A DESCRIPTIVE STUDY OF DEVELOPMENTAL MATHEMATICS STUDENTS' BELIEFS AND AFFECTS

A Dissertation

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ABSTRACT

This qualitative study investigates and describes the beliefs and affects that three developmental mathematics students have about mathematics and about themselves as do-ers of mathematics. This study also investigates and describes how these students' beliefs and affects influence their learning of mathematics and influence mathematics teaching. Participants in this study are community college students enrolled in an elementary algebra class. The participants were purposefully selected and signed informed consent forms before data collection began. Methods of data collection included personal interviews, task interviews, and classroom observations. The results are presented in three case stories. Primary affective responses discovered in this study were: 1) students believe that the mathematics they experience in the classroom has no connection to the real world, 2) students believe that mathematics instruction should actively involve them, and 3) students desire to know how mathematics is applied, by whom, and for what purposes. The mapping metaphor for the development of affective responses is also presented. The recommendations that emerged from this study are: 1) teachers should clearly illustrate how various skills, procedures, and concepts taught in class relate to a non-school environment; 2) researchers need to understand how affective responses are formed in students because affective responses impact students' understanding,

motivation, and learning; and 3) teachers need to allow students to experience the full problem-solving process including the struggle and the feelings of doubt.

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APPROVAL OF THE DISSERTATION

This dissertation, A Descriptive Study of Developmental Mathematics Students' Beliefs and Affects, has been approved by the Graduate Faculty of the Curry School of Education in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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DEDICATION

This dissertation is dedicated to my high school mathematics teacher,

Ms. Elizabeth Bly,

whose enthusiasm for mathematics was contagious.

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I have many people to acknowledge for the completion of this dissertation and this degree.

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To Cal, Chris, and Joan: Thank you for your time, your honesty, and your willingness to participate. I am eternally grateful!

Finally, if anyone is reading this who is currently writing a dissertation or contemplating the thought, I leave you with the following words of advice. In the words of Andrew Wiles, the mathematician credited with solving Fermat's Last Theorem, "You have to have tremendous faith that you can do it, tremendous faith that you are right and you have to never, ever give up."

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CHAPTER 1: INTRODUCTION

This study focuses on developmental mathematics students, and consequently, on developmental mathematics education. This chapter provides a brief overview of the study including basis, intent, and justification. Chapter Two provides a description of developmental students as well as an overview that includes the nature, purposes, impact, and reform of developmental mathematics education.

Nature of this Study

Basis of the Study

Even though developmental mathematics education comprises a large part of college mathematics courses, "developmental mathematics students, teachers, and curricula have been the focus of little sustained research and scholarship" (West, Garofalo, & Perdue, 1993, p. 81). Because developmental mathematics students have not been the focus of much research, I believe there is a gap in the field of developmental mathematics education. This gap is not so much a lack of information as it is a lack of *detailed* information. Although literature dealing with students' affect and mathematics education abounds, many research results are reported in terms of quantity not quality. In other words, many of these past studies surveyed large numbers of developmental mathematics students about how they felt about certain aspects of mathematics and then assigned scores to Likert-scale-type responses so statistical calculations could be made. A few quantitative studies incorporated some qualitative aspects. Even research that includes qualitative components can sometimes yield results that are not very detailed however. As one such researcher noted, "This exploratory study, due to its breadth, may offer other investigators interested in beliefs about mathematics several starting points for further investigation. The investigations *may* be of a more in-depth nature of a particular aspect or of a confirmatory or non-confirmatory nature about a conclusion from this study" (Willis, 1992, p. 16, [italics added]). An "in-depth" study is exactly what this study is.

Research Question

The purpose of this in-depth study is to investigate and describe the beliefs developmental college students have about the nature of mathematics and its teaching and to explore how these beliefs affect the students' thinking about mathematics and their doing mathematics. The research questions of this study are:

1. What are the beliefs and affects developmental college students have about the nature of mathematics and its teaching? This question includes:
    a. What is mathematics?
    b. What does it mean to "do" mathematics?

c. What constitutes a "math problem"?

2. What beliefs and affects do developmental college students have about themselves as do-ers of mathematics (including issues related to control and metacognition)?

