INFLUENCE LINE

Reference:

Structural Analysis

Third Edition (2005)

By

Aslam Kassimali

DEFINITION

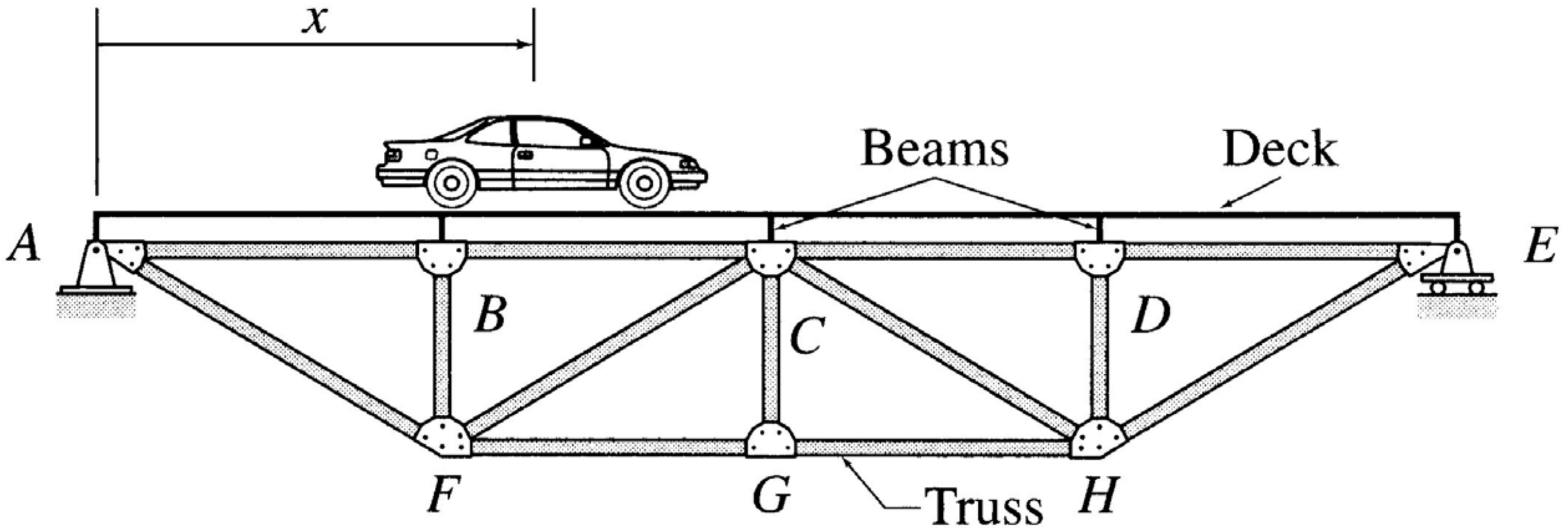


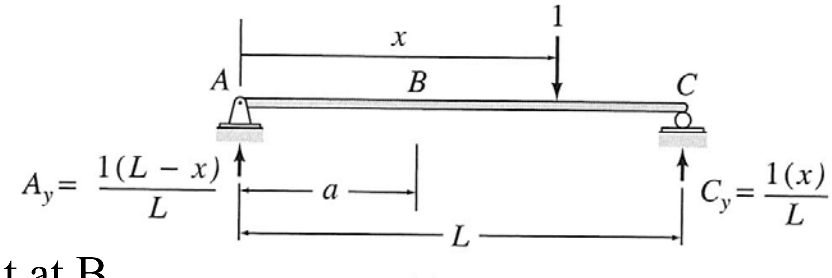
FIG. 8.1

An influence line is a graph of a response function of a structure as a function of the position of a downward unit load moving across the structure

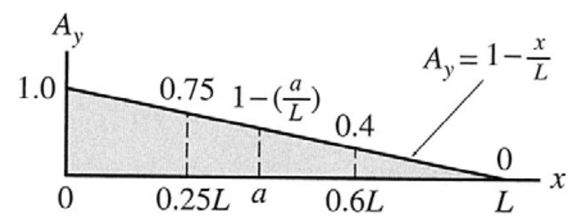
INFLUENCE LINES FOR BEAMS AND FRAMES BY EQUILIBRIUM METHOD

Influence Line for:

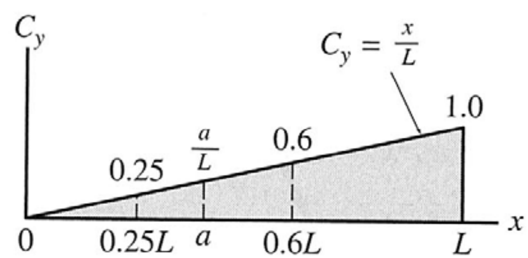
a) Reactions
b) Shear at B
c) Bending moment at B



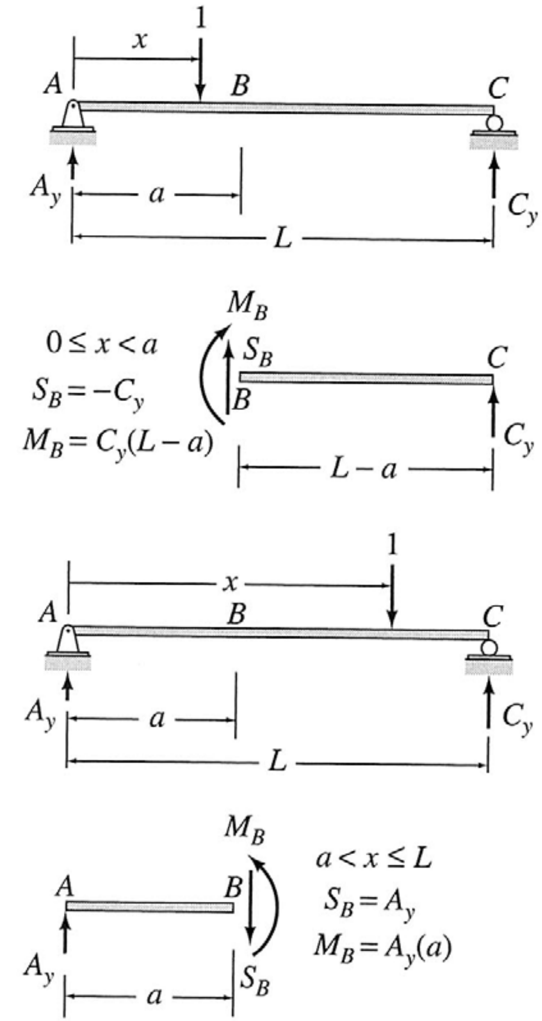
(a)



(b) Influence Line for A_y

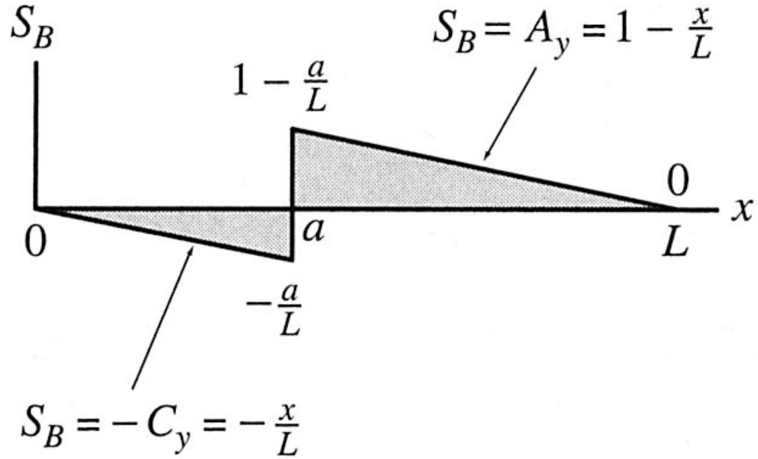


(c) Influence Line for C_y



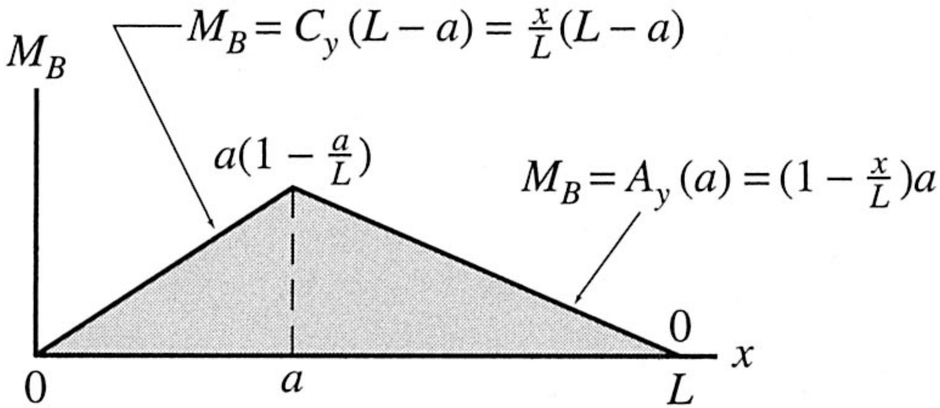
(d)

FIG. 8.2



(e) Influence Line for S_B

$$S_B = \begin{cases} -C_y = -\dfrac{x}{L}, & 0 \le x < a \\ A_y = 1 - \dfrac{x}{L}, & a < x \le L \end{cases}$$



(f) Influence Line for M_B

$$M_B = \begin{cases} C_y(L-a) = \dfrac{x}{L}(L-a), & 0 \le x \le a \\ A_y(a) = \left(1 - \dfrac{x}{L}\right)a, & a \le x \le L \end{cases}$$

FIG. 8.2 (contd.)

Example 8.1

Draw the influence lines for the vertical reactions at supports A and C, and the shear and bending moment at point B, of the simply supported beam shown in Fig. 8.3(a).

Influence line for A_y

$$\sum M_c = 0:$$

$$-A_y(20) + 1(20 - x) = 0$$

$$A_y = \frac{1(20-x)}{20} = 1 - \frac{x}{20} \longrightarrow \text{Fig. 8.3(c)}$$

Influence line for C_y

$$\sum M_A = 0:$$

$$-1(x) + C_y(20) = 0$$

$$C_y = \frac{1(x)}{20} = \frac{x}{20} \longrightarrow \text{Fig. 8.3(d)}$$

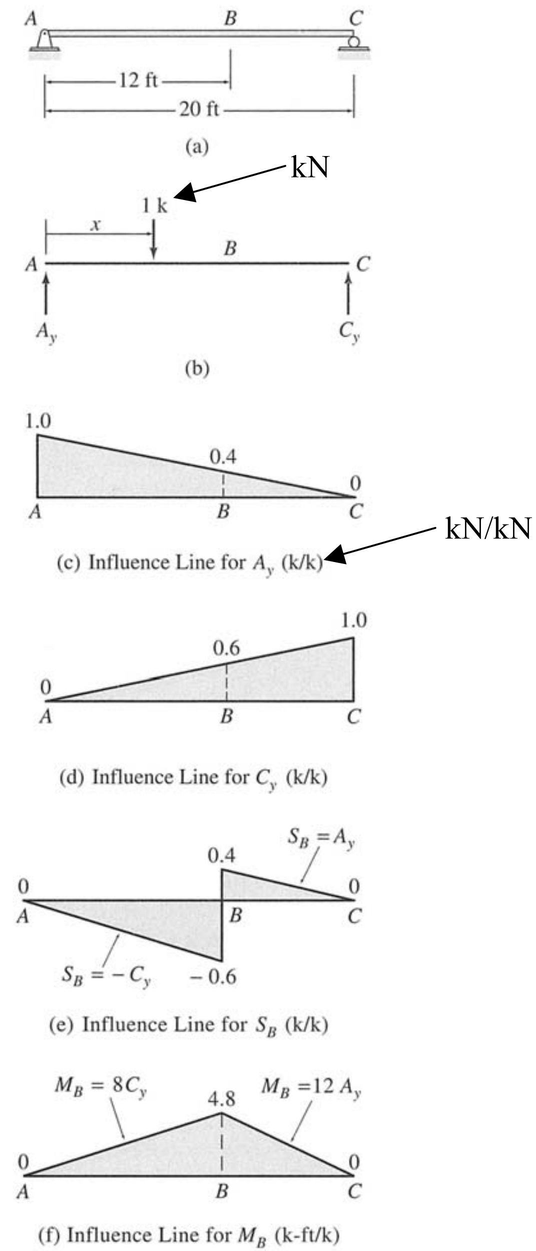


FIG. 8.3

Influence line for S_B

Place the unit load to the left of point B, determine the shear at B by using the free body of the portion BC:

$$S_B = -C_y \qquad 0 \le x < 12\,ft$$

Place the unit load to the right of point B, determine the shear at B by using the free body of the portion AB:

$$S_B = A_y \qquad 12\,ft < x \le 20\,ft$$

gives

$$S_B = \begin{cases} -C_y = -\dfrac{x}{20}, & 0 \le x < 12\,ft \\ A_y = 1 - \dfrac{x}{20}, & 12\,ft < x \le 20\,ft \end{cases} \longrightarrow \text{Fig. 8.3(e)}$$

