}}{\partial s}|\psi\rangle + \langle\psi|\widehat{\mathcal{A}}|\frac{\partial\psi}{\partial s}\rangle\,. \tag{3.3}$$

Considering probability eigenvalue equations

$$|\frac{\partial\psi}{\partial s}\rangle = \frac{i}{\hbar}|\widehat{w}\psi\rangle\,, \qquad \langle\frac{\partial\psi}{\partial s}| = -\frac{i}{\hbar}\langle\widehat{w}^\dagger\psi|\,, \tag{3.4}$$

we obtain

$$\frac{\partial}{\partial s}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{\partial\widehat{\mathcal{A}}}{\partial s}\rangle - \frac{i}{\hbar}[\langle\widehat{w}^\dagger\psi|\widehat{\mathcal{A}}|\psi\rangle - \langle\psi|\widehat{\mathcal{A}}\widehat{w}|\psi\rangle]\,. \tag{3.5}$$

Here $\mathcal{A}$, defined by $\mathcal{A} = \langle\psi|\widehat{\mathcal{A}}|\psi\rangle$, is a *dynamical* [28] — an observable-valued-function of system's variables — $\mathcal{A}(q_i,t)$ as distinct from observables. Since probability is a real aspect of nature, i.e., in operator representation, it must be hermitian[3],

$$\langle\widehat{w}^\dagger\psi|\widehat{\mathcal{A}}|\psi\rangle = \langle\psi|\widehat{w}\widehat{\mathcal{A}}|\psi\rangle\,, \tag{3.6}$$

which yields

$$\frac{\partial}{\partial s}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{\partial\widehat{\mathcal{A}}}{\partial s}\rangle - \frac{i}{\hbar}\langle[\widehat{w},\widehat{\mathcal{A}}]_-\rangle\,. \tag{3.7}$$

[2] As Probability does not exist in the limit $(-\infty,0)\,\mathbb{U}\,(1,+\infty)$, we have omitted integration over this limit. It does not create trouble in formalism.

[3] It also follows from counter-intuitive behavior of probability operator $\widehat{w}$.

This is first order *quantum dynamical equation.* Following the analogy, we further obtain second and third order quantum dynamical equations

$$\frac{\partial^2}{\partial s^2}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{\partial^2\widehat{\mathcal{A}}}{\partial s^2}\rangle - \frac{i}{\hbar}\left\langle\left\{[\widehat{w}, \frac{\partial\widehat{\mathcal{A}}}{\partial s}]_- + \frac{\partial}{\partial s}[\widehat{w}, \widehat{\mathcal{A}}]_- - \frac{i}{\hbar}[\widehat{w}, [\widehat{w}, \widehat{\mathcal{A}}]_-]_-\right\}\right\rangle , \tag{3.8}$$

and

$$\begin{aligned}\frac{\partial^3}{\partial s^3}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{\partial^3\widehat{\mathcal{A}}}{\partial s^3}\rangle - \Bigg\langle\Bigg\{&[\widehat{w}, \frac{\partial^2\widehat{\mathcal{A}}}{\partial s^2}]_- + \frac{\partial}{\partial s}[\widehat{w}, \frac{\partial\widehat{\mathcal{A}}}{\partial s}]_- + \frac{\partial^2}{\partial s^2}[\widehat{w}, \widehat{\mathcal{A}}]_- \\ &- \frac{i}{\hbar}\Bigg([\widehat{w}, [\widehat{w}, \frac{\partial\widehat{\mathcal{A}}}{\partial s}]_-]_- + [\widehat{w}, \frac{\partial}{\partial s}[\widehat{w}, \widehat{\mathcal{A}}]_-]_- + \frac{\partial}{\partial s}[\widehat{w}, [\widehat{w}, \widehat{\mathcal{A}}]_-]_- \\ &- \frac{i}{\hbar}[\widehat{w}, [\widehat{w}, [\widehat{w}, \widehat{\mathcal{A}}]_-]_-]_-\Bigg)\Bigg\}\Bigg\rangle .\end{aligned} \tag{3.9}$$

For operators ($\frac{\partial^n\widehat{\mathcal{A}}}{\partial s^n}$, $n = 0, 1, 2, ...$) compatible with $\widehat{w}$, these equations follow Ehrenfest's theorem

$$\frac{\partial^n\langle\widehat{\mathcal{A}}\rangle}{\partial s^n} = \langle\frac{\partial^n\widehat{\mathcal{A}}}{\partial s^n}\rangle . \tag{3.10}$$

It holds good for observables having simultaneous eigenstates with probability w.