3. What effect do these beliefs and affects have on students' doing mathematics and on their preferences for teaching methods, including: (a) student problem–solving strategies, (b) strategies students used to learn mathematics, and (c) teacher actions that help and hinder that learning process?

Answering these questions in a meaningful way will require developmental college students to tell us their "stories" of mathematics experiences in qualitative case studies.

Potential Outcomes and Significance

I suspect, after reading the stories told by developmental mathematics students, we will discover that they "know" many things about what mathematics is and how it should be taught, but that many of these ideas will not be accurate interpretations about the nature of mathematics or the real definition of what it means to "do" mathematics. I believe the outcome of this study will benefit mathematics educators by shedding a highly focused beam of light on students' beliefs including how those beliefs affect mathematics teaching and learning.

The participants in this study were developmental mathematics students in a community college and the results of this research will benefit developmental mathematics educators. Because of the similarities in course content and student characteristics, however, there are also implications for high school and four-year college educators. The results of this study are applicable immediately because any developmental mathematics educator (secondary or post-secondary) can gain insights into these students' beliefs, knowledge, and attitudes. The case studies generated by this research add to the depth of the research base in developmental mathematics education. As one investigator observed, "before this building or rebuilding can take place, increased knowledge of the current mathematical belief structure of the students is a feasible requirement" (Willis, 1992, p. 12). Very little research conducted on developmental mathematics students focused on their beliefs about mathematics (Hart, 1989; West, Garofalo, & Perdue, 1993; Willis, 1992). The lack of research in this area and its qualitative nature add to the importance of this study.

Therefore, if this study of developmental students' beliefs about the nature of mathematics and its teaching can add to the knowledge of mathematics educators so they may encourage their students to further "build beliefs about what mathematics is, about what it means to know and do mathematics, and about . . . themselves as mathematics learners" (NCTM, 1989, pp. 16-17), then it is justified.

CHAPTER TWO: DEVELOPMENTAL MATHEMATICS EDUCATION

Nature of Developmental Mathematics Education

Definition of developmental mathematics. Developmental mathematics, also known as introductory college mathematics or remedial mathematics, is defined in a variety of ways. Most definitions take the traditional view of calculus as the cornerstone of a mathematics program. The American Mathematics Association of Two-Year Colleges (AMATYC) (1994) defines introductory college mathematics as "the curriculum taught at two-year colleges and in the lower division at four-year colleges and universities below the level of mathematical sophistication traditionally associated with the study of calculus or other mathematics courses at that level" (p. iv). The National Research Council (NRC) (1991) defines "school-level" mathematics as those courses "below the level of calculus" (p. 9). Koch (1992) differs from these calculus-referenced definitions for developmental mathematics when she indicates that "developmental mathematics programs" are for those students "who are deficient in basic mathematical skills" (p. 12). Albers (1983) defines college "remedial mathematics" as "high school level courses that are taught in colleges and universities" like "arithmetic, general mathematics, and intermediate algebra" (p. 191). He adds, "Some universities and private colleges tend to regard courses in college algebra and trigonometry as being remedial" (Albers, 1983, p. 191). I will refer to the terms developmental mathematics and introductory college mathematics interchangeably in this document. The term remedial mathematics will not be used because of negative connotations I do not wish to convey.

Developmental mathematics courses. The operational definition that specifies which courses fall under the umbrella of developmental mathematics is consistent with the definitions of developmental mathematics. According to AMATYC (1994), the introductory college mathematics curriculum "includes college algebra, trigonometry, precalculus, finite mathematics, and introductory statistics, as well as all courses presently characterized as developmental mathematics" (p. iv). McDonald (1988) includes "Computation / Arithmetic / Basic / Developmental Math, Beginning / Elementary Algebra, . . . Intermediate Algebra, . . . General Trigonometry, Vocational Math, Business Math, High School Geometry, . . . Pre-Algebra, . . . College Algebra and Pre-Calculus" in her list of developmental mathematics courses (p. 9).

The courses that have been included in developmental mathematics education programs have ranged from basic and vocational mathematics to pre-calculus. Typically, all of these are offered in high school. For the most part, developmental mathematics programs "do little more than mimic high school programs, both in content and in methodology" (Garfield, 1991, cited in Koch, 1992). Considering the national call for widespread curricular reform in high school (NCTM, 1989; NRC, 1989; NRC, 1991), the similarities between high school and developmental mathematics should be a concern for all mathematics educators (Koch, 1992; AMATYC, 1994).