Influence line for M_B

Place the unit load to the left of point B, determine the bending moment at B by using the free body of the portion BC:

$$M_B = 8C_y \qquad 0 \le x \le 12\,ft$$

Place the unit load to the right of point B, determine the bending moment at B by using the free body of the portion AB:

$$M_B = 12A_y \qquad 12\,ft \le x \le 20\,ft$$

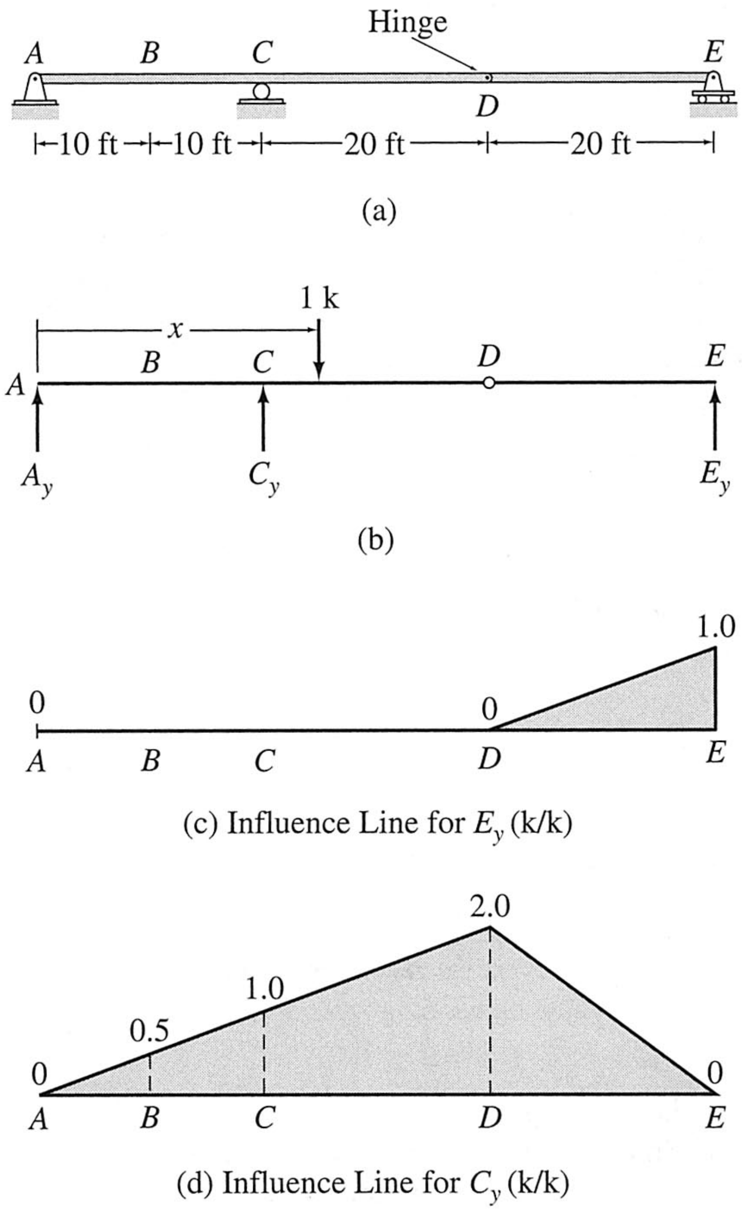
gives

$$M_B = \begin{cases} 8C_y = \dfrac{2x}{5}, & 0 \le x \le 12\,ft \\ 12A_y = 12 - \dfrac{3x}{5}, & 12\,ft \le x \le 20\,ft \end{cases}$$

⟶ Fig. 8.3(f)

Example 8.3

Draw the influence lines for the vertical reactions at supports A, C, and E, the shear just to the right of support C, and the bending moment at point B of the beam shown in Fig. 8.5(a).



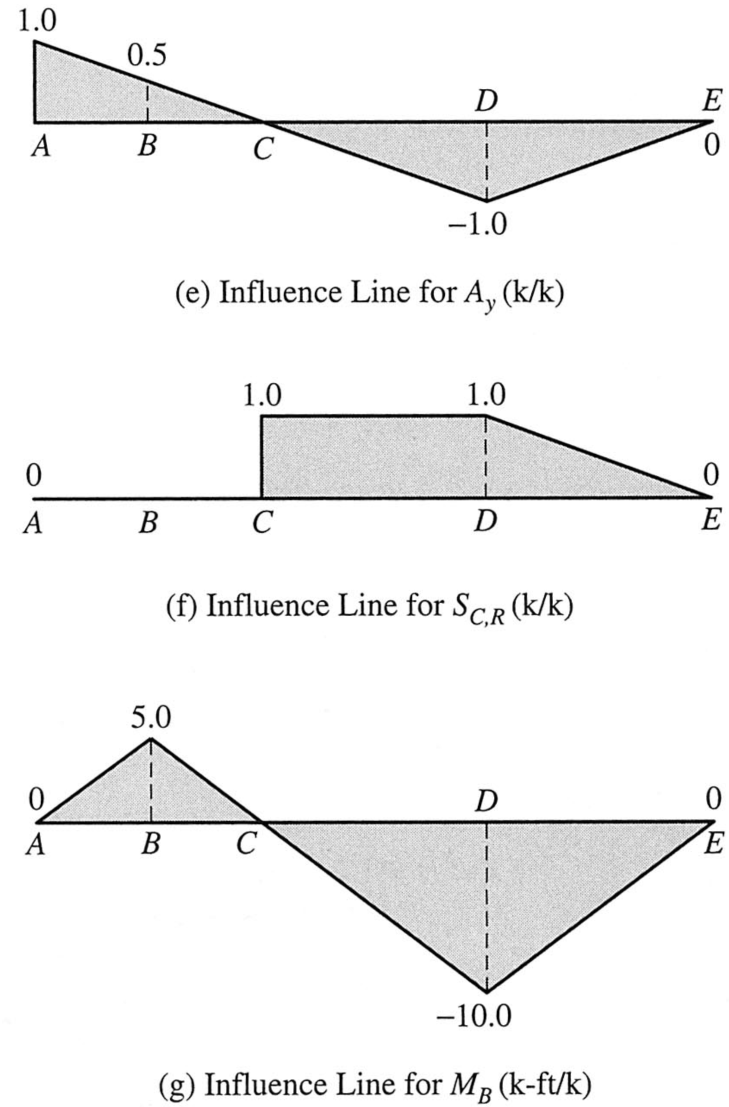


FIG. 8.5

Influence line for E_y

Place the unit load at a variable position x to the left of the hinge D and consider free body diagram DE:

$$\sum M_D^{DE} = 0$$

$$E_y(20) = 0$$

$$E_y = 0 \qquad 0 \le x \le 40\,ft$$

Next, the unit load is located to the right of hinge D:

$$\sum M_D^{DE} = 0$$

$$-1(x-40) + E_y(20) = 0$$

$$E_y = \frac{1(x-40)}{20} = \frac{x}{20} - 2 \qquad 40\,ft \le x \le 60\,ft$$

$$E_y = \begin{cases} 0 & 0 \le x \le 40\,ft \\ \dfrac{x}{20} - 2 & 40\,ft \le x \le 60\,ft \end{cases} \longrightarrow \text{Fig. 8.5(c)}$$

Influence line for C_y

$$\sum M_A = 0$$

$$-1(x) + C_y(20) + E_y(60) = 0$$

$$C_y = \frac{x}{20} - 3E_y$$

By substituting the expressions for E_y, we obtain

$$C_y = \begin{cases} \dfrac{x}{20} - 0 = \dfrac{x}{20} & 0 \le x \le 40\,ft \\ \dfrac{x}{20} - 3\left(\dfrac{x}{20} - 2\right) = 6 - \dfrac{x}{10} & 40\,ft \le x \le 60\,ft \end{cases}$$

⟶ Fig. 8.5(d)

Influence line for A_y

$$\sum F_y = 0$$

$$A_y - 1 + C_y + E_y = 0$$

$$A_y = 1 - C_y - E_y$$

By substituting the expressions for C_y and E_y, then

$$A_y = \begin{cases} 1 - \dfrac{x}{20} - 0 = 1 - \dfrac{x}{20} & 0 \le x \le 40\,ft \\ 1 - \left(6 - \dfrac{x}{10}\right) - \left(\dfrac{x}{20} - 2\right) = \dfrac{x}{20} - 3 & 40\,ft \le x \le 60\,ft \end{cases}$$

→ Fig. 8.5(e)

Influence line for Shear at Just to the Right of C, $S_{C,R}$

$$S_{C,R} = \begin{cases} -E_y & 0 \le x \le 20\,ft \\ 1 - E_y & 20\,ft \le x \le 60\,ft \end{cases}$$

By substituting the expressions for E_y, we obtain

$$S_{C,R} = \begin{cases} 0 & 0 \le x < 20\,ft \\ 1 - 0 = 1 & 20\,ft < x \le 40\,ft \\ 1 - \left(\dfrac{x}{20} - 2\right) = 3 - \dfrac{x}{20} & 40\,ft \le x \le 60\,ft \end{cases}$$

→ Fig. 8.5(f)

Influence line for M_B

$$M_B = \begin{cases} 10A_y - 1(10-x) & 0 \le x \le 10\,ft \\ 10A_y & 10\,ft \le x \le 60\,ft \end{cases}$$

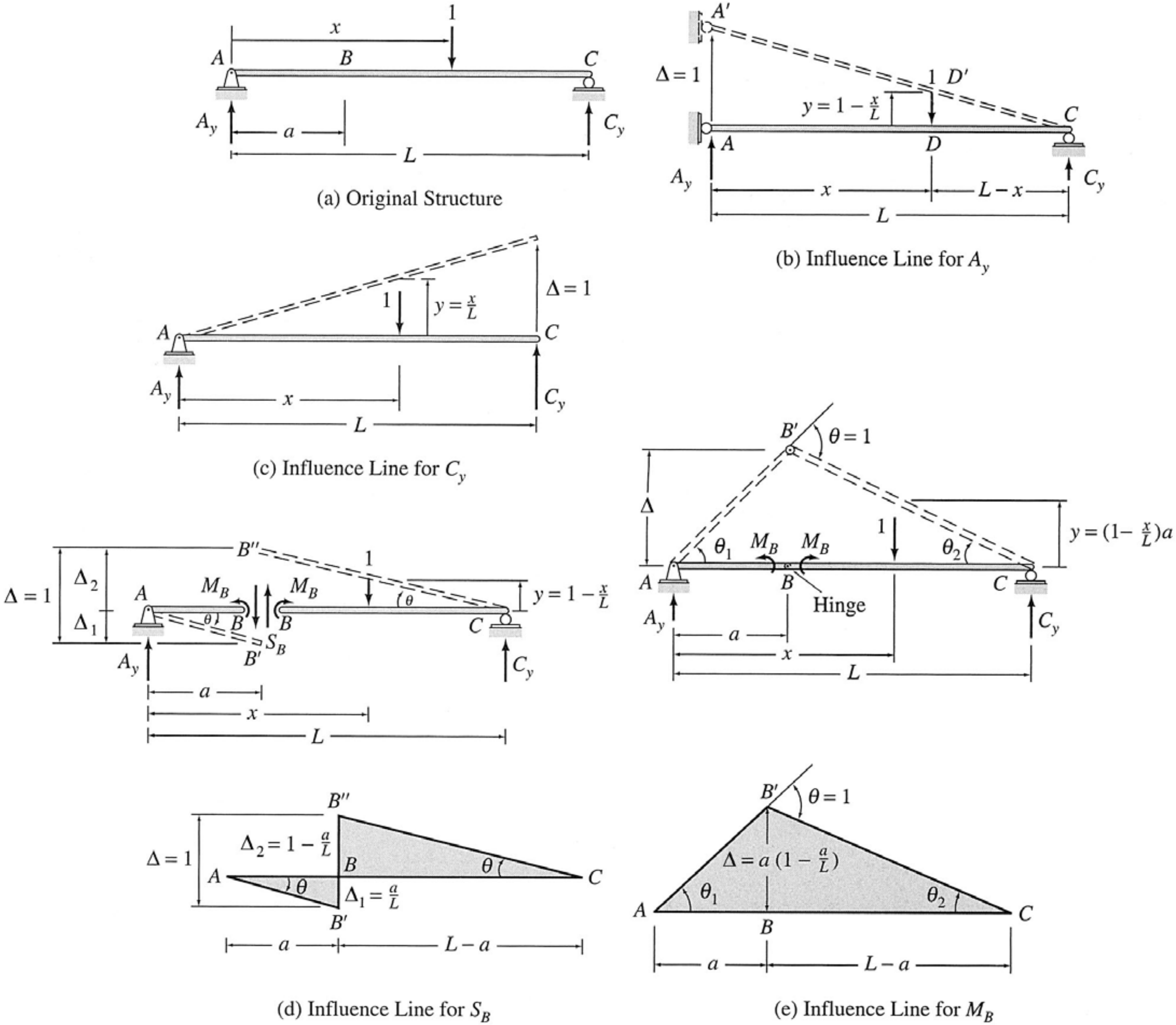
By substituting the expressions for A_y, we obtain

$$M_B = \begin{cases} 10\left(1-\dfrac{x}{20}\right) - 1(10-x) = \dfrac{x}{2} & 0 \le x \le 10\,ft \\ 10\left(1-\dfrac{x}{20}\right) = 10-\dfrac{x}{2} & 10\,ft \le x \le 40\,ft \\ 10\left(\dfrac{x}{20}-3\right) = \dfrac{x}{2}-30 & 40\,ft \le x \le 60\,ft \end{cases}$$

$\longrightarrow$ Fig. 8.5(g)

MULLER-BRESLAU'S PRINCIPLE AND QUALITATIVE INFLUENCE LINES

- Developed by Heinrich Muller-Breslau in 1886.
- <u>Muler-Breslau's principle:</u> *The influence line for a force (or moment) response function is given by the deflected shape of the released structure obtained by removing the restraint corresponding to the response function from the original structure and by giving the released structure a unit displacement (or rotation) at the location and in the direction of the response function, so that only the response function and the unit load perform external work.*
- Valid only for influence lines for response functions involving forces and moments, e.g. reactions, shears, bending moments or forces in truss members, not valid for deflections.



