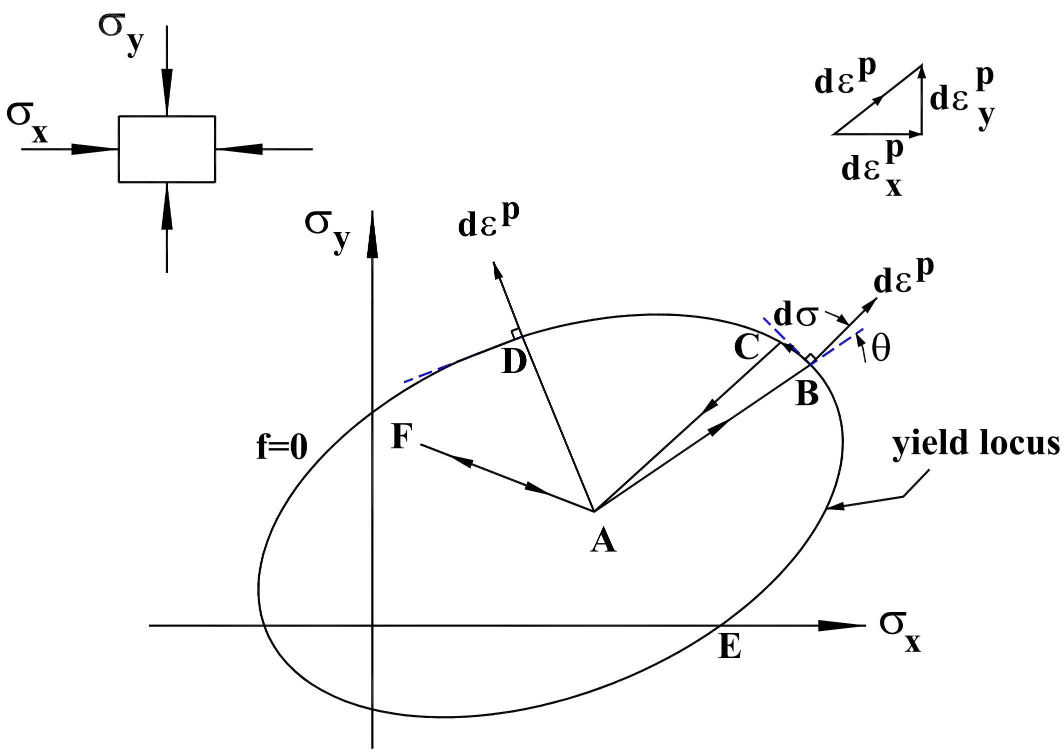
PLASTIC THEORY

YIELD LOCUS AND PLASTIC THEORY



Stress state $(\sigma_x, \sigma_y, \tau_{xy})$ inside the yield locus correspond elastic behaviour of material

If the external load is applied to the material so that the stress state moves to point B on the yield locus, the material is yielding and starts plastic flow / plastic property.

$$\delta\varepsilon^p = (\delta\varepsilon_x{}^p, \delta\varepsilon_y{}^p, \delta\gamma_{xy}{}^p)$$

When the stress state moves from point B to C :

Plastic Work Increment, $dW^p = \left(\sigma_{ij}^C - \sigma_{ij}^B\right) d\varepsilon_{ij}^p \geq 0$

$dW^p = d\varepsilon_{ij}^p d\sigma_{ij} \geq 0$ ⟶ Drucker's stability postulate

To ensure Drucker's stability postulate is obeyed:

1) $\delta\varepsilon_{ij}^p = \lambda \dfrac{\partial f}{\partial \sigma_{ij}}$ ⟵ The plastic strain increment vector, $\delta\varepsilon^p$, is normal to yield locus (normality rule)