4 Generalized Uncertainty Principle

The hermitian observables s and w are *incompatible* as operators $\widehat{s}$ and $\widehat{w}$ do not commute. As a consequence of incompatibility, s and w can not have simultaneous (shared) eigenfunction. The eigenvalue equations are $\widehat{s}\psi_s = s\psi_s$ and $\widehat{w}\psi_w = w\psi_w$ provided $\psi_s \neq \psi_w$. Thus, s and w could not be measured *simultaneously* for a given eigenstate. However, there appears uncertainty between *existence* and *measure of $\mathcal{A}$ction.* The expectation value of their commutator is given by inner-product

$$\langle\phi|[\widehat{s}, \widehat{w}]_-\phi\rangle = i\hbar\langle\phi|\phi\rangle , \tag{4.1}$$

for some arbitrary wavefunction $\phi := \phi(s)$. The commutator

$$[\widehat{s}, \widehat{w}]_- = i\hbar \tag{4.2}$$

yields measure of uncertainty between s and w by virtue of generalized uncertainty principle [30]

$$\sigma_s^2\sigma_w^2 \geq \left(\frac{\langle[\widehat{s}, \widehat{w}]_-\rangle}{2i}\right)^2 , \tag{4.3}$$

or

$$\sigma_s\sigma_w \geq \frac{\hbar}{2} . \tag{4.4}$$

This is similar to Heisenberg's uncertainty relations [32, 33]

$$\begin{cases} \Delta q\Delta p \geq \hbar/2 , \\ \Delta E\Delta t \geq \hbar/2 . \end{cases} \tag{4.5}$$

As consequence: for a pure state ($w = 1, \sigma_w = 0$) the spread (uncertainty) in measurement of s is $\sigma_s = \infty$. However, for a pure state, $\mathcal{A}$ction s can not be measured with certainty, and wavefunction (2.1) can not be constructed out of it. It does hold for forbidden state too. On the contrary, for a mixed state (a poorly or statistically defined state) ($w \,\epsilon\, (0, 1), \sigma_w \,\epsilon\, (0, \infty)$) $\mathcal{A}$ction can be measured with maximum precision limit

$$\boxed{\Delta s\Delta w \geq \frac{\hbar}{2}} . \tag{4.6}$$

However, for a mixed state, wavefunction can be constructed with a maximum precision (4.6). It follows that in Ontic theory mixed state is more predictable than a pure state, despite its poor statistical definition.

My uncertainty relation (4.6) is not contrary to Heisenberg's assertions for measurement of "*momentum* and *position*" or "*energy* and *time*". By inspection, if measurement of $\mathcal{A}$ction may be certain, then all dynamical quantities (energy, momentum,...) may be derived *simultaneously* from $\mathcal{A}$ction using *Euler-Lagrange* and *Hamilton-Jacobi* equations. However, it would destroy all uncertainty relations (4.5) (as quantities appear simultaneously) that contradicts usual quantum theory. Our assertion that *$\mathcal{A}$ction can not be certainly (strictly) measured for a system* followed by uncertainty limit (4.6) overcomes this contradiction. $\mathcal{A}$ction is a *hidden variable* for systems in pure state and eventually:

$$\boxed{\psi \textit{ is hidden variable for systems in pure state.}}$$

The generalized uncertainty relation (4.6) can be decisive to deal with problems underlying mixed state (as there is higher predictability than pure state). For example, quantum phase transition. In transition period, state of system is mixed and a (pre or pro) pure state is not certain (strictly defined) during transition. We can contemplate (4.4) to deal with quantum phase transitions.

5 Ultimate Indeterminism

A quantum is neither a *granule* nor a *wave*. These features are its *partial* characteristics that show agreement with particular situations (experiments) when any of it is more apparent (dominant). On the contrary, we can still look for another peculiar kind of such feature, alien to physics.