(a) Original Structure

(b) Influence Line for A_y

(c) Influence Line for C_y

(d) Influence Line for S_B

(e) Influence Line for M_B

FIG. 8.8

Qualitative Influence Lines

In many practical applications, it is necessary to determine only the general shape of the influence lines but not the numerical values of the ordinates. *A diagram showing the general shape of an influence line without the numerical values of its ordinates is called a qualitative influence line*. In contrast, an influence line with the numerical values of its ordinates known is referred to as a *quantitative influence line*.

Example 8.6

Draw the influence lines for the vertical reactions at supports B and D and the shear and bending moment at point C of the beam shown in the Figure 8.9(a).

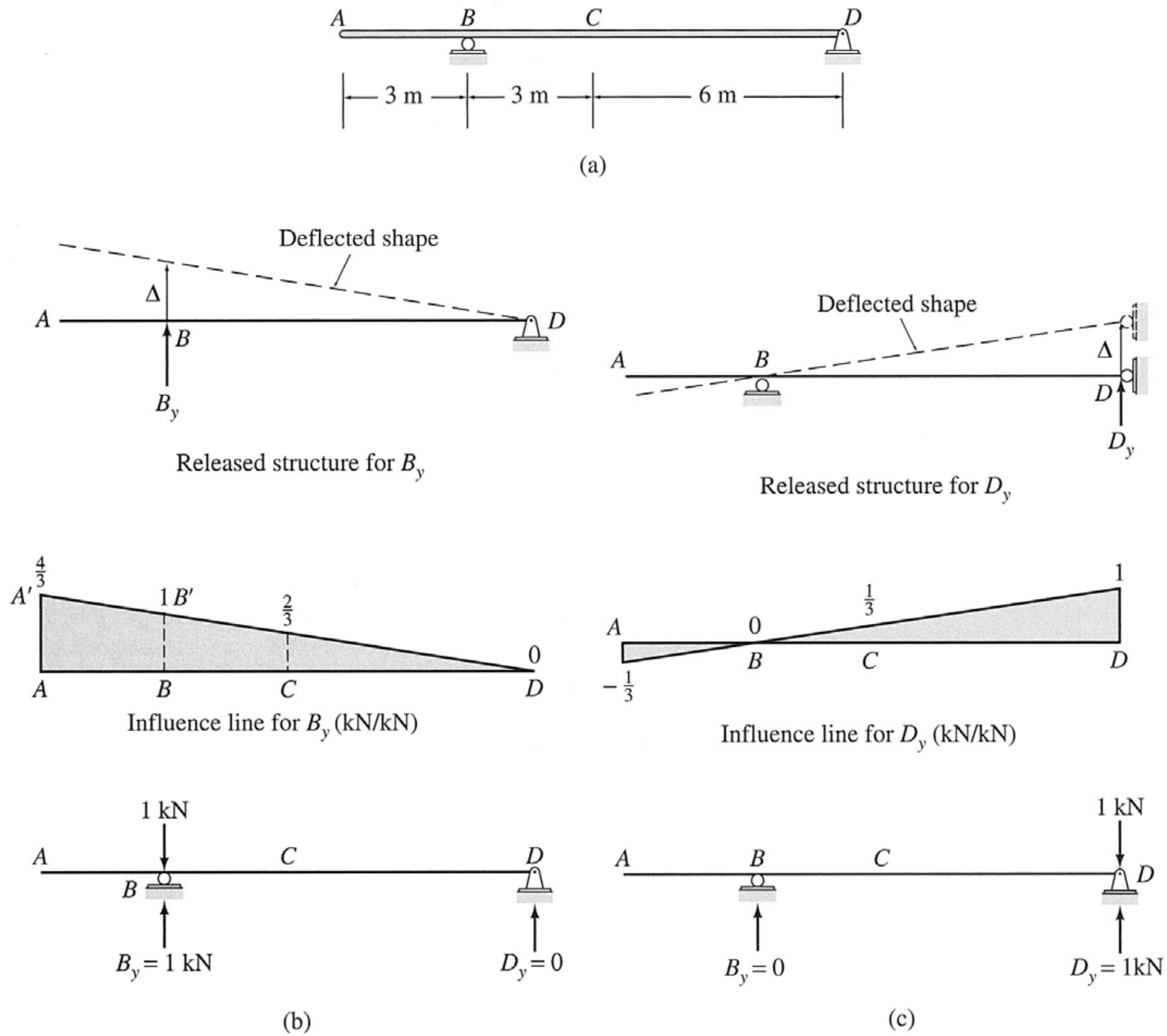
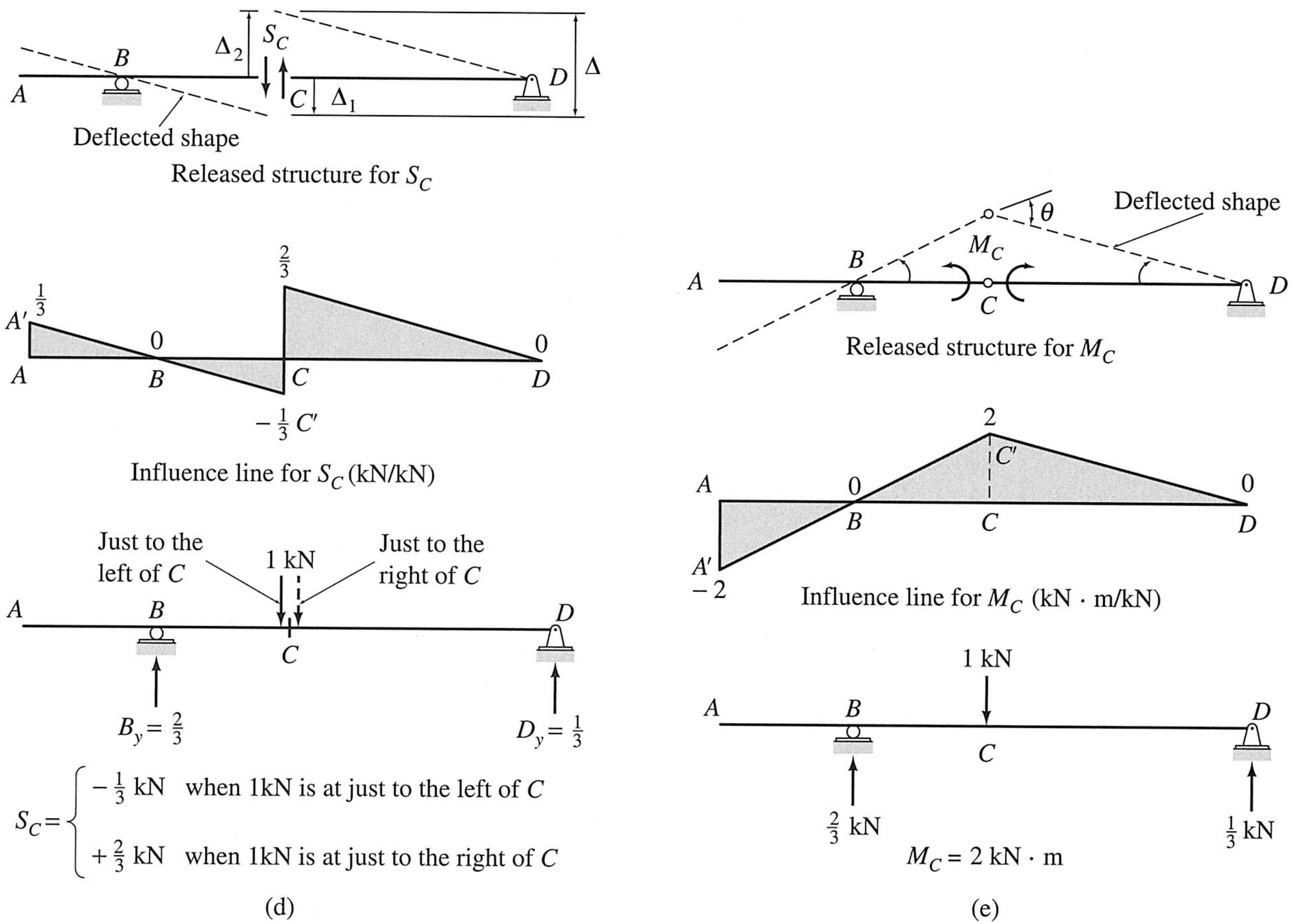


FIG. 8.9



$$S_C = \begin{cases} -\frac{1}{3}\text{ kN} & \text{when 1kN is at just to the left of } C \\ +\frac{2}{3}\text{ kN} & \text{when 1kN is at just to the right of } C \end{cases}$$

FIG. 8.9 (contd.)

Example 8.7

Draw the influence lines for the vertical reactions at supports A and E, the reaction moment at support A, the shear at point B, and the bending moment at point D of the beam shown in Fig. 8.10(a).