where $\lambda>0$ is a positive scalar proportionality factor

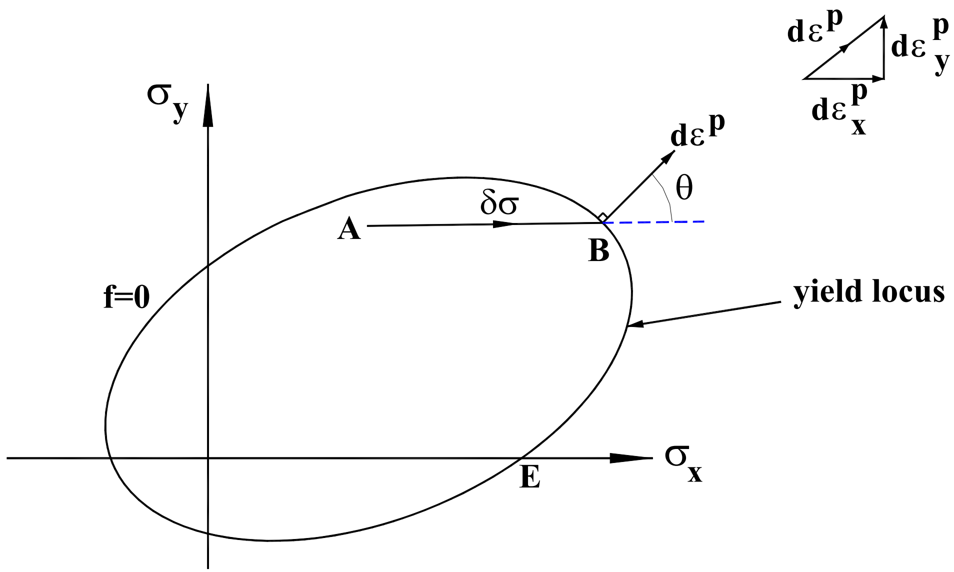
2) Yield locus should convex

Under perfectly plastic model:

$$d\varepsilon_{ij}^p d\sigma_{ij} = 0$$

Thus, stress increment vector, $d\sigma_{ij}$, is perpendicular to the strain increment vector, $d\varepsilon_{ij}{}^p$ and its direction is