Wavemechanics elaborated *equivalence* between Hamilton's principle (particle, granule) and Fermat's principle (wave, ray) in a unified manner: matter-wave, and replaced the constant under study with planck's constant [43, Schrödinger 1933]. Nevertheless, the theory thus evolved was more visualized and explained phenomena adequately, but still uses classical ideas. A wave does not introduce a new kind of *entity*. It is based upon classical idea of a particle [in the sense of localization]. Thus wavemechanics is an Ontic theory that assumes absolute reality; state of being. The reason for its enormous success in quantum domain is that *wavelength* introduced by matter-wave hypothesis is of *order* of atomic diameter [43]. On the contrary, matrixmechanics was less visualized [34] but yet more valid ideologically.

Praxic theory entails fundamental physical entity — *action* — which is quantized. A *measurement* exchanges *quanta* (nh) with the system. A subsequent measurement exchanges few more quanta and we are led to a new — *perturbed* — observation. Uncertainty arises from the theory itself, not from perturbation of measurement by observer [45, Weizsäcker 2006, ch. 9, p. 252]. A *minimum* uncertainty is *indispensable* because it appears due to *quantization of action* [34]. In classical domain *discreteness* goes over to *smoothness* and exchange of quanta during measurement vanishes with good approximation, and we are led to precisely same observation in repeated measurements. In quantum domain a *minimum uncertainty* is *inevitable* even though measurement were ultimately *precise* [Bohr]. Heisenberg suggested to exclude (minimum) uncertainty by merging object, observation, observer into one (whole) object-observation-observer [34, Heisenberg 1933]. It cries out for another relativization [23, Finkelstein 1996]. Nevertheless, I urge that (a minimum) uncertainty is indispensable and can only be excluded by means of changing postulates of the theory; not by relativizing it. Such a change would lead to yet another theory, and it remains that uncertainty can be eliminated from quantum theory.

Einstein suggested *hidden-variables* at sub-quantum level that proceed more rapidly than quantum, in order to overcome with uncertainty, but no such theory has yet been developed satisfactorily [7, Bohm 1980, ch. 4].

6 Conclusion

In *objective vision* we can learn about a system without interacting with it. It is similar to paint a portrait in a dark room. In this context — objective reality — seems orthodoxical. We ought to change the definition of reality, rather than fitting a theory with element of reality previously defined. Heisenberg called his quantum theory non-objective. In quantum domain *measurement* occurs via *interaction* with the system. The fundamental *entity* of interaction is a *signal*, which is quantized. A reality comes out to be what we *observe*, not what *exists*. In this connection, observation with observer itself is not *separate* from the system being observed. An *observation* depends upon the *frame of experiment* (explicitly on observer), not system's *state of being*. Epistemology changes. The fundamental physical entities are *actions*, *operations* and *processes*, not state of being. Instead of claiming *incompleteness* of the wavefunction [19], we emphasize that *state* can not be constructed out of it. Wavefunction is a catalogue of knowledge (probability) not objective reality [45]. The system does not has ψ whether it be interpreted as *state vector* [Almost majority], *mode vector* [23, Finkelstein 1996] or *channel* [4, Blatt and Weisskopf 1952].

Appendix

A Unit Operator

Unit operator (eigenoperator), analogous to *identity matrix*, is deduced as a zero-order (ordinary or partial) differential operator (irrespective of with respect to what) defined as

$$\mathcal{I} := \partial_x^0 = \frac{\partial^0}{\partial x^0}\,; \qquad (x = q, p, t, ...)\,. \tag{A.1}$$

We have observed in *mathematical analysis* that a zero-order differential operator does not change the function to which it is applied which leads to deduce it unit operator satisfying $\mathcal{I}f = f$. For example, in Ostrogradsky transformation, zero-order prime of generalized co-ordinate $\overset{(n)}{q}$, $(n = 0, 1, 2, 3, ...)$ for $n = 0$ is given by q. It may be extended to $\overset{(n)}{q} = \mathcal{I}q = q$ for $n = 0$ with $\mathcal{I} := \partial_t^0$. The deduction is less applicable in mathematical analysis but is very important to deal with quantum problems. Unit operator is quantized to $\widehat{\mathcal{I}} := -i\hbar\frac{\partial}{\partial s}$ satisfying unit eigenoperator equation $\widehat{\mathcal{I}}|\psi\rangle = \mathcal{I}|\psi\rangle$ while treating quantum problems. For example, a quantum transformation with $\overset{(n)}{\psi}$, $(n = 0, 1, 2, 3, ...)$ (being n^{th}-order partial derivative of ψ w.r.t. any variable x) is extended for $n = 0$, $\overset{(n)}{\psi} = \mathcal{I}\psi = \psi$ with $\mathcal{I} := \partial_x^0$. This is a quantum problem and we quantize $\mathcal{I}$ to $\widehat{\mathcal{I}}$ which yields $\overset{(n)}{\psi} + i\hbar\frac{\partial\psi}{\partial s} = 0$, for $n = 0$.