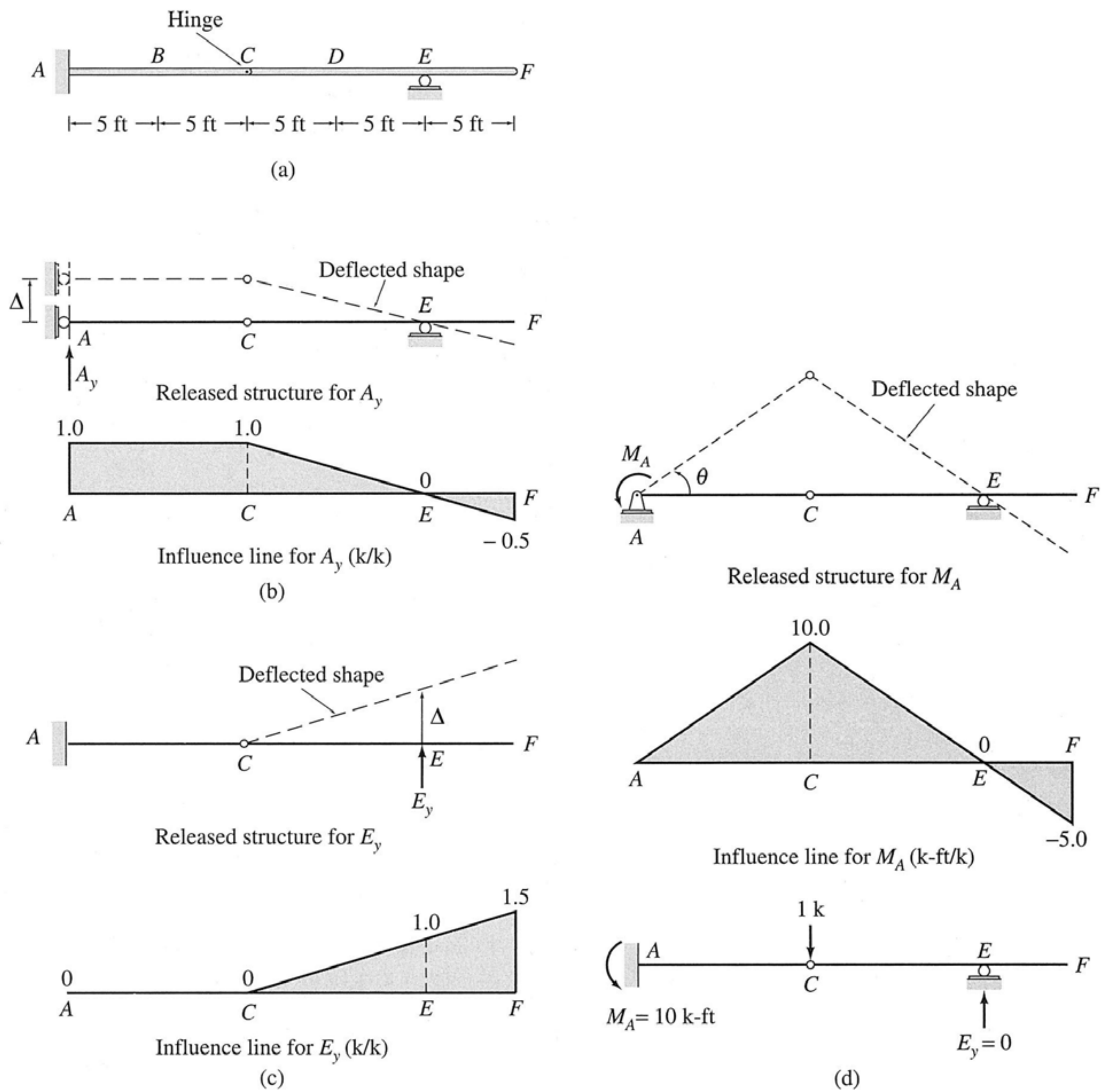


FIG. 8.10

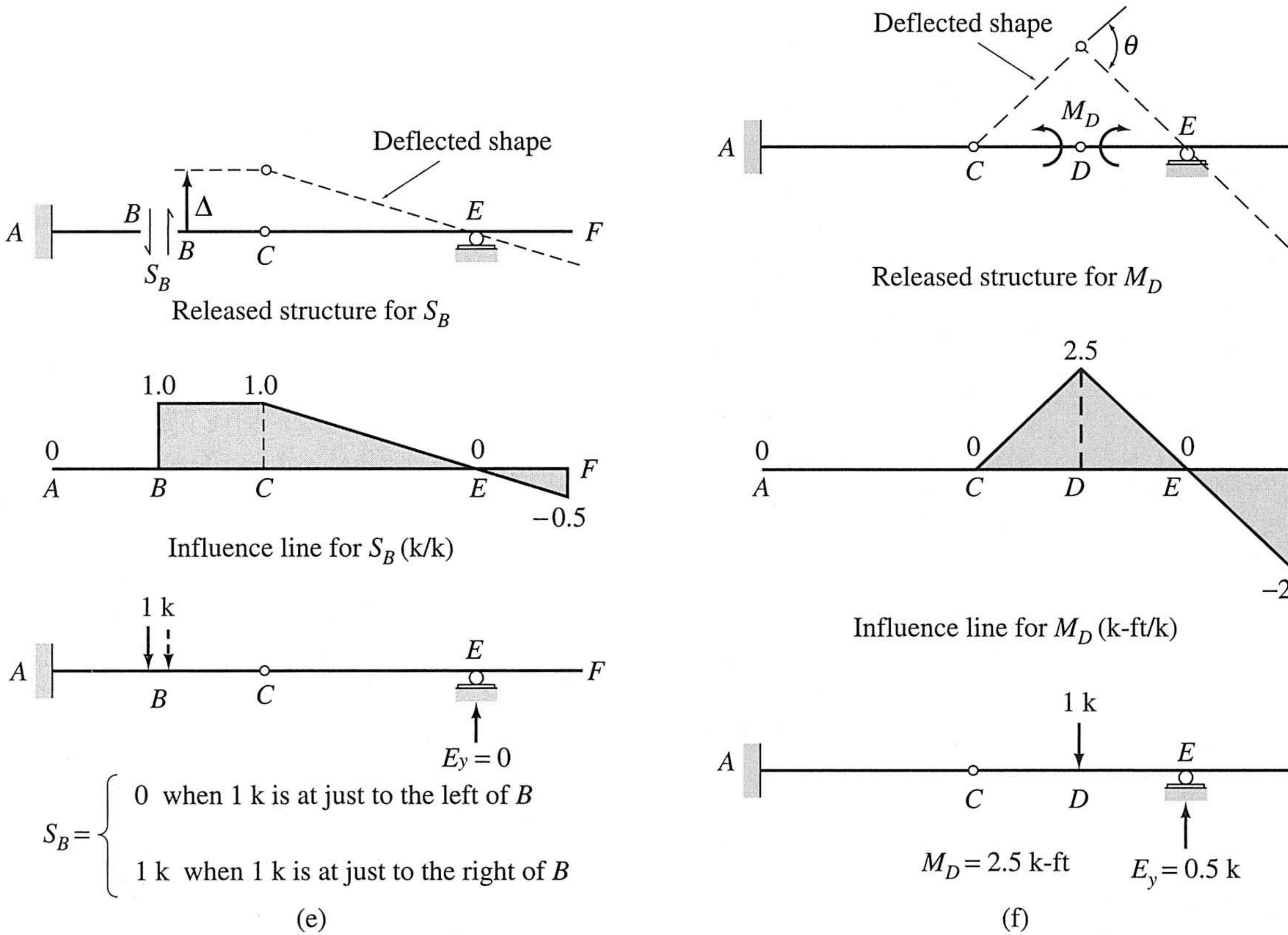


FIG. 8.10 (contd.)

Example 8.8

Draw the influence lines for the vertical reactions at supports A and C of the beam shown in Fig. 8.11(a).

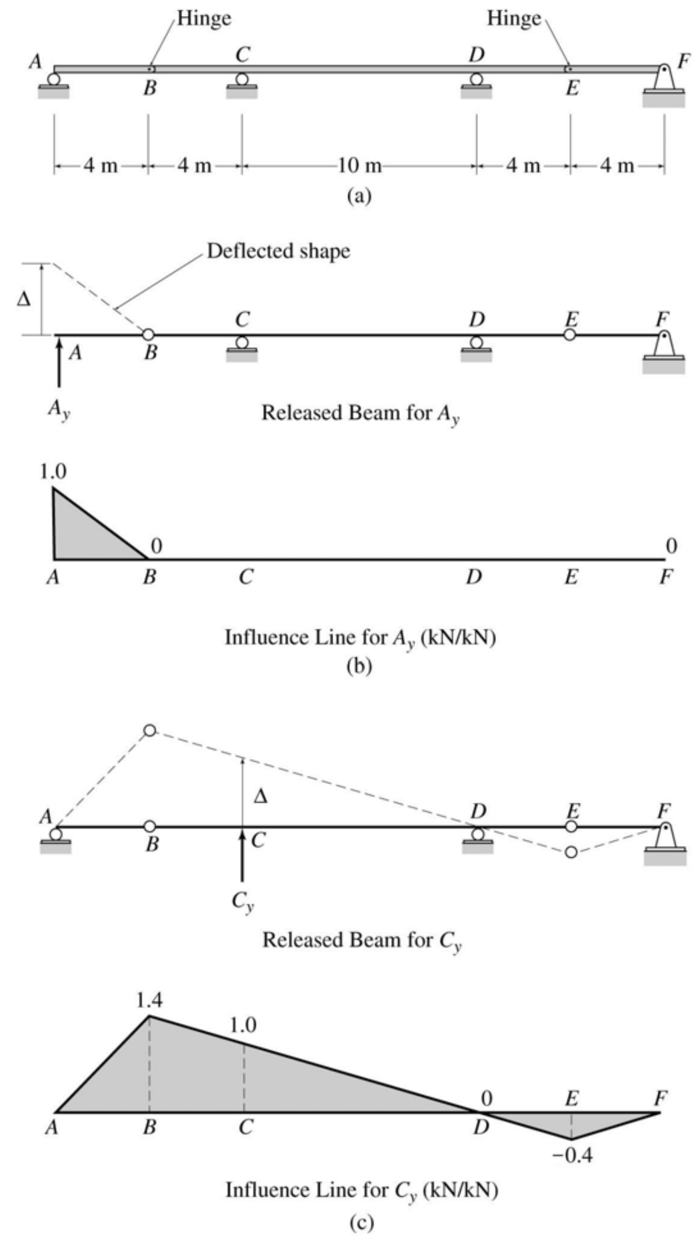
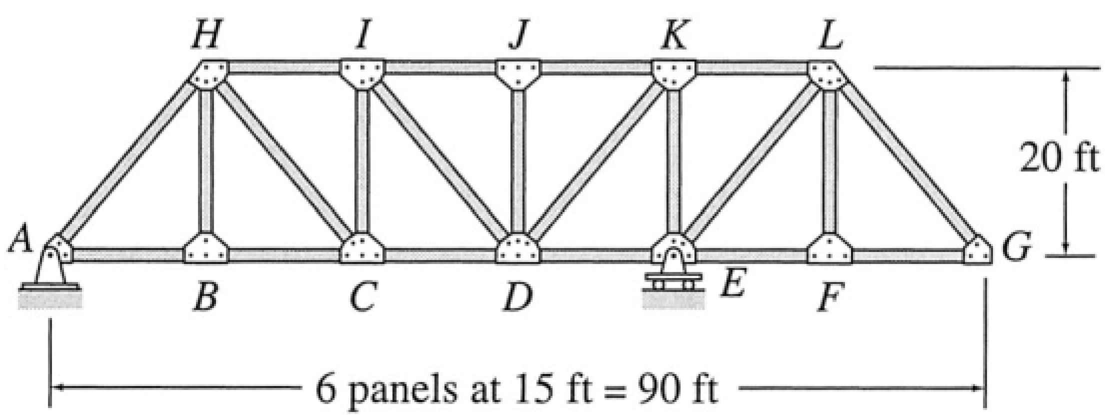
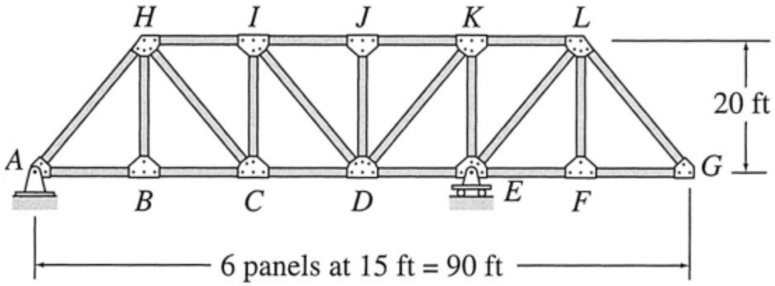


FIG. 8.11

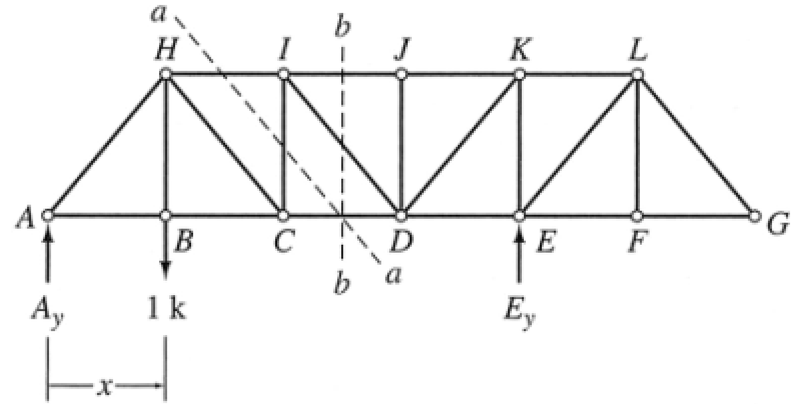
INFLUENCE LINES FOR TRUSSES

Consider the Pratt bridge truss shown. A unit load moves from left to right. Suppose that we wish to draw the influence lines for the vertical reactions at supports A and E and for the axial forces in members CI, CD, DI, IJ and FL of the truss.





Influence Lines for Reactions

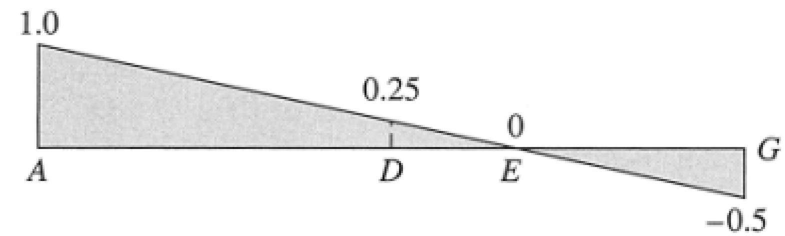


(b)

$$\sum M_E = 0$$

$$-A_y(60) + 1(60 - x) = 0$$

$$A_y = 1 - \frac{x}{60}$$

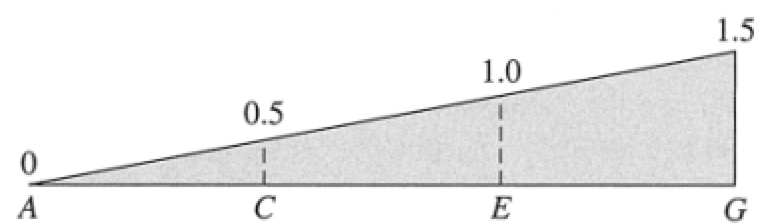


(c) Influence Line for A_y (k/k)

$$\sum M_A = 0$$

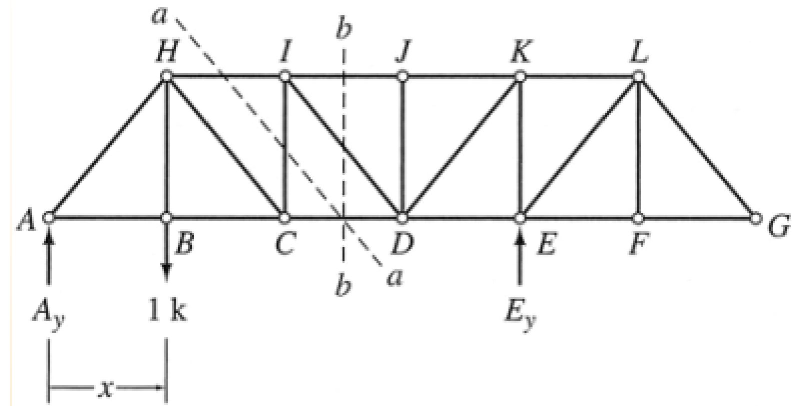
$$-1(x) + E_y(60) = 0$$

$$E_y = \frac{x}{60}$$



(d) Influence Line for E_y (k/k)

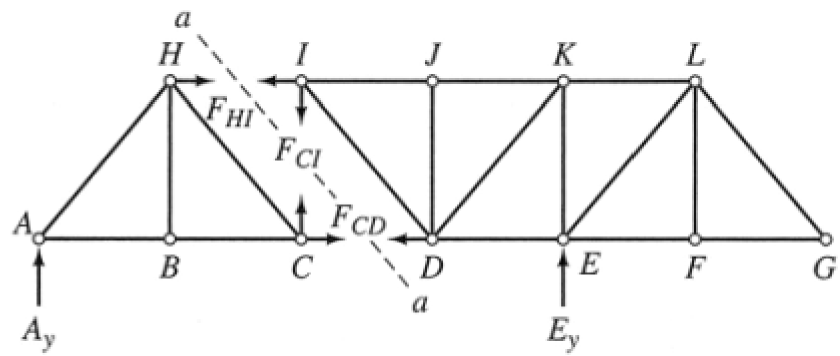
Influence line for force in Vertical Member CI