parallel to tangential line on the yield locus → stress state can only moves along the yield locus and can not excluded beyond the region of yield locus.

Proof: Normality Rule



$$dW^p = d\varepsilon_{ij}^p \delta\sigma_{ij}$$

$$= d\varepsilon . \delta\sigma$$

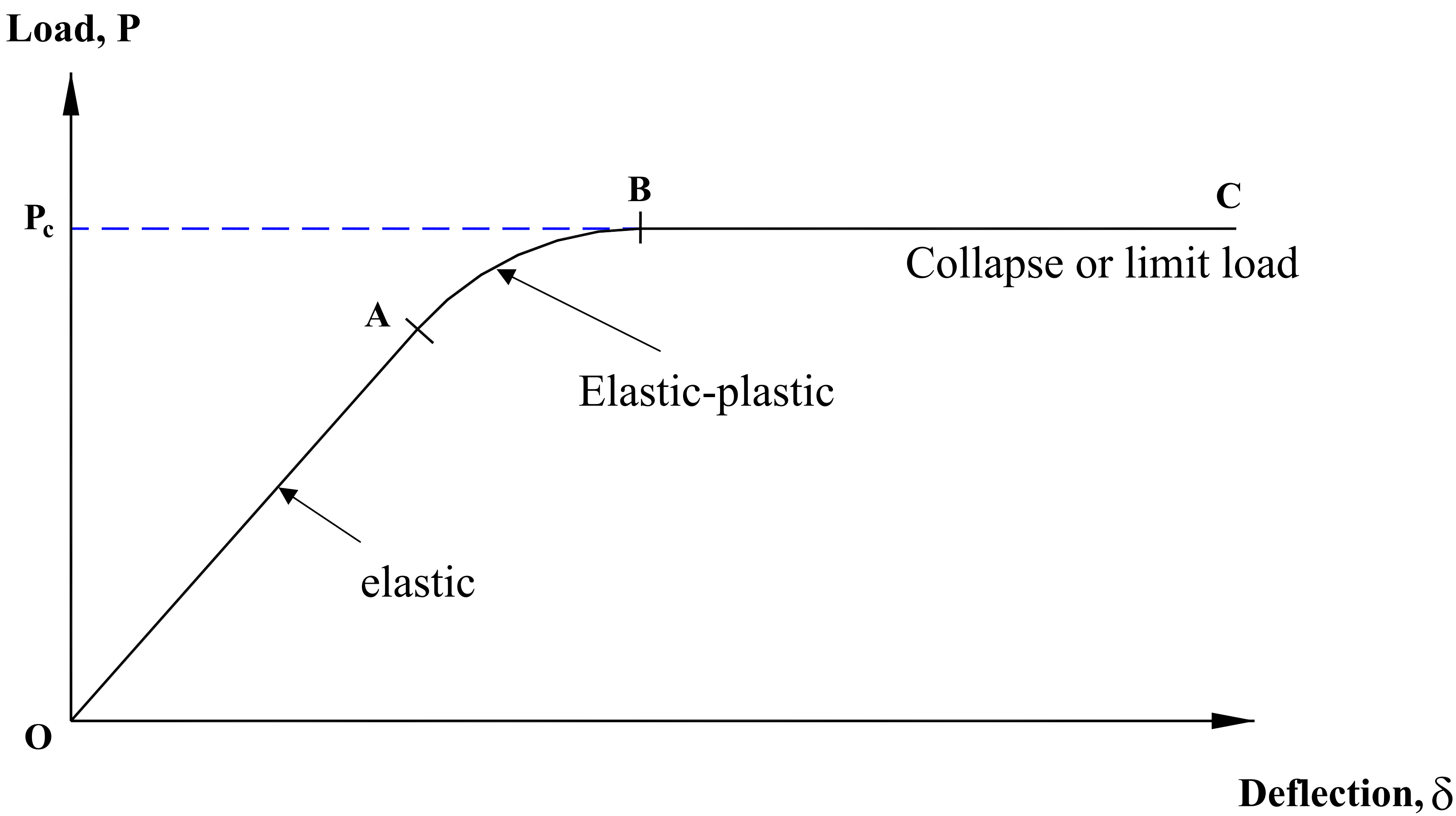
$$= |d\varepsilon||\delta\sigma|\cos(\theta)$$