Developmental programs need reform as much, or more than, high school programs. According to Albers (1983), "Redesign of the first two years of the college mathematics curriculum might have a wonderful impact on remedial mathematics, for at present remediation at the college level seems to mean teaching the same old high-school stuff—only LOUDER" (p. 191). Davis (1989) agrees that developmental courses tend to repeat high school content only with methods that are "faster" and "louder" than the methods employed before. Algebra, one of the most prominent courses in high school, "was the principal developmental math course reported" in McDonald's (1988) survey of 145 developmental mathematics programs (p. 10). McDonald (1988) describes the courses in this way:

> At both levels, beginning/elementary algebra courses placed emphasis on operations with signed numbers and real numbers, with one-half of all courses continuing to emphasize computational skills. It is in this course that a decline is seen in the amount of time and effort spent studying estimation and approximation skills, basic geometry, and the use of math in everyday situations, and the interpretation of charts and graphs. Little or no effort is spent on elementary statistics or probability. (p. 10).

The emphasis on computation is not restricted to algebra courses; it permeates the developmental mathematics curriculum and is reflected in the method of instruction.

Development and Goals of Mathematics Education

Development of Developmental Mathematics Education

One understands the purposes of developmental mathematics education best when one realizes why developmental mathematics education developed.

Need for developmental mathematics education. In order to best describe the specific goals and purposes of developmental mathematics education, it seems beneficial to examine how mathematics education arrived in its present condition. There are many reasons, among them, secondary school programs that do not seem to be producing graduates with skills in mathematical communication, reasoning, or problem–solving. The National Research Council (NRC) states, "About two-thirds of all college mathematics enrollments are in school-level courses" (NRC, 1991, p. 9). The NRC uses the term "school level" to denote content usually taught in high school. In fact, generally speaking, "the profile of mathematics in higher education is not much different from that of mathematics in high school" (NRC, 1991, p. 9). According to Koch (1992), "A large number of students enter college each year not knowing basic mathematical concepts such as fractions, decimals, and percents" (p. 12). Furthermore, "although many students take mathematics for at least 9 years in elementary school, junior high school, and high school they lack the fundamental processes necessary to be successful in college-level mathematics courses" (Koch, 1992, p. 12).

This is not a new problem. Over a decade ago, Tucker (1984) observed that "most problems facing colleges, two-year and four-year, seem to be the result of problems in the secondary schools" (p. 31). He cites "lack of basic skills in high-school graduates" as one of the major reasons college courses are not college-level (Tucker, 1984, p. 31). Furthermore, the courses designed to help the student "catch up" to the college level are very frustrating, both to the student and to the faculty. Often these courses are "a trying experience . . . because the pace is too fast (remedial courses tend to have a lot of ground to cover, but if students did not learn the material when taught at a normal pace, teaching it faster is not going to help)" (Tucker, 1984, p. 31).

Goals of Developmental Mathematics Education

There are several goals of developmental mathematics education. These include giving students a second chance to learn high school mathematics, preparing students for college-level mathematics courses, and many of the goals of mathematics education in general as well as vocational education.

Goal: Second chance. Many studies and reports in the last 15 years tell the same sad story: students entering college are not prepared for traditional college-level mathematics. The American Mathematics Association of Two-Year Colleges

(AMATYC) (1994) reveal that more and more, the students who are "seeking entrance to postsecondary education" are "mathematically underprepared" (p. iv). The NRC (1989) tells the grim fact that "most students leave school without sufficient preparation in mathematics to cope . . . with college requirements for mathematical literacy" (p. 2). Dossey, Mullis, Lindquist, and Chambers (1988) explain that "too many students leave high school without the mathematical understanding that will allow them to participate fully as workers and citizens in contemporary society" (p. 9). This sad trend has continued into the twentieth century as evidenced by this finding from the 1992 National Assessment of Educational Progress (NAEP): according to the "overall average mathematics proficiency and achievement level results for the nation" only 16% of twelfth graders are at or above the "proficient" level (NAEP, 1993, p. 34). Those realizations led to one of the major goals and purposes of developmental mathematics education: a second (or third or more) chance for a student in college to learn the mathematics s/he was supposed to have learned in high school but did not.