B Mode Vector

An state $|\psi\rangle \;\epsilon\; \mathbb{H}$ in Ontic theory is entailed *definite* for an operator $\widehat{\mathcal{A}}$, that transforms it into a *hermitian observable* α via linear equation $\widehat{\mathcal{A}}|\psi\rangle = \alpha|\psi\rangle$. On the contrary, $|\psi\rangle$ does not transform operator $\widehat{\mathcal{B}}$ (non-commutative with $\widehat{\mathcal{A}}$; $[\widehat{\mathcal{A}}, \widehat{\mathcal{B}}]_- \neq 0$) into its *trace* β : $\widehat{\mathcal{B}}|\psi\rangle \neq \beta|\psi\rangle$. $|\psi\rangle$ is not a definite state for $\widehat{\mathcal{B}}$. Instead of assigning a *definite state* to an operator, we entail *channel* (or *mode of action*) $|\psi\rangle$ that transforms both operators, being definite state of neither, in following manner:

$$\begin{aligned} \widehat{\mathcal{A}}|\psi\rangle &= \{\alpha \pm \sigma_\alpha\}|\psi\rangle\,; \\ \widehat{\mathcal{B}}|\psi\rangle &= \{\beta \pm \sigma_\beta\}|\psi\rangle\,, \end{aligned} \tag{B.1}$$

satisfying

$$\sigma_\alpha \sigma_\beta \geq \frac{\langle[\widehat{\mathcal{A}}, \widehat{\mathcal{B}}]_-\rangle}{2i}\,. \tag{B.2}$$

However, we exclude *definite state* with *channel*, (much as *mode vectors* excluded *state vectors* [23, 24, 25, 26, 27]) without changing postulates of the theory. The rest of result agrees with usual quantum theory. It transforms *Ontic theory* to *Praxic theory*. Nevertheless, quantum systems have neither *state vectors* nor *mode vectors* [27], nor *channels* [4].

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References

[1] George B. Arfken and Hans J. Weber. *Mathematical Methods for Physicists*. Academic Press, New York, 6th edition, 2005.

[2] Bell et al. *John S. Bell on the Foundations of Quantum Mechanics*. World Scientific, Singapore, 2001.

[3] P. G. Bergmann. The quantum state vector and physical reality. chapter 3. In M. Bunge, editor, *Quantum Theory and Reality*, volume 2 of *Studies in the Foundations, Methodology, and Philosophy of Sciences*. Springer, Heidelberg, 1967.

[4] J. M. Blatt and V. F. Weisskopf. *Theoretical Nuclear Physics*. Wiley, 1952.

[5] David Bohm. *Quantum Theory*. Prentice Hall, New York, 1951.

[6] David Bohm. *Causality and Chance in Modern Physics*. Routledge and Kegan Paul, London, 1957.

[7] David Bohm. *Wholeness and the Implicate Order*. Routledge and Kegan Paul, London, 1980.

[8] Niels Bohr. The structure of the atom. *Nobel Lecture*, 1922.

[9] N. Bohr. Quantum mechanics and physical reality. *Phys. Rev.*, 48:696, 1935.

[10] Max Born and P. Jordan. *Zeitschrift für Physik*, 34:858, 1925.

[11] Max Born and N. Wiener. *Zeitschrift für Physik*, 36:174, 1926. In this work authors replaced matrix by the general concept of an operator, and thus made it possible to describe aperiodic processes.

[12] Max Born. The statistical interpretation of quantum mechanics. *Nobel Lecture*, 1954. See further references therein.

[13] Louis de Broglie. The wave nature of the electron. *Nobel Lecture*, 1929.

[14] P.A.M. Dirac. Theory of electrons and positrons. *Nobel Lecture*, 1933.

[15] P.A.M. Dirac. *The Principles of Quantum Mechanics*. Oxford University Press, 1947.

[16] Michael Drieschner. *Int. J. Theor. Phys.*, 31(1), 1992.