To ensure $dW^p \geq 0$ ➔ $\theta \leq 90°$ ➔ Yield locus should convex and $d\varepsilon^p$ should normal to yield locus

THEOREM OF LIMIT LOAD

- There exists physical collapse load
- Displacements can increase without limit while the load is held constant.
- A load computed on the basis of this ideal situation is called plastic limit load or collapse load.
- This hypothetical limit load usually gives a good approximation to the physical plastic collapse load or the load at which deformations become excessive.

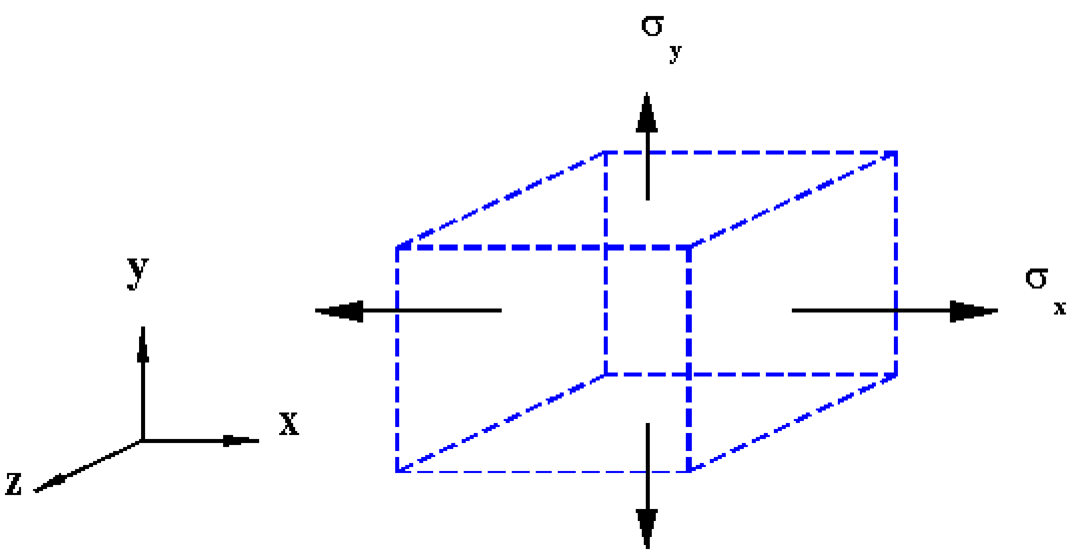
THEOREM OF LIMIT LOAD

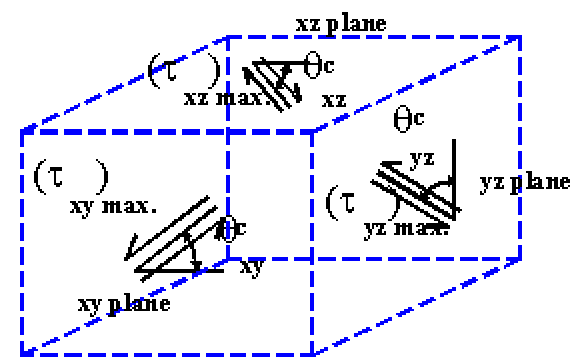


The curve consists of an elastic portion, a region of transition from mainly elastic to mainly plastic behaviour, a plastic region, in which the load increases very little while the deflection increases manifold.

TRESCA CRITERIA

The plastic yielding is assumed take placed when the shear stress at any points in the structural element reach a critical value. This type of criteria is suitable to use for isotropic material.