Goal: College preparation. The underprepared population of students entering colleges and universities helped create another of the major goals and purposes of developmental mathematics education: to prepare students for college mathematics courses (usually calculus) that they are required to take in order to graduate. Colleges "reported a 10- to 30-percent rise in demand . . . for remedial coursework in mathematics for incoming freshmen" (Dossey, Mullis, Lindquist, &

Chambers, 1988, p. 9). That demand has continued to increase; "in fact, the proportion of college students enrolled in mathematics who studied calculus or advanced mathematics courses remained fairly constant over the 20 year period 1970-1990, while the proportion of students studying remedial mathematics greatly increased" (Albers, Loftsgaarden, Rung, and Watkins, 1992 cited in AMATYC, 1994, p. iv).

Garofalo (1988) addressed the issue of why developmental mathematics exists by discussing the question "What is the main purpose of developmental mathematics programs at colleges and universities?" (p. 89). He summarized the reasons that included, "To give students another chance to learn the mathematics they should have learned in high school," to allow students the "opportunity to learn the mathematics" they will need to succeed in college, and in order "to prepare students for calculus" (Garofalo, 1988, p. 89). Although a major reason for the existence of developmental mathematics courses has been to prepare students for other classes, usually calculus, that reality is changing because of the impact of reform efforts by NCTM, NRC, and AMATYC. Garofalo (1988) echoes those sentiments of reform by stating, "Preparation for calculus should not be the *sole* determinant of developmental mathematics curricula" (p. 90). He recommends that decisions about what topics to include and omit in developmental mathematics be made on the "actual and specific needs" of the students rather than their "assumed and general needs" (Garofalo, 1988, p. 90).

Specifically, the goals and purposes of developmental mathematics education are to given students another chance to learn high school mathematics and to prepare students for college mathematics (generally calculus) required for graduation. Although these are the primary reasons developmental mathematics programs exist, other goals and purposes are included, namely those of mathematics education in general.

General Goals of Mathematics Education

Goal: Numeracy. Some of the goals and purposes of developmental mathematics education are the same as the goals and purposes of mathematics education in general. One of those goals is "numeracy" (Paulos, 1988), demonstrated by mathematically literate individuals who can and do make use of mathematics in their lives. We want mathematics to prepare our students for the future: getting jobs, choosing and obtaining careers, succeeding in higher-level mathematics education, and preparing them to be better citizens of our world. We want our students to meet the goals of the National Council of Teachers of Mathematics' (NCTM) (1989) *Standards*: we want them to value mathematics, become confident mathematical problem solvers, and to communicate and reason mathematically. It should be mentioned that although "the recommendations of the [NCTM (1989)] *Standards* were directed to the teaching and learning of mathematics in grades K-12, they are also applicable to developmental mathematics courses" because the content is the same (Skinner, 1993, p. 2). We want the same goals the NRC (1989) does for mathematics education: "to learn practical skills for daily lives, to understand quantitative aspects of public policy, to develop problem-solving skills, and to prepare for careers" (p. 6). In short, we want our students to learn mathematics because we feel it is important.

Historical goals. Historically, many reasons have been given for why the study of mathematics is important. Smith (1989) recounts, "since the days of the ancient Greeks, mathematics has been seen not only as useful and playful but also as possessing a certain kind of magic, a magic that might provide a way to unlock the mysteries of the universe" (p. 448). Unlocking universal mystery was only the beginning of beliefs surrounding mathematical power however. It was also believed to aid "spiritual development" by the Pythagorean Brotherhood, to be "an aid to morality" in the 1800's, and, at "the turn of the century," it was revered because of "the exercise it gave to the brain," which was supposed to improve thinking in general (Driver, 1987, pp. 479-480). Throughout much of its history, mathematics was learned and revered because of the "assumption that ability or training in mathematics will transfer to other areas" of learning. (Smith, 1989, p. 449). Although bits and pieces of these previous beliefs survive today, most people view mathematics as important because of its practical uses. Dossey, Mullis, Lindquist, and Chambers (1988) report "mathematical understanding" as the device that will allow students "to participate fully as workers and citizens in contemporary society" (p. 9).