Considering the right portion of the truss (unit load at left portion)

$$\sum F_y = 0$$

$$-F_{CI} + E_y = 0 \qquad F_{CI} = E_y \qquad 0 \le x \le 30\,ft$$



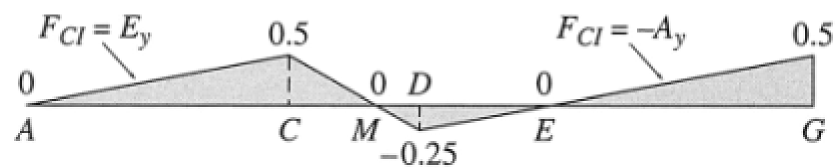
(e) Section *aa*

Considering the left portion of the truss (unit load at right portion)

$$\sum F_y = 0$$

$$A_y + F_{CI} = 0 \qquad F_{CI} = -A_y \qquad 45\,ft \le x \le 90\,ft$$

Unit load is located between C and D:



(f) Influence Line for F_{CI} (k/k)

$$\sum F_y = 0$$

$$A_y - \left(\frac{45 - x}{15}\right) + F_{CI} = 0$$

$$F_{CI} = -A_y + \left(\frac{45 - x}{15}\right) \qquad 30\,ft \le x \le 45\,ft$$

Influence line for force in Bottom Chord Member CD

$$\sum M_I = 0$$

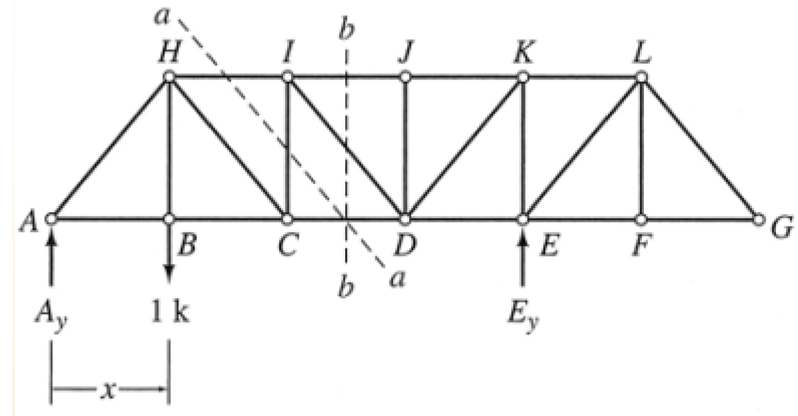
$$-F_{CD}(20) + E_y(30) = 0$$

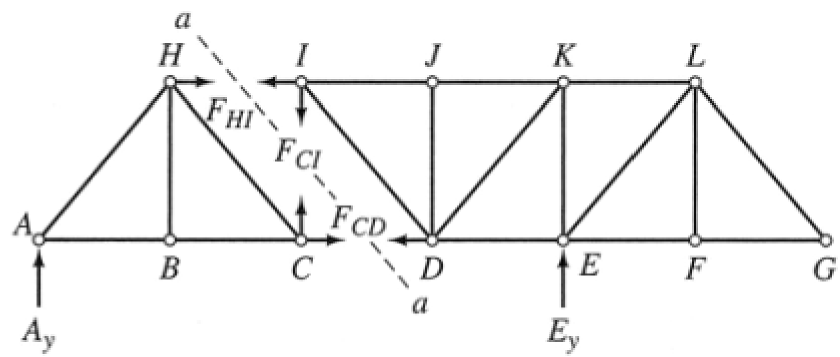
$$F_{CD} = 1.5E_y \qquad 0 \le x \le 30\,ft$$

$$\sum M_I = 0$$

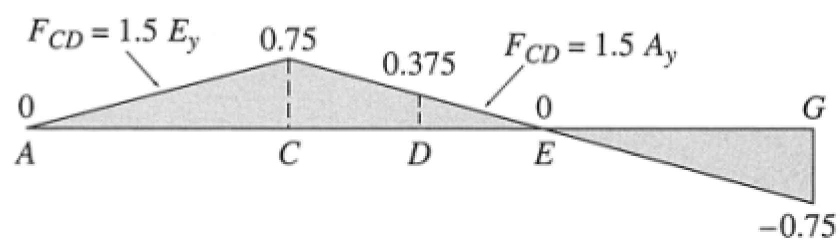
$$-A_y(30) + F_{CD}(20) = 0$$

$$F_{CD} = 1.5A_y \qquad 30\,ft \le x \le 90\,ft$$





(e) Section *aa*



(g) Influence Line for F_{CD} (k/k)

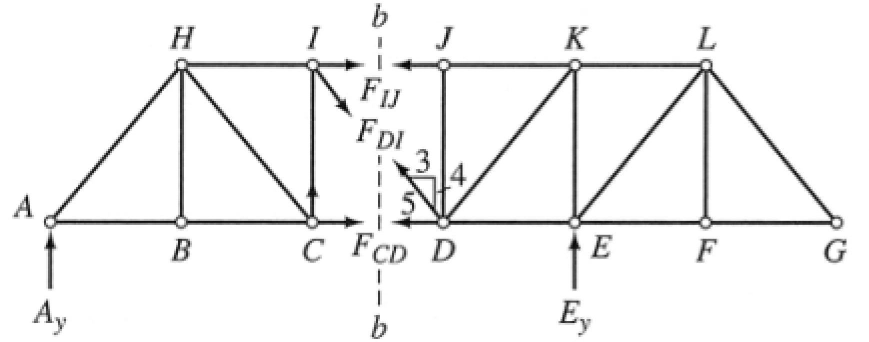
Influence line for force in Diagonal Member DI

$$\sum F_y = 0: \qquad \frac{4}{5}F_{DI} + E_y = 0$$

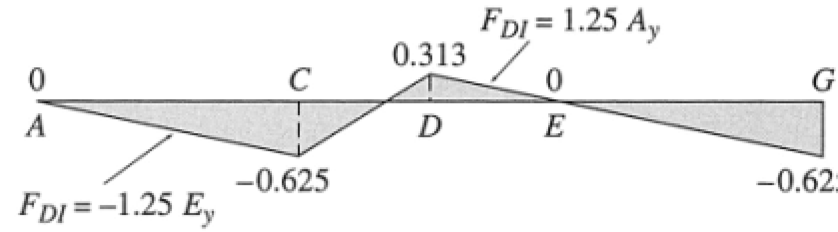
$$F_{DI} = -1.25E_y \qquad 0 \le x \le 30\,ft$$

$$\sum F_y = 0: \qquad A_y - \frac{4}{5}F_{DI} = 0$$

$$F_{DI} = 1.25A_y \qquad 45\,ft \le x \le 90\,ft$$



(h) Section *bb*



(i) Influence Line for F_{DI} (k/k)

Influence line for force in Top Chord Member IJ

$$\sum M_D = 0:$$

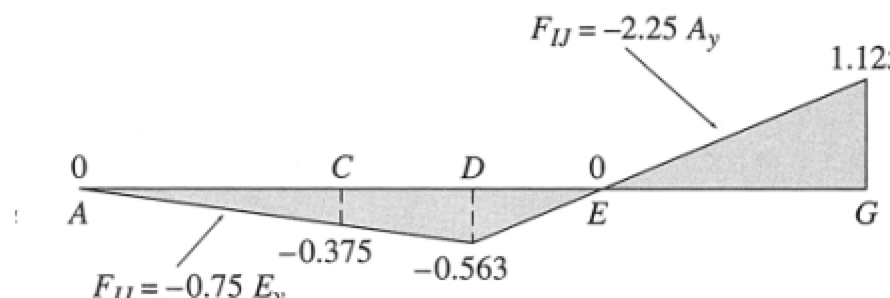
$$F_{IJ}(20) + E_y(15) = 0$$

$$F_{IJ} = -0.75E_y \qquad 0 \le x \le 45\,ft$$

$$\sum M_D = 0:$$

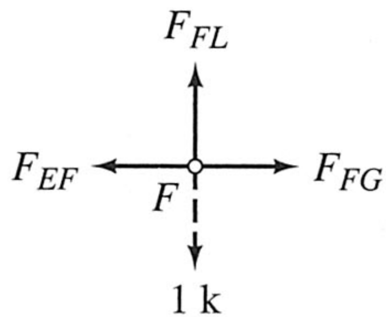
$$-A_y(45) - F_{IJ}(20) = 0$$

$$F_{IJ} = -2.25A_y \qquad 45\,ft \le x \le 90\,ft$$

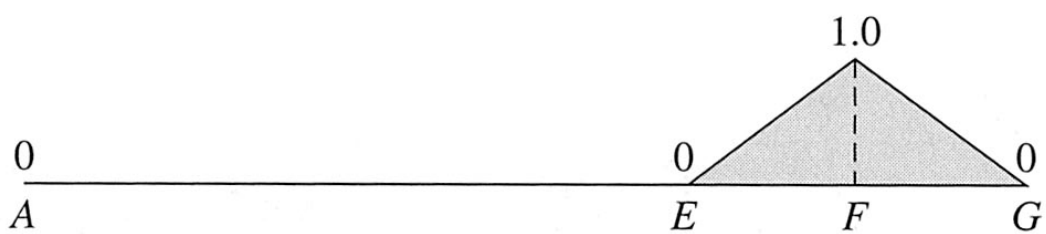


(j) Influence Line for F_{IJ} (k/k)

Influence line for force in Vertical Member FL



(k)



(1) Influence Line for F_{FL} (k/k)

FIG. 8.18 (contd.)

Example 8.12

Draw the influence lines for the forces in members AF, CF, and CG of the Parker truss shown in Fig. 8.19(a). Live loads are transmitted to the bottom chord of the truss.

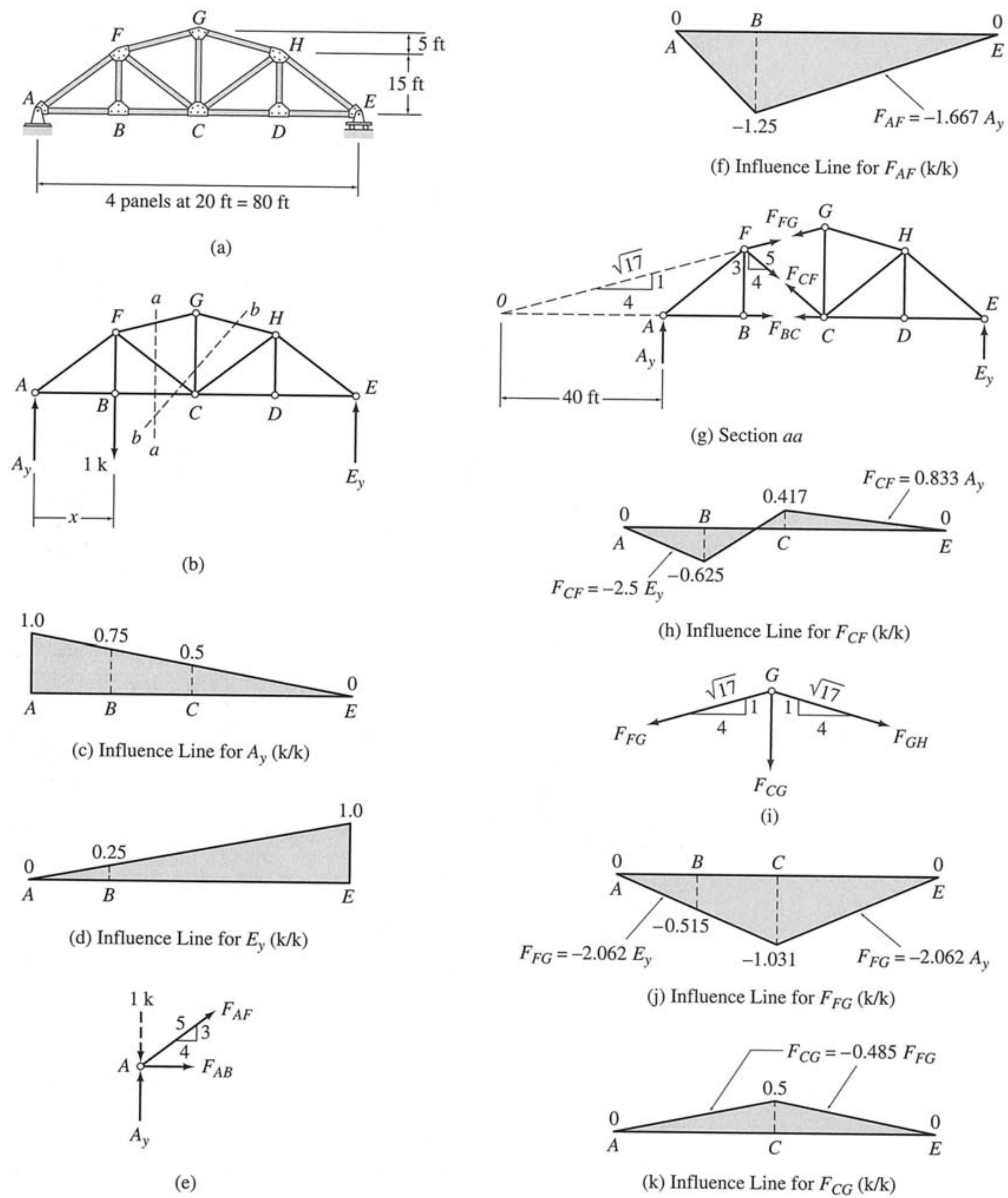
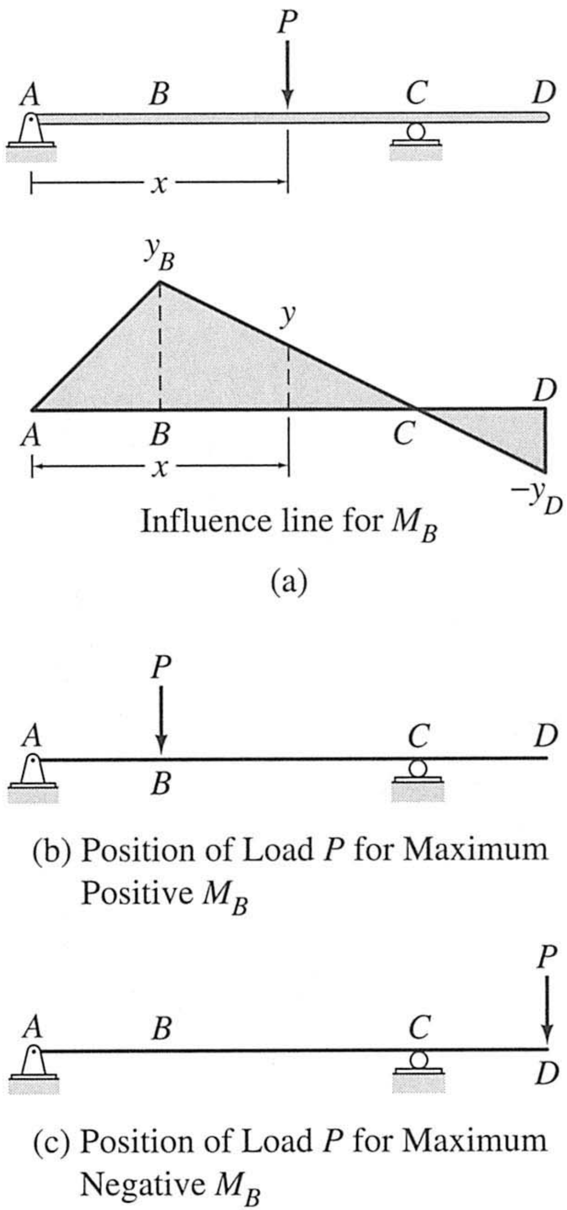