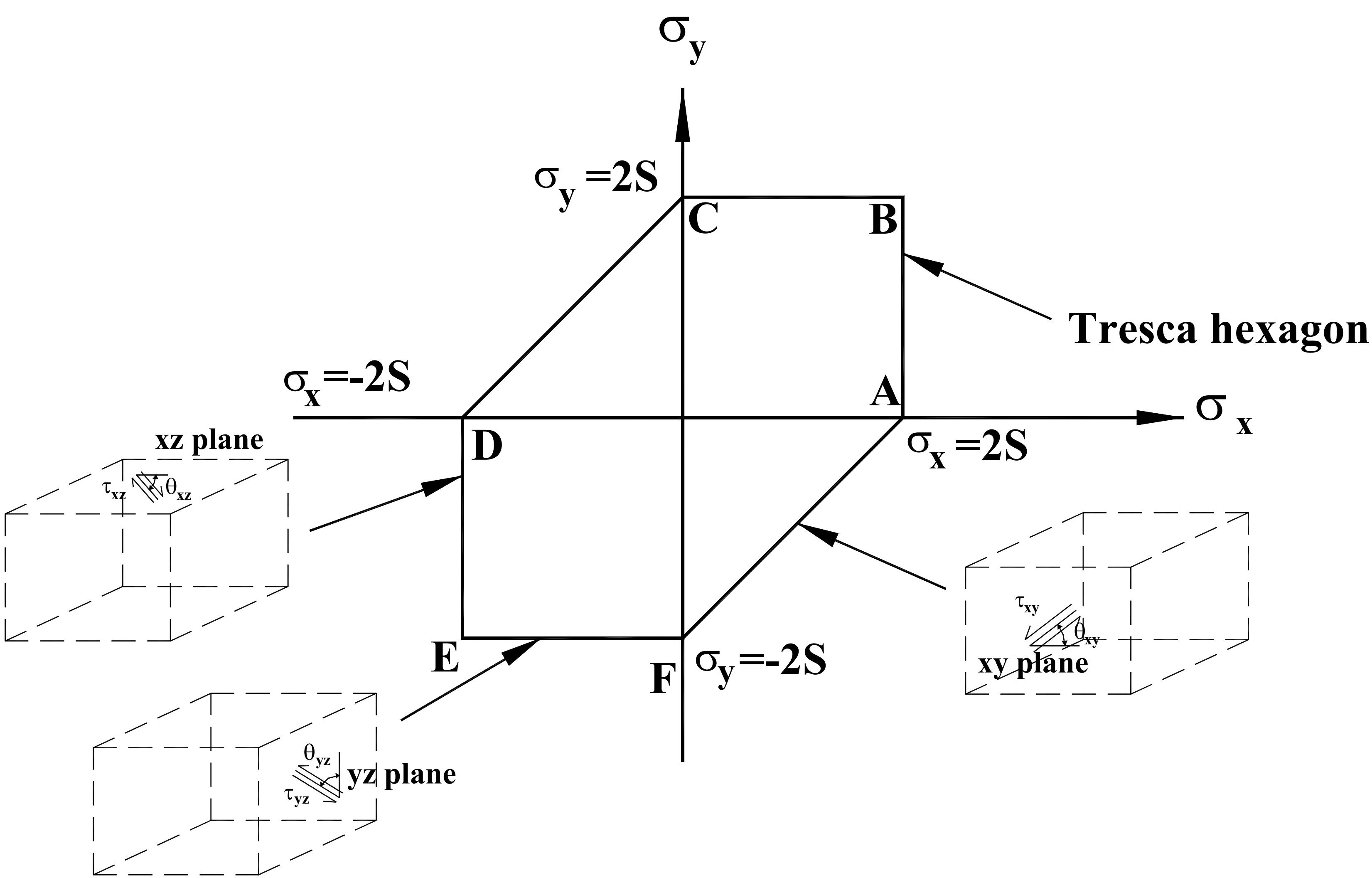


Maximum shear stresses in xy, yz and xz planes:

$$(\tau_{xy})_{max.} = \frac{1}{2}(\sigma_x - \sigma_y) \qquad (\tau_{yz})_{max.} = \frac{1}{2}\sigma_y \qquad (\tau_{xz})_{max.} = \frac{1}{2}\sigma_x$$

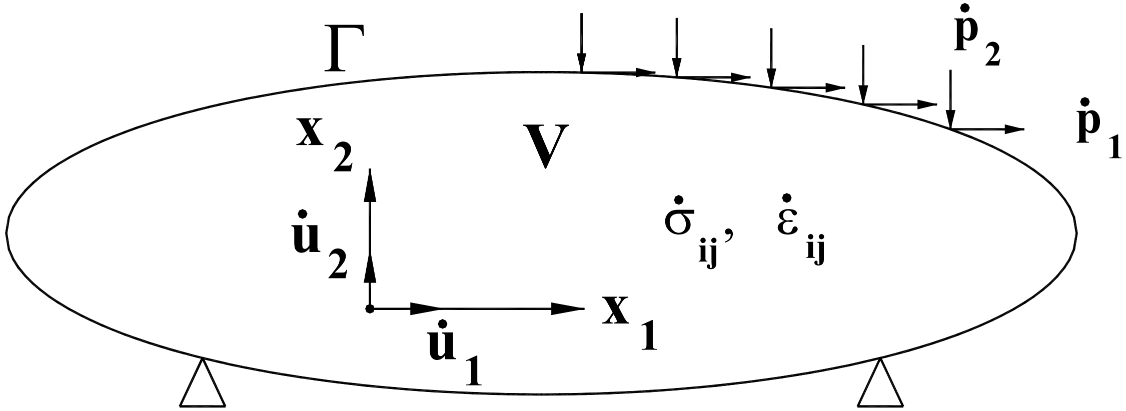
All $\theta_{xy}{}^c$, $\theta_{yz}{}^c$, $\theta_{xz}{}^c = 45°$