[17] Saurav Dwivedi. The eigenoperator formalism. 2005. submitted to *Int. J. Theor. Phys.* [IJTP **437**]. Online: http://www.dwivedi.bravehost.com/data/p9.pdf.

[18] Saurav Dwivedi. Probabilistic interpretation of quantum mechanics with schrödinger quantization rule. 2005. submitted to *Int. J. Theor. Phys.* [IJTP **524**]. Preprint: http://www.vixra.org/abs/1006.0019.

[19] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47:777, 1935.

[20] R.P. Feynman. The development of the space-time view of quantum electrodynamics. *Nobel Lecture*, 1965.

[21] R.P. Feynman and A.R. Hibbs. *Quantum Mechanics and Path Integrals.* McGraw-Hill, New York, 1965.

[22] R.P. Feynman, R.B. Leighton, and M. Sands. *The Feynman Lectures on Physics*, volume 3. Addison-Wesley, 1965.

[23] David Finkelstein. *Quantum Relativity.* Springer, Heidelberg, 1996.

[24] David Ritz Finkelstein. Action physics. *Int. J. Theor. Phys.*, 38(1), 1999.

[25] David Ritz Finkelstein. The qubits of qunivac. *Int. J. Theor. Phys.*, 42(2), 2003. arXiv: hep-th/0206036.

[26] D. R. Finkelstein. The state of quantum physics. *Notes*, 2005.

[27] David Ritz Finkelstein. General quantization. *Int. J. Theor. Phys.*, 45(8), 2006. arXiv: quant-ph/0601002.

[28] David Ritz Finkelstein. *Whither Quantum Theory?* Essays in Honour of David Speiser, Two Cultures. Birkhäuser Verlag, Berlin, 2006.

[29] H. Goldstein. *Classical Mechanics.* Addison-Wesley, Reading, MA., 1980.

[30] David J. Griffiths. *Introduction to Quantum Mechanics.* Pearson Education, 2nd edition, 2005.

[31] Stanley Gudder. *Int. J. Theor. Phys.*, 31(1), 1992.

[32] Werner Heisenberg. *Zeitschrift für Physik*, 33:879, 1925.

[33] Werner Heisenberg. *Zeitschrift für Physik*, 43:172, 1927.

[34] Werner Heisenberg. The development of quantum mechanics. *Nobel Lecture*, 1933.

[35] Larry Horwitz. Private communication. 2010.

[36] Max Jammer. *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective.* John Wiley & Sons Inc, 1974.

[37] J. von Neumann. *Mathematical Foundations of Quantum Mechanics.* Princeton University Press, 1955.

[38] Roger G. Newton. *Quantum Physics, A Text for Graduate Students.* Springer International Edition, 2002.

[39] Wolfgang Pauli. Exclusion principle and quantum mechanics. *Nobel Lecture*, 1946.

[40] Max Planck. The genesis and present state of development of the quantum theory. *Nobel Lecture*, 1920.

[41] M. Reed and B. Simon. *Methods of Modern Mathematical Physics.* Academic Press, New York, 1978.

[42] Erwin Schrödinger. *Annalen der physik*, 79, 80, 81, 1926. These papers naively ascertain de Broglie's ideas of wave-corpuscle duality under the influence of a constraint, and thus, are advent of Wave-Mechanics.

[43] Erwin Schrödinger. The fundamental idea of wave mechanics. *Nobel Lecture*, 1933.

[44] Gerald Teschl. *Mathematical Methods in Quantum Mechanics; With Applications to Schrödinger Operators.* Graduate Studies in Mathematics. Amer. Math. Soc., Providence, 2009.

[45] Carl Friedrich von Weizsäcker, Thomas Görnitz, and Holger Lyre. *The Structure of Physics.* Fundamental Theories of Physics. Springer, Netherlands, 2006. Specially ch. 9. The problem of the interpretation of quantum theory.

[46] Günther Ludwig and Gérald Thurler. *A New Foundation of Physical Theories*. Springer-Verlag, Berlin, 2006.

[47] Matej Pavsic. *Nobody Really Understands Quantum Mechanics, Ch. 12.* The Landscape of Theoretical Physics: A Global View. Kluwer Academic Publishers, Dordrecht, Netherlands, 2001. Preprint: arXiv: gr-qc/0610061.