FIG. 8.19

APPLICATION OF INFLUENCE LINES

Response at a particular location due to a single moving concentrated load

- The value of a response function due to any single concentrated load can be obtained by multiplying the magnitude of the load by the ordinate of the response function influence line at the position of the load
- To determine the maximum positive value of a response function due to a single moving concentrated load, the load must be placed at the location of the maximum positive ordinate of the response function influence line, whereas to determine the maximum negative value of the response function, the load must be placed at the location of the maximum negative ordinate of the influence line.



Influence line for M_B

(a)

(b) Position of Load P for Maximum Positive M_B

(c) Position of Load P for Maximum Negative M_B

FIG. 9.1

Suppose that we wish to determine the bending moment at B when the load P is located at a distance x.

$M_B = Py$

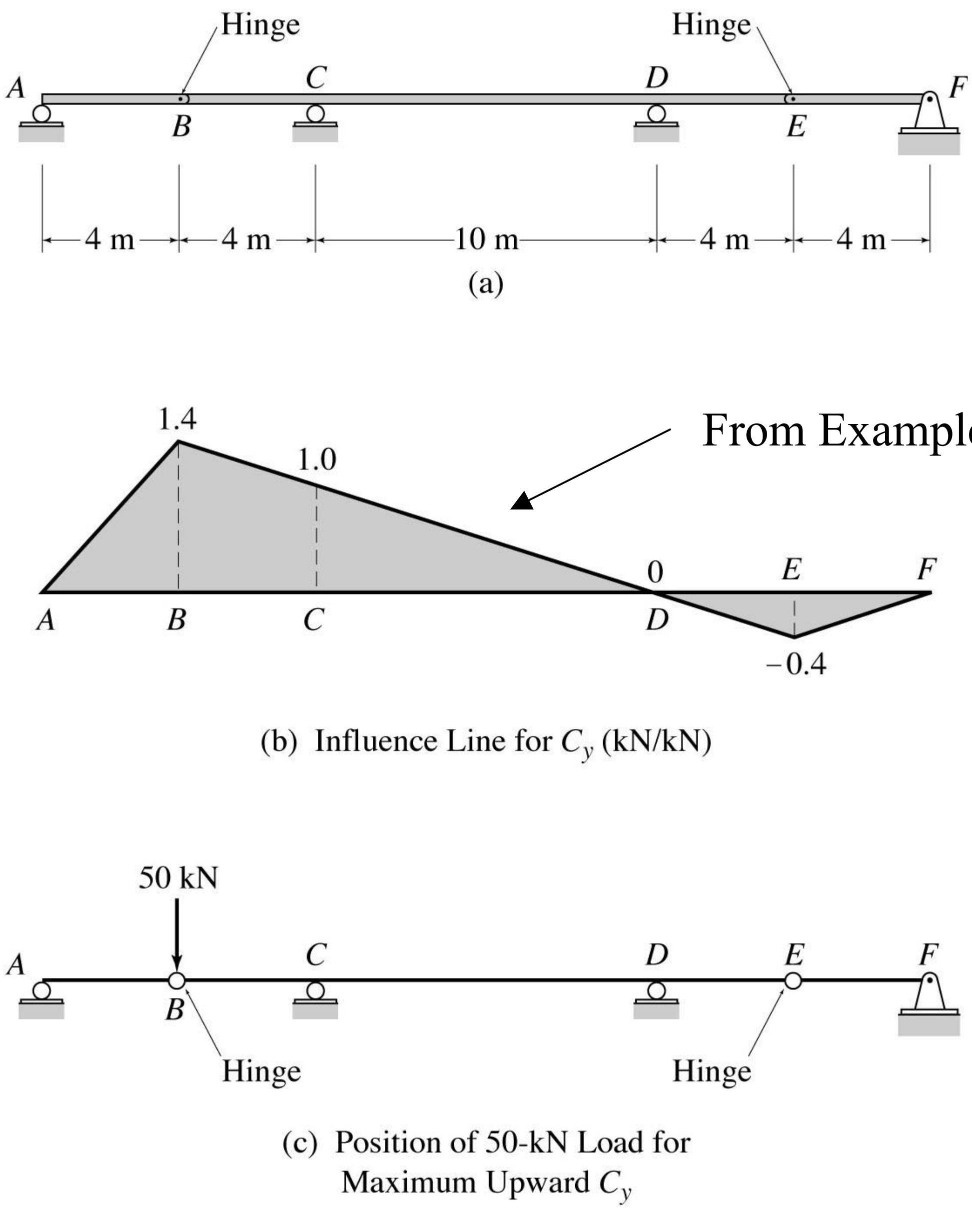
<u>Maximum Positive bending moment at B</u>

* Place the load P at point B
* $M_B = Py_B$

<u>Maximum Negative bending moment at B</u>

* Place the load P at point D
* $M_B = -Py_D$

Example 9.1



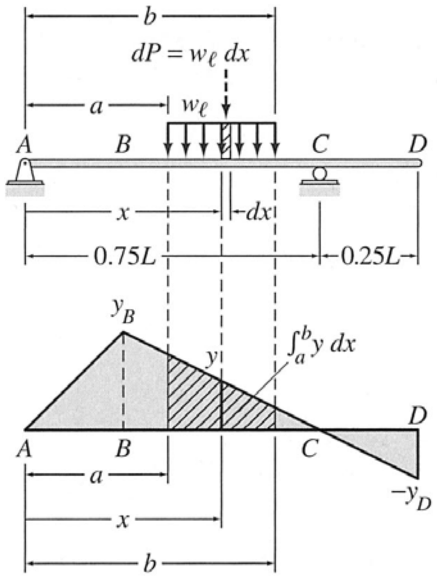
From Example 8.8

FIG. 9.2

For the beam shown in Fig. 9.2(a), determine the maximum upward reaction at support C due to a 50 kN concentrated live load.

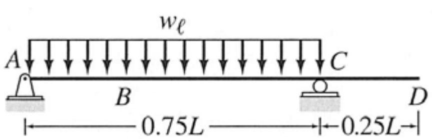
Maximum upward reaction at C:

$$C_y = 50(+1.4) = +70\,kN = 70\,kN \uparrow$$

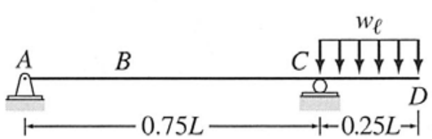


Influence line for M_B

(a)



(b) Arrangement of Uniformly Distributed Live Load w_ℓ for Maximum Positive M_B



(c) Arrangement of Uniformly Distributed Live Load w_ℓ for Maximum Negative M_B

FIG. 9.3

Response at a particular location due to a uniformly distributed live load

Consider, for example, a beam subjected to a uniformly distributed live load w_l. Suppose that we want to determine the bending moment at B when the load is placed on the beam, from x=a to x=b.

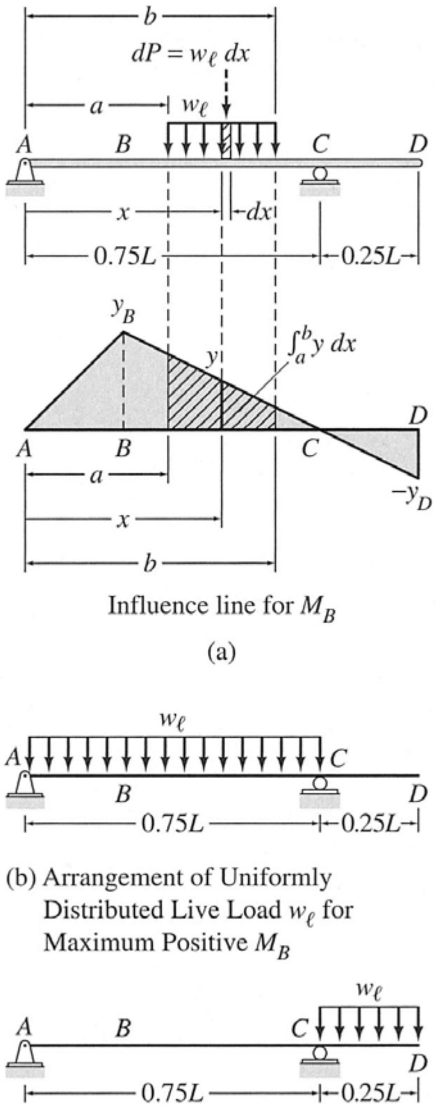
The bending moment at B due to the load dP as

$$dM_B = dPy = (w_l dx)\, y$$

The total bending moment at B due to distributed load from x=a to x=b:

$$M_B = \int_a^b w_l y dx = w_l \int_a^b y dx$$

The value of a response function due to a uniformly distributed load applied over a portion of the structure can be obtained by multiplying the load intensity by the net area under the corresponding portion of the response function influence line.



Influence line for M_B

(a)

(b) Arrangement of Uniformly Distributed Live Load w_ℓ for Maximum Positive M_B

(c) Arrangement of Uniformly Distributed Live Load w_ℓ for Maximum Negative M_B

FIG. 9.3

$$M_B = \int_a^b w_l y dx = w_l \int_a^b y dx$$

This equation also indicates that the bending moment at B will be maximum positive if the uniformly distributed load is placed over all those portions of the beam where the influence-line ordinates are positive and vice versa.

<u>Maximum positive bending moment at B</u>

$$M_B = w_l \left(area\, under\, the\, \inf luence\, line\, A \rightarrow C \right)$$

$$= w_l \left(\frac{1}{2} \right) (0.75L)(y_B) = 0.375 w_l y_B L$$

<u>Maximum negative bending moment at B</u>

$$M_B = w_l \left(area\, under\, the\, \inf luence\, line\, C \rightarrow D \right)$$

$$= w_l \left(\frac{1}{2} \right) (0.25L)(-y_D) = -0.125 w_l y_D L$$

To determine the maximum positive (or negative) value of a response function due to a uniformly distributed live load, the load must be placed over those portions of the structure where the ordinates of the response function influence line are positive (or negative).