Furthermore, mathematics is viewed "as a way of thinking, communicating, and resolving problems" (Dossey, Mullis, Lindquist, & Chambers, 1988, p. 13). It is precisely this viewpoint that has resulted in the current furor over problem solving in our schools.

Additional goals: Vocational education and job training. Having stated that some of the goals of developmental mathematics education are the same as those for mathematics education in general, let me now clarify the area of difference between the two. The goals of general mathematics education are *included* in the goals of developmental mathematics education, but developmental mathematics also has additional goals. In the 1980's, Taylor (1984) remarked, "Until now the secondary school mathematics program has tended to concentrate on two areas: preparation for four-year colleges, and basic skills instruction" (p. 468). The NRC (1989) illustrate how the foci have changed over the years:

> Historically, schools in the United States were designed with a dual mission: to teach all students basic skills required for a lifetime of work in an industrial and agricultural economy and to educate thoroughly a small elite who would go to college en route to professional careers. As the needs of society have changed—as the fraction of students preparing to work in factories or on farms has declined — the balance of these two goals has shifted. Today's schools labor under the legacy of a structure designed for the industrial age misapplied to educate children for the information age. (p. 11).

One important point must be made here. The "basic skills required for a lifetime of work" in our current age of technology have changed from the basic skills of yesterday's age of agriculture and industry (NRC, 1989, p. 11). Computational skills are no longer the main concern of employers. The NCTM (1989) gives the following list of "mathematical expectations for new employees in industry" based on the 1987 remarks by "a noted industrial mathematician," Henry Pollack:

- The ability to set up problems with the appropriate operations
- Knowledge of a variety of techniques to approach and work on problems
- Understanding of the underlying mathematical features of a problem
- The ability to see the applicability of mathematical ideas to common and complex problems
- Preparation for open problem situations, since most real problems are no well formulated
- Belief in the utility and value of mathematics (p. 4).

Since developmental programs are designed to teach mathematics usually taught in high school, it is natural that the two programs would have similar goals. This implies it was only recently, late 1980's, that developmental mathematics programs, like general mathematics education programs, began to realize acquisition of basic skills was not enough preparation for the workplace and that not every student was destined to continue their education at a four-year institution. As a result, additional goals for developmental education were set. Grubb and Kalman (1994) list a few of the additional goals for developmental courses, like using them as prerequisites "for vocational education and job training" (p. 54). Because of the advances made in today's age of technology, students with only a high school education have a very slim job market to choose from, especially if the goals of high school are not met.

Many students do not realize the importance of mathematics in the job market. Studies show students who must take "high- school level mathematics courses in college" are "severely limiting his/her vocational choices" (Albers, 1983, p. 193). One of the myths exposed by the NRC (1991) is that "most jobs require little mathematics" (p. 11). The truth, however, is that "although a working knowledge of arithmetic may have sufficed for jobs of the past, it is clearly not enough for today, for the next decade, or for the next century" (NRC, 1991, p. 11). The widespread use of computers in society has also influenced goals of developmental mathematics education. Jobs, which in the past required only minimal skills, like cashiers, today require technological experience. The NRC (1989) sums up the differences in this way:

> Jobs that contribute to this world economy require workers who are mentally fit—workers who are prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns, and to solve unconventional problems. It is *these* needs, not just the needs for calculation (which is now done mostly by machines), that make mathematics a prerequisite to so many jobs. More than ever before, Americans need to think for a living; more than ever before, they need to think mathematically. (p. 1).

Therefore, developmental mathematics education, like school mathematics programs, must teach mathematics that will prepare our students for the job market—after all, "today's mathematics opens doors to tomorrow's jobs" (NRC, 1989, p. 2).