TRESCA CRITERIA

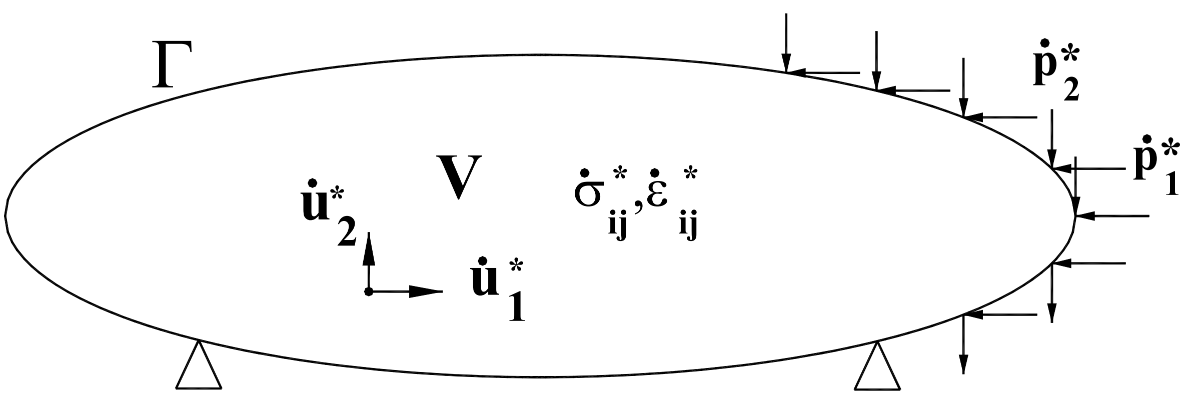


Under a system of stresses σ_x and σ_y, the material is assumed yield if the maximum shear stress at any planes reaches a critical shear stress value, S.

VIRTUAL WORK METHOD (BASIC PLASTIC ANALYSIS)



(a) Actual System



(b) Virtual System

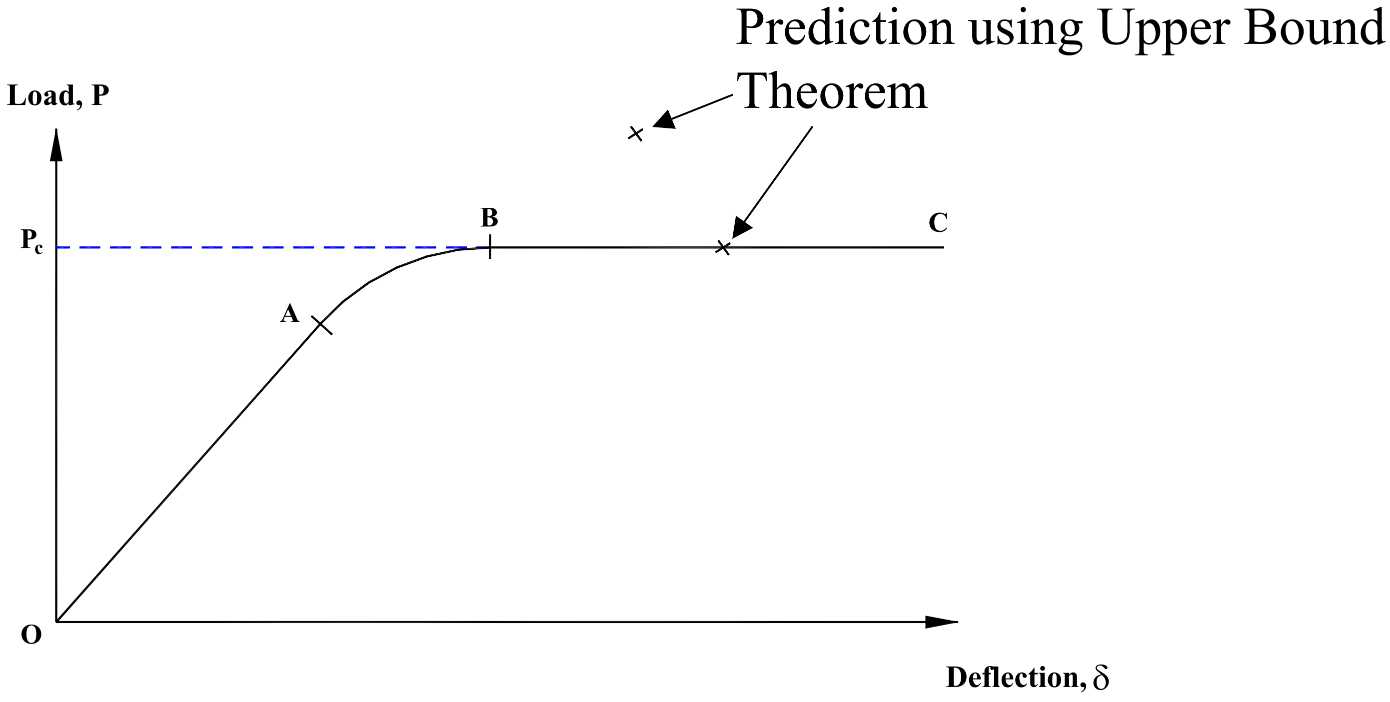
External Virtual Work = Internal Virtual Work

$$\int_S \dot{p}_i \dot{u}_i^* dS + \int_V \dot{b}_i \dot{u}_i^* dV = \int_V \dot{\sigma}_{ij} \dot{\varepsilon}_{ij}^* dV$$