Example 9.2

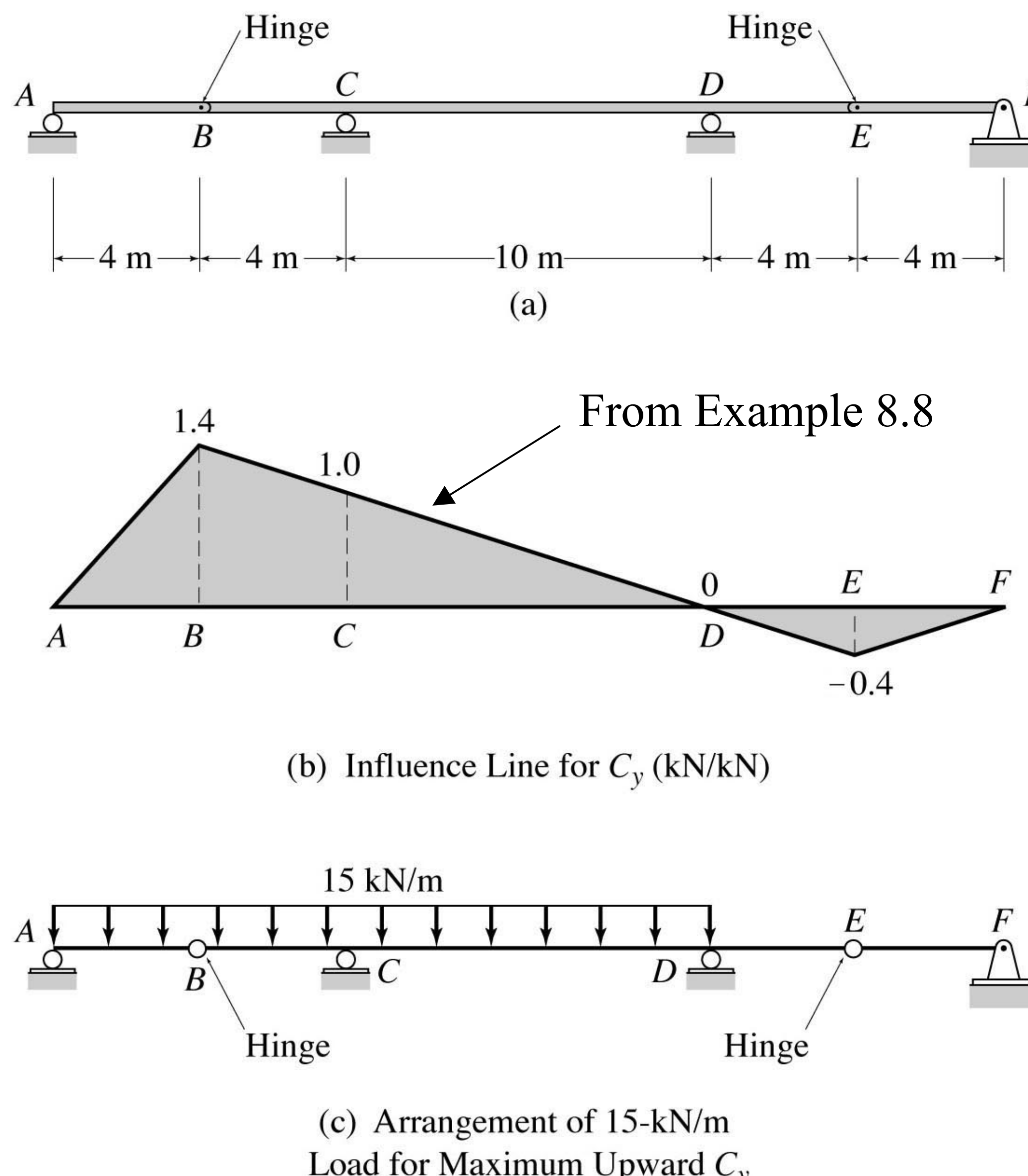
For the beam shown in Fig. 9.4(a), determine the maximum upward reaction at support C due to a 15 kN/m uniformly distributed live load (udl).

To obtain the maximum positive value of C_y, we place the 15 kN/m udl over the portion AD of the beam, Fig. 9.4(c).

Maximum upward reaction at C:

$$C_y = 15\left[\frac{1}{2}(+1.4)(18)\right]$$

$$= +189\,kN = 189\,kN \uparrow$$



(a)

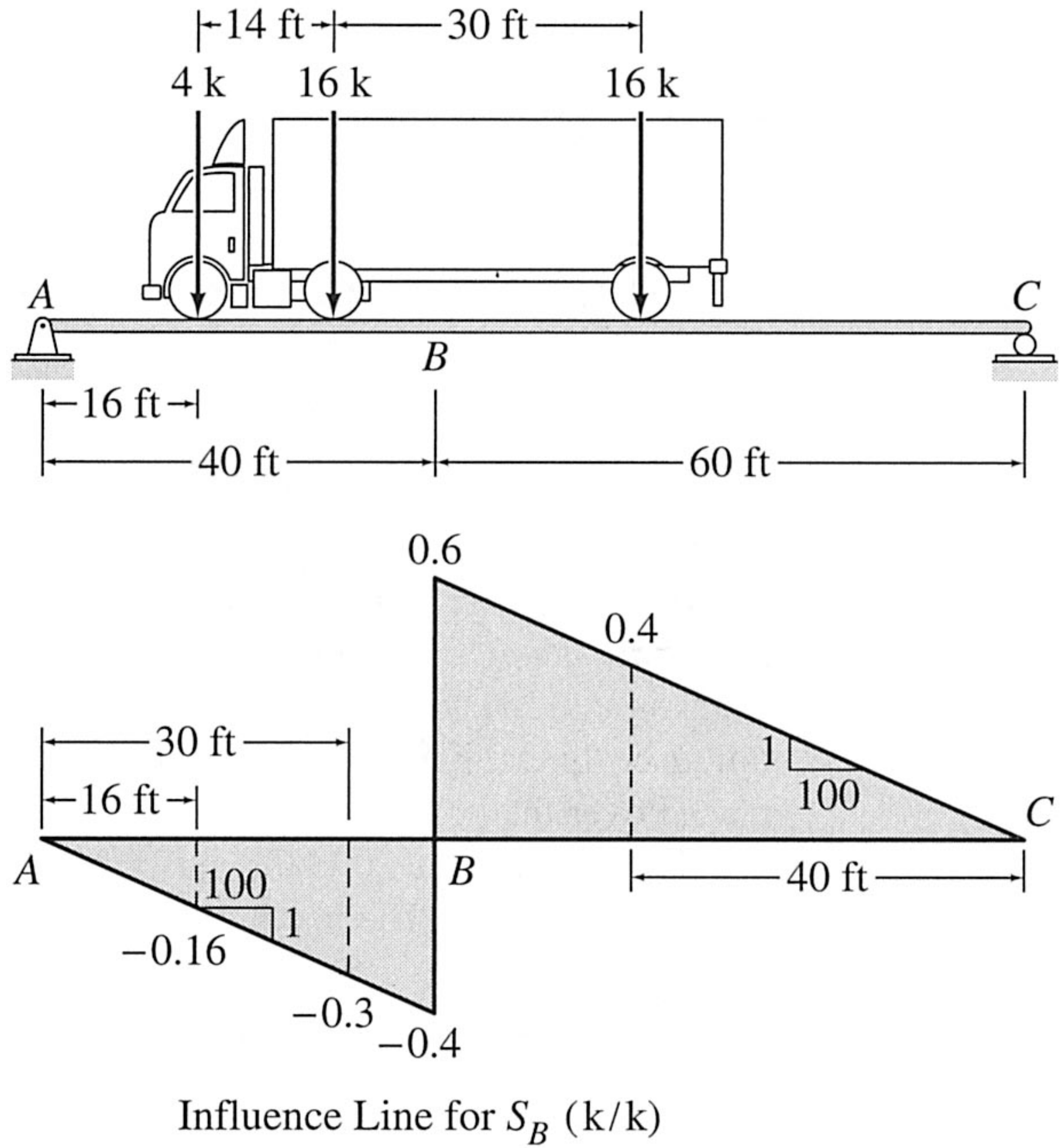
(b) Influence Line for C_y (kN/kN)

(c) Arrangement of 15-kN/m Load for Maximum Upward C_y

FIG. 9.4

Response at a particular location due to a series of moving concentrated loads

Suppose we wish to determine the shear at B of the beam due to the wheel loads of a truck when the truck is located as in figure

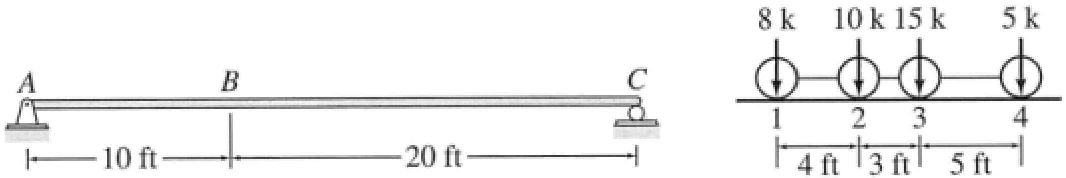


$$S_B = -4(0.16) - 16(0.3) + 16(0.4)$$
$$= 0.96k$$

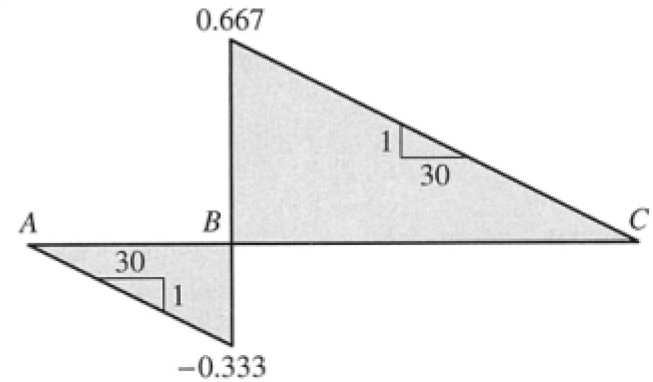
FIG. 9.6

Influence lines can also be used for determining the maximum values of response functions at particular locations of structures due to a series of concentrated loads.

Suppose that our objective is to determine the maximum positive shear at B due to the series of four concentrated loads.



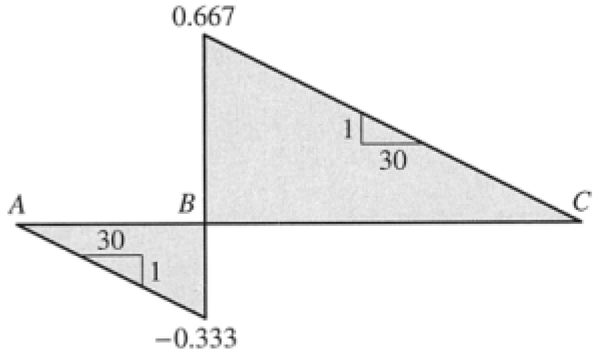
(a)



(b) Influence Line for S_B (k/k)

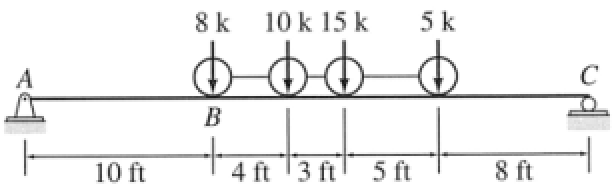
During the movement of the series of loads across the entire length of the beam, the (absolute) maximum shear at B occurs when one of the loads of the series is at the location of the maximum positive ordinate of the influence line for S_B.

We use a trial-and-error procedure



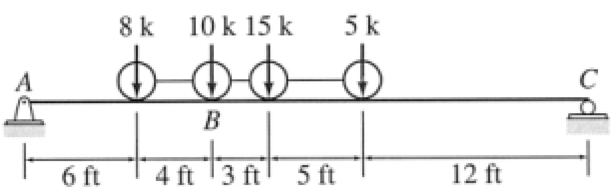
(b) Influence Line for S_B (k/k)

Let the loads move from right to left, the 8k load placed just to the right of B:



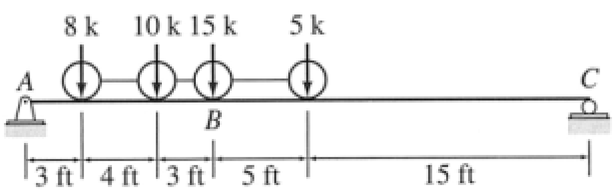
(c) Loading Position 1

$$S_B = 8(20)\left(\frac{1}{30}\right) + 10(16)\left(\frac{1}{30}\right) + 15(13)\left(\frac{1}{30}\right) + 5(8)\left(\frac{1}{30}\right)$$
$$= 18.5k$$



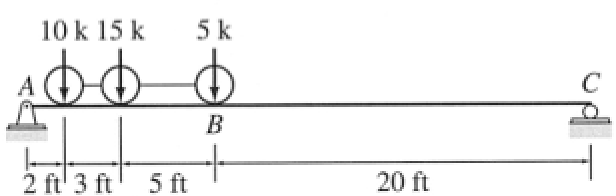
(d) Loading Position 2

$$S_B = -8(6)\left(\frac{1}{30}\right) + 10(20)\left(\frac{1}{30}\right) + 15(17)\left(\frac{1}{30}\right) + 5(12)\left(\frac{1}{30}\right)$$
$$= 15.567k$$



(e) Loading Position 3

$$S_B = -8(3)\left(\frac{1}{30}\right) - 10(7)\left(\frac{1}{30}\right) + 15(20)\left(\frac{1}{30}\right) + 5(15)\left(\frac{1}{30}\right)$$
$$= 9.367k$$



(f) Loading Position 4

$$S_B = -10(2)\left(\frac{1}{30}\right) - 15(5)\left(\frac{1}{30}\right) + 5(20)\left(\frac{1}{30}\right)$$
$$= 0.167k$$

∴ Maximum positive S_B=18.5k → Fig.(c)

Example 9.4

Determine the maximum axial force in member BC of the Warren truss due to the series of four moving concentrated loads shown in Fig. 9.8(a).

We move the load series from right to left, successively placing each load of the series at point B, where the maximum ordinate of the influence line for F_{BC} is located (see Fig. 9.8(c)-(f)).