Developmental Mathematics Students

Developmental mathematics students are a diverse group of students, yet they do have something in common. The diversity is obvious since students from different nationalities, all races, both genders, all levels of socio-economic status, and all kinds of backgrounds may be found in developmental mathematics courses. They all share at least one thing, however: a past filled with mathematics failure, and, often, anxiety and frustration that results in avoidance of the subject whenever possible. Benander, Cavanaugh, and Rubenzahl (1990) describe the developmental mathematics students at Greenfield Community College as "reserved, anxious, hesitant, and sometimes sullen" (p. 26). This is really not a surprising description when one realizes the "12 years of misconceptions and procedural bugs" which many of these students have built during their, often unsuccessful, school careers (Brown & Burton, 1978 cited in Koch, 1992, p. 12). The vast majority of students who are in developmental mathematics classes did not experience the K-12 curriculum as envisioned by NCTM (1989)— specifically, they never experienced opportunities to learn to reason and communicate mathematically through problem–solving activities. As a result, they develop coping strategies to deal with the traditional teaching method of imposition. They "do not apply rules and procedures correctly" because, instead of a conceptual understanding of the process, they "rely heavily on memory," which often fails them (Koch, 1992, p. 12).

AMATYC (1995) characterizes the student population served by developmental mathematics courses in this way:

> Some are traditional full-time students who are recent high school graduates. Others, particularly those at two-year colleges, are from widely diverse populations and fall into one or more of the following categories. They are older, work a full- or part-time job while attending college, manage a household, are returning to college after an interruption in their education of several years, intend to enter the work force after obtaining an associate degree, intend to work toward a bachelor's degree either at a transfer institution or in the upper division of their present four-year college or university, are studying for a degree as a part-time student, have English as a second language, need formal developmental work in a variety of disciplines and in study skills, have no family history in postsecondary education, or have disabilities that require special accommodations (p. 4).

Benander, Cavanaugh, and Rubenzahl's (1990) description of their developmental students at Greenfield Community College sound very much like the AMATYC's (1995). The Greenfield students consist of a variety of students who "lack confidence in their mathematical ability" and whose "average age is 30" (Benander, Cavanaugh, & Rubenzahl, 1990, p. 26). "Sixty percent of the students are women," and some of them "are returning to college after many years, often as single mothers" (Benander, Cavanaugh, & Rubenzahl, 1990, p.26). In interviews, a few developmental mathematics teachers described "money problems, car problems, family problems, and court problems" as issues that many of their students must deal with in addition to school-related issues (West, Garofalo, & Perdue, 1993, p. 82). Many of the developmental mathematics instructors interviewed described their students as lacking maturity, determination, confidence, and persistence (West, Garofalo, & Perdue, 1993). Their students, like the other developmental mathematics students discussed, often have harmful beliefs about mathematics and themselves as mathematicians, which cause them to "act and think in unproductive ways" (West, Garofalo, & Perdue, 1993, p. 83). It is apparent that students with these additional burdens—parenthood, careers, harmful beliefs, etc.—may not have the time and other resources available to the traditional college student. Developmental mathematics programs must take these factors into account.

Impact of Developmental Mathematics Education

In spite of the monumental effort, "relatively little is accomplished by remediation programs" (NRC, 1989, p. 13). Steen (1991) agrees, reporting that, for the most part, efforts to remediate mathematics are a failure. Because of their "lack of mathematical power, many of today's students are not prepared for tomorrow's jobs. In fact, many are not even prepared for today's jobs" (NRC, 1989, p. 1). An instructor at Columbus State Community College in Columbus, Ohio, Edward Laughbaum cites an example of the failure he sees in remedial mathematics programs:

"During the 1989 summer quarter, all 62 Intermediate Algebra students at Columbus State Community College were tested on most of the traditionally taught skills at the remedial level. The study . . . shows that 48% of the students couldn't factor polynomials at the beginning of the course, nor could they factor the same polynomials at the end of the course. . . . Even worse, seventy percent failed to learn how to solve radical and quadratic equations during the course!" (p. 7).

The National Research Council (1989) explains the failure in this way: "No one—not educators, mathematicians, or researchers—knows how to reverse a consistent early pattern of low achievement and failure. Repetition rarely works; more often than not, it simply reinforces previous failure" (p. 13). The reality is "the best time to learn mathematics is when it is first taught; the best way to teach mathematics is to teach it well the first time" (NRC, 1989, p. 13). Demana (1988) states the grim fact that "far fewer students who enter college needing remedial work graduate than nonremedial students, even in the most successful programs" (p. 289). Therefore, he concludes, "the most effective way to reduce the remediation in college is to accomplish the remediation in high school" to "ensure that students are adequately prepared for college-preparatory mathematics courses when they take them" (p. 289).