THEOREM OF PLASTIC THEORY

THE 'UPPER BOUND' (UNSAFE) OR KINEMATICAL THEOREM

"If an estimate of the plastic collapse load of a body is made by equating internal rate of dissipation of energy to the rate at which external forces do work in any postulated mechanism of deformation of the body, the estimate will be either be higher, or correct" *(Calladine)*

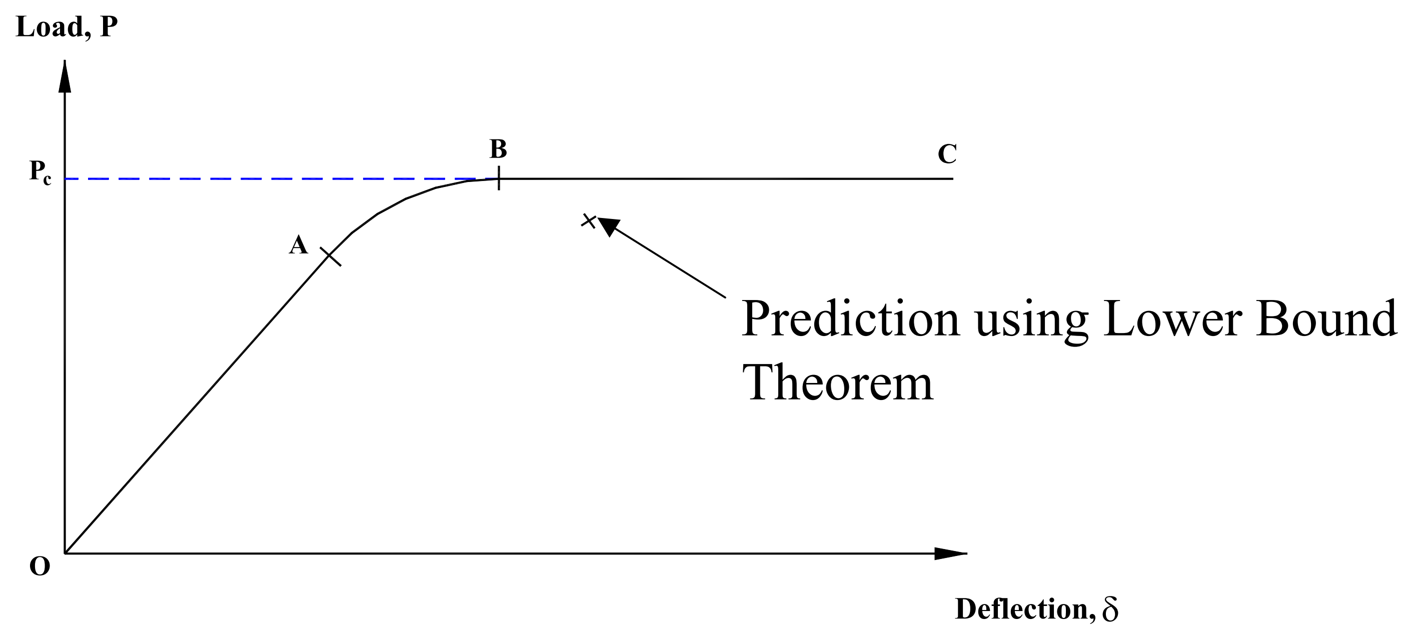


Proof: refer to "Limit Analysis and Soil Plasticity" by Wai Fah Chen

THEOREM OF PLASTIC THEORY (CONTINUED)

THE 'LOWER BOUND' (SAFE) OR STATICAL THEOREM

"If any stress distribution throughout the structure can be found which is everywhere in equilibrium internally and balances certain external loads at the same time does not violate the yield condition, those loads will be carried safely by the structure" *(Calladine)*