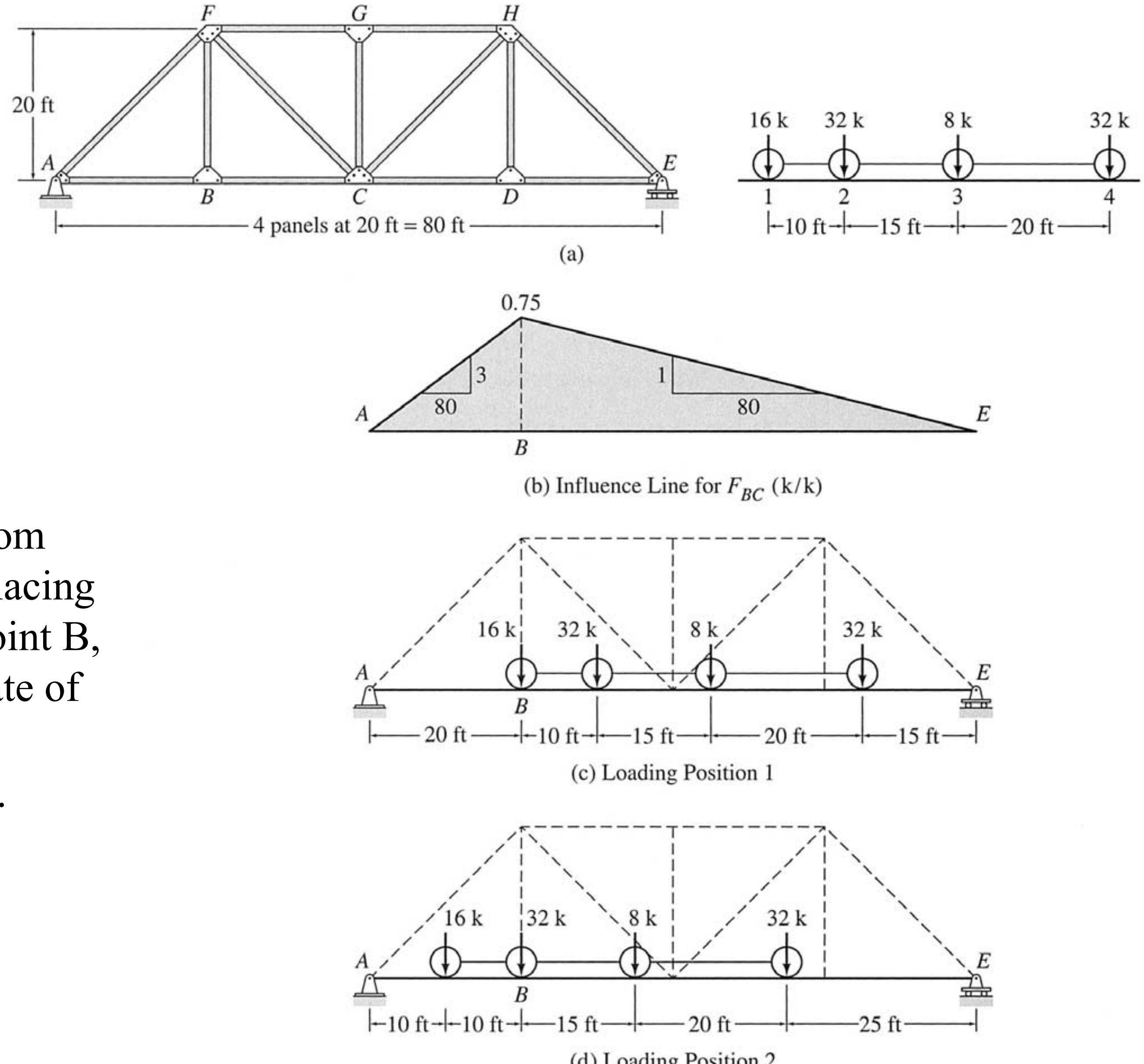
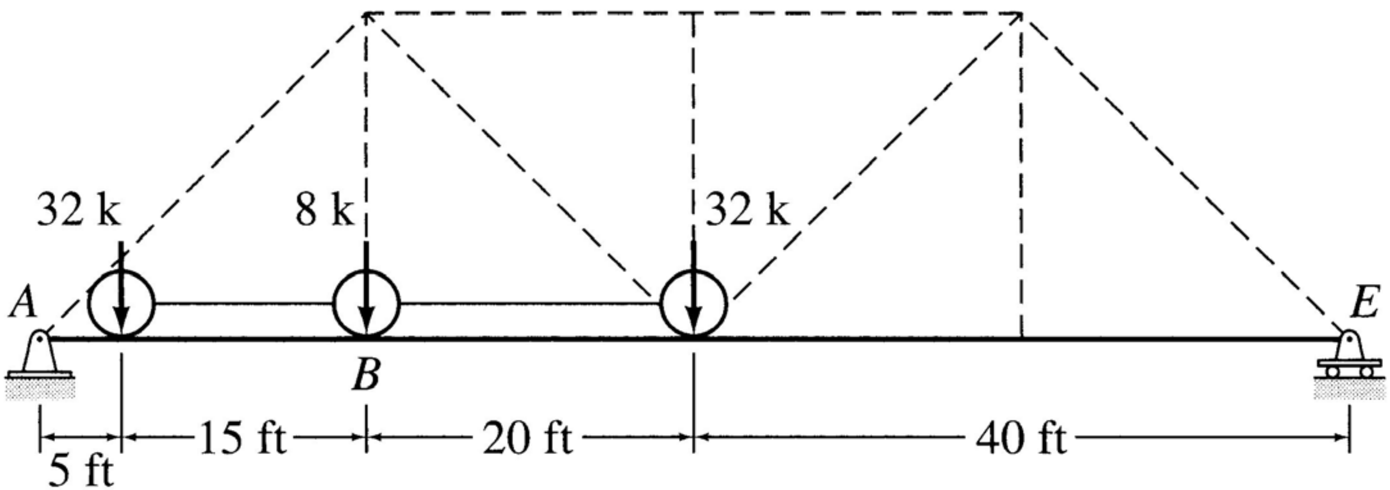
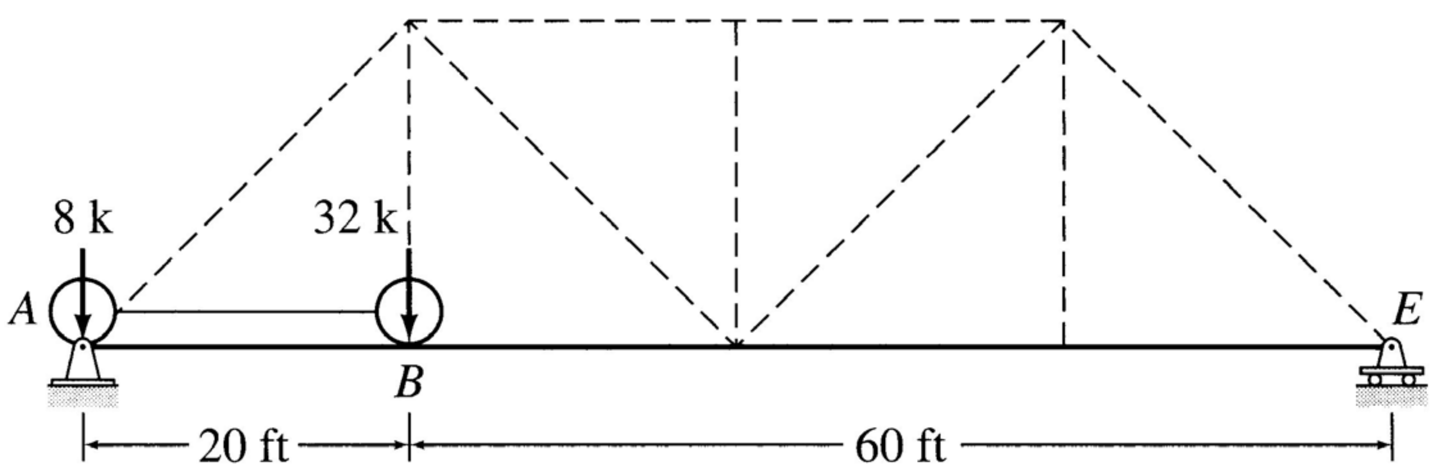


FIG. 9.8



(e) Loading Position 3



(f) Loading Position 4

FIG. 9.8 (contd.)

For loading position 1 (Fig. 9.8(c)):

$$F_{BC} = [16(60) + 32(50) + 8(35) + 32(15)]\left(\frac{1}{80}\right) = 41.5k\,(T)$$

For loading position 2 (Fig. 9.8(d)):

$$F_{BC} = 16(10)\left(\frac{3}{80}\right) + [32(60) + 8(45) + 32(25)]\left(\frac{1}{80}\right) = 44.5k\,(T)$$

For loading position 3 (Fig. 9.8(e)):

$$F_{BC} = 32(5)\left(\frac{3}{80}\right) + [8(60) + 32(40)]\left(\frac{1}{80}\right) = 28.0k\,(T)$$

For loading position 4 (Fig. 9.8(f)):

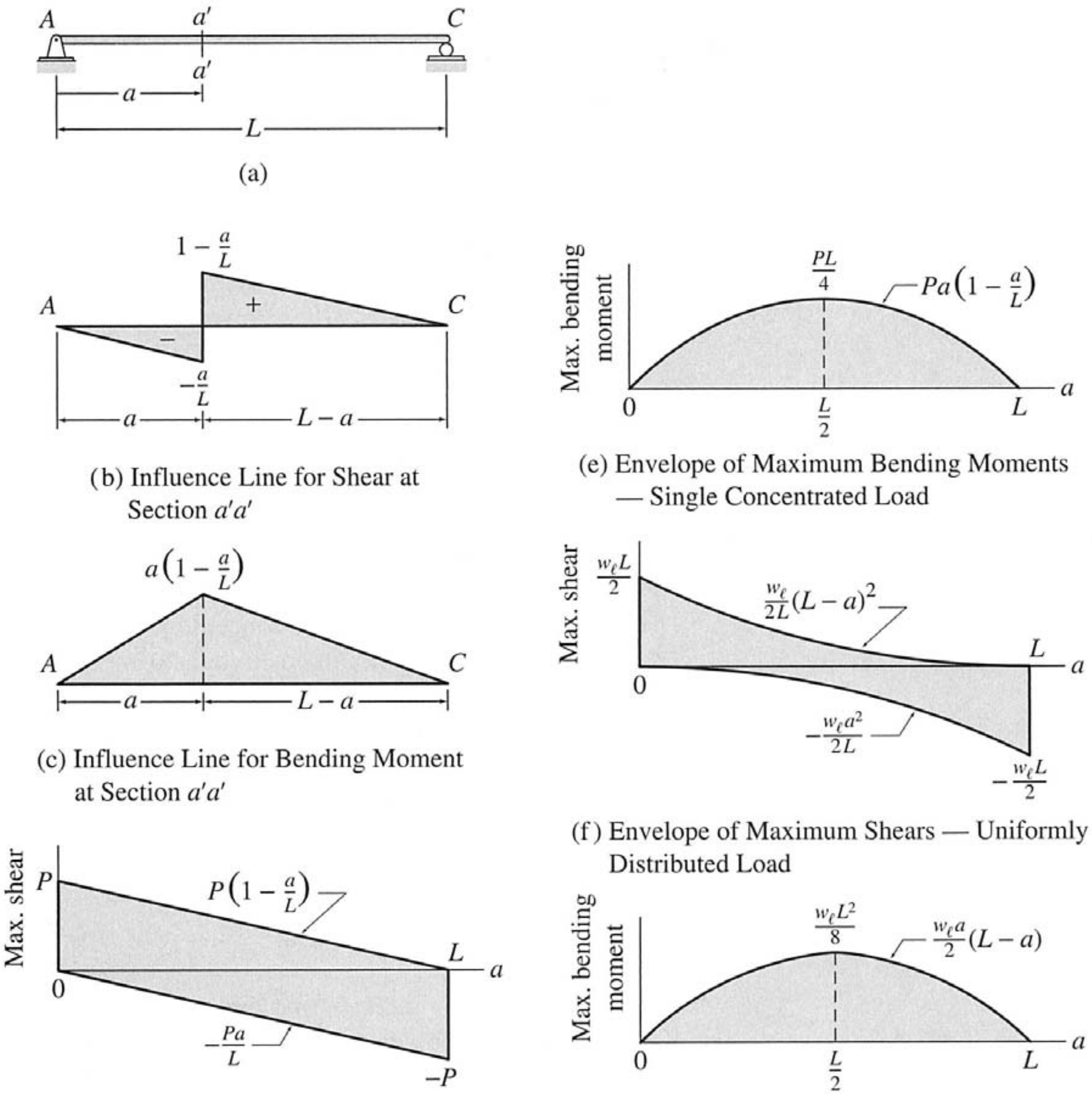
$$F_{BC} = 32(60)\left(\frac{1}{80}\right) = 24.0k\,(T)$$

Maximum F_{BC} = 44.5 k (T)

Absolute maximum response

Thus far, we have considered the maximum response that may occur at a particular location in a structure. In this section, we discuss how to determine the *absolute maximum* value of a response function that may occur at any location throughout a structure. Although only simply supported beams are considered in this section, the concepts presented herein can be used to develop procedures for the analysis of absolute maximum responses of other types of structures.

Single Concentrated Load



(a)

(b) Influence Line for Shear at Section $a'a'$

(c) Influence Line for Bending Moment at Section $a'a'$

(d) Envelope of Maximum Shears — Single Concentrated Load

(e) Envelope of Maximum Bending Moments — Single Concentrated Load

(f) Envelope of Maximum Shears — Uniformly Distributed Load

(g) Envelope of Maximum Bending Moments — Uniformly Distributed Load

FIG. 9.9