Small programs or few course offerings cannot be the blame of students' failures in mathematics education, however. According to the NRC (1989), "mathematics education takes place in each of 16,000 public school districts, in another 25,000 private schools, in 1,300 community colleges, 1,500 colleges, 400 comprehensive universities, and 200 research universities" (p. 38). It is disheartening to realize that all of this results in very little mathematical success—success being defined as the ability to do more than simply compute. Unfortunately, "the Educational Testing Service- (ETS) administered National Assessment of Educational Progress (ETS, 199), which continually surveys American education, reports that 'The number of students mastering the basic skills has been rising, but the number who display higher-order skills has not increased' (p.2)" (Chaffee, 1992, p. 2). Although, "critical thinking is rarely taught explicitly and systematically at *any* level of education," the need for it "is particularly acute in developmental education" (Chaffee, 1992, p. 2).

West, Garofalo, and Perdue (1993) found that "although a few instructors had individual success stories, overall they reported very low student success rates" in developmental courses (p. 82). The instructors interviewed in that study described their low student success rates, and the effectiveness of developmental mathematics courses in general, by revealing, "Many students dropped out of courses; many students failed the courses and needed to repeat them (but often did not do so); and many of those students who passed the courses did not do well in subsequent mathematics courses or courses that used mathematics" (West, Garofalo, & Perdue, 1993, p. 82).

Reform of Developmental Mathematics Education

Because developmental mathematics education has not achieved its goals and because of the *Standards* (NCTM, 1989) and other reform documents, reform *is* taking place. The teaching strategies, goals of instruction, and methodology of developmental mathematics educators, partly because of reform in high school mathematics teaching, is one of the major areas of developmental mathematics education where reform is evident.

> It is, in fact, nothing short of a miracle that the modern methods of instruction have not yet entirely strangled the holy curiosity of inquiry; for this delicate little plant, aside from stimulation, stands mostly in need of freedom; without this it goes to wreck and ruin without fail. It is a very grave mistake to think that the enjoyment of seeing and searching can be promoted by means of coercion and a sense of duty. (Einstein, cited in Smith 1989, p. 454).

As discussed in the previous sections, developmental mathematics is very similar to high school mathematics. Silver, Kilpatrick, and Schlesinger (1990) provide this description of the mathematics high school experience:

> For years, teachers have complained that students are bored by high school mathematics. By the time students graduate, they have forgotten most of what they were supposed to learn, and they proclaim, often with a hint of misplaced pride, "I never was any good at mathematics." The mathematics they say they were no good at was, by and large, the mathematics of routine computation and manipulation (usually with some formal proof thrown in).

> Mathematics was little more than a collection of recipes that students were expected to memorize and practice. (pp. 5-6).

Garofalo (1988), confirming the aforementioned *mathematics of routine computation,* makes this observation of developmental mathematics instruction: "Spending less time on the more important and harder-to-learn topics, and more time on the less-useful and easier-to-learn topics makes little sense" (p. 92). Developmental students use the strategy of memorization also, and, probably, they learned it when they were in high school. This strategy works well since the method of instruction used in most developmental classrooms is lecture (McDonald, 1988). Because most teachers believe it to be effective, they use lecture to transmit their knowledge to their students. Recently, however, "mathematics teachers are realizing that learning is not receiving and remembering a transmitted message" (Silver, Kilpatrick, & Schlesinger, 1990, p. 6).

There is some indication that developmental instructors are realizing the error in their previous thinking. Interview results from developmental mathematics instructors have revealed that they "feel they were doing the best they could" to get across to their students but with very little success (West, Garofalo, & Perdue, 1993, p. 83). Many of these instructors believed "that better ways to teach this students existed," but they did not know what they were and felt "helpless" because of it (West, Garofalo, & Perdue, 1993, p. 83). Recent research has also shown that the lecture method is not very effective in teaching developmental mathematics (Lamm, 1993).