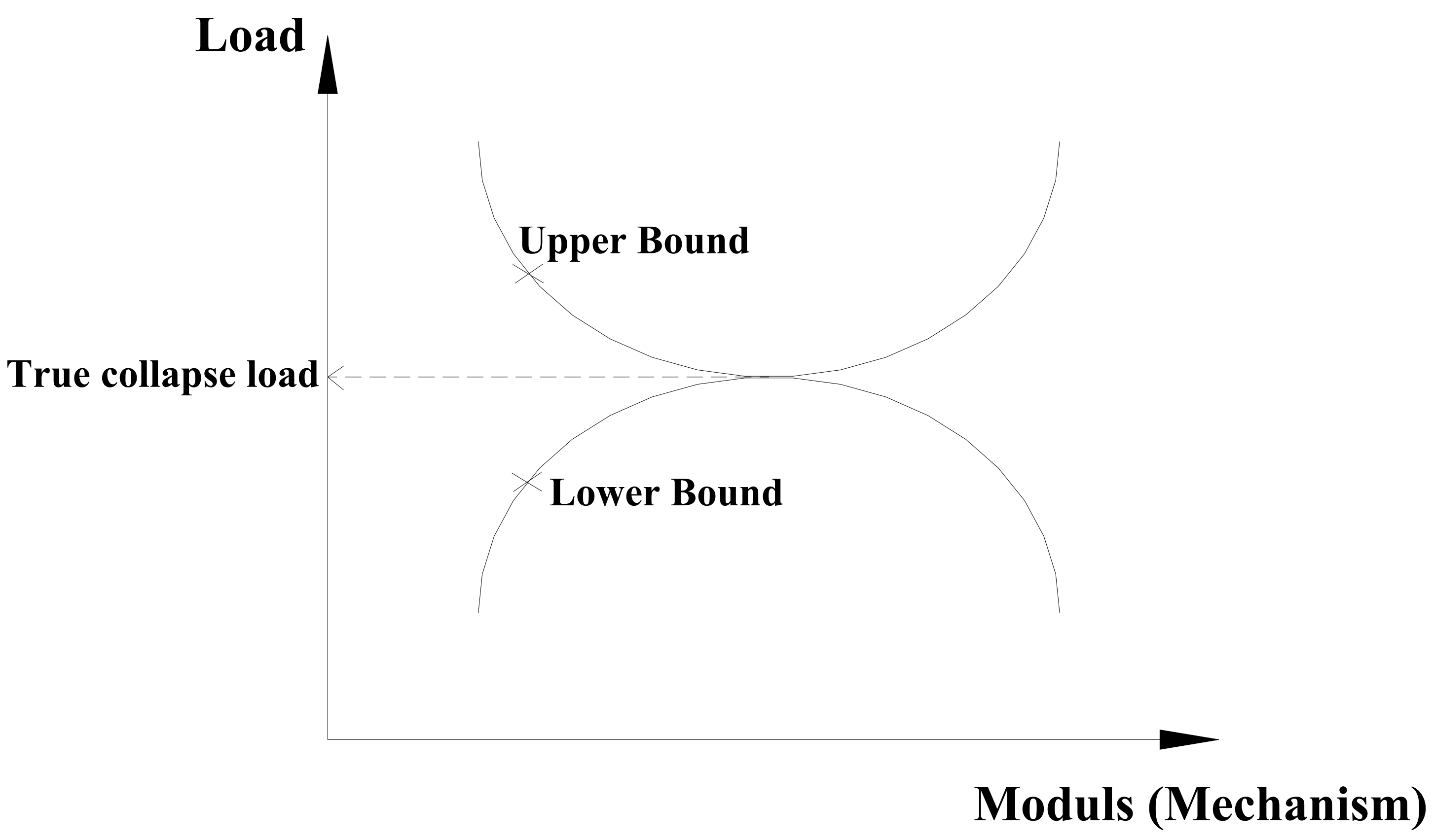


Proof: refer to "Limit Analysis and Soil Plasticity" by Wai Fah Chen

THEOREM OF PLASTIC THEORY (CONTINUED)

THE ‘UNIQUENESS’ OR EXACT THEOREM

When the two solutions coincide or equal then the solution is an exact one.



YIELD LINE ANALYSIS

- Slab and Plate
- Loading: Transverse load
- Analysis for ultimate load or collapse load
- Material behave as rigid plastic