The AMATYC and other organizations have made issues of reform their concern as evidenced by the development of the *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* (AMATYC, 1995). Much of the reform taking place in developmental mathematics education stems from the K-12 reform movement begun by the NCTM and NRC in the 1980's. Many of the suggestions revolve around the idea that mathematics, to be meaningful, must be constructed by the individual learning it rather than simply be transmitted from teacher to student. It is my belief that this construction process is influenced by the individual's beliefs about the nature of mathematics and its learning. Furthermore, I think we cannot effectively study the complex nature of these beliefs with the quantitative, statistics-based studies reported in mathematics education journals in the past. The case stories generated by this study give much-needed insight into the complexities of developmental mathematics students' beliefs and affects.

CHAPTER THREE: RELATED RESEARCH IN DEVELOPMENTAL EDUCATION

Introduction

The previous chapter described developmental mathematics students and offered brief descriptions of the nature, purposes, impact, and reform of developmental mathematics programs. In this chapter and in the next chapter, a review of the literature that addresses the topics that appertain to the research questions is presented. These topics include related research in developmental mathematics education, problem–solving, and affective issues in mathematics education (e.g. students' beliefs and attitudes).

Related Research in Developmental Education

Overview of Research in Developmental Education

West, Garofalo, & Perdue (1993) concluded that more research was needed in the field of developmental mathematics, especially "how students learn mathematics" and how to help students with a history of failure and negative experiences in mathematics to "develop confidence in their abilities to learn mathematics" (p. 84).

Other researchers have come to the same conclusion: developmental mathematics is not very commonplace as a subject of research. In their review of current research, O'Hear and MacDonald (1995) sought to answer the question, "What is the state of research in developmental education?" (p. 2). They reviewed 52 randomly selected studies—including research articles, conference presentations, and research reviews—conducted over the last decade (1985 - 1995). The results were significant though not especially surprising. Of the 52 studies, only 3 (about 6%) were on the subject of mathematics compared to 20 (almost 39%) on the subject of reading (O'Hear & MacDonald, 1995, p. 3).

Their review of research in developmental education also revealed that the vast majority of the research was quantitative (40 out of the 52 studies, or about 77%) rather than qualitative (7 out of the 52 studies, or about 13%) in nature (O'Hear & MacDonald, 1995, p. 3). Although the authors conceded that the number of research studies in their sample is relatively small, they maintained that "the numbers *do* reflect the incidence of research in the field" (O'Hear & MacDonald, 1995, p. 4, [italics added]). They made that assertion because one of the stated purposes of their review was "to assemble a representative sample of research practices current in the profession" (O'Hear & MacDonald, 1995, p. 2).

The following overview of developmental mathematics education studies and results reflect the larger trend of the research: most are quantitative rather than qualitative. By the small number of studies included, this review also supports the claim that developmental mathematics education has not received the attention in research that other subject areas have enjoyed. This is a situation that desperately needs to be addressed and corrected since "the quality of knowledge and its responsible applications influence both the success of developmental programs and the academic community's perception of and knowledge about developmental education" (O'Hear & MacDonald, 1995, p. 2). Considering the modicum of success of developmental mathematics education efforts as discussed in the previous chapter, it is a reasonable assumption that the amount and quality of research in the field is at least partly responsible for that lack of success. O'Hear and MacDonald (1995) state the obvious conclusion, "Clearly, the field could greatly benefit from more research studies and more researchers" (p. 4).

Findings of Research in Developmental Mathematics Education

As most of the research in developmental mathematics education, and developmental education as a whole, is quantitative in nature, those types of studies will be discussed first. Skinner (1993) sought to determine that of two types of instructional methods were most influential on developmental algebra students' achievement. The design was a common one in quantitative research: a pre- and post-test were administered to both the control and treatment groups in order to compare the two. The Assessment and Placement Test (APT) was used as the pretest. The variable of interest between the two groups was how type of instruction received affected student test scores. The control group, consisting of 29 community college students, was taught by the more traditional procedure-based method of instruction while the treatment group, consisting of 36 community college students, was taught conceptually. The groups were selected randomly.

The "procedural instruction" given to the control group was characterized by the emphasis "on the learning of procedures"; and, "the teaching style was that of teacher example" (Skinner, 1993, p. 3). The treatment group received "conceptual instruction" which emphasized "the building of understanding of the underlying concepts behind procedures;" and, the teaching style "was that of providing situations from which students could construct their own knowledge" (Skinner, 1993, p. 3). The sample of 65 students (both groups) received instruction for four weeks (total time of instruction was 20 hours). The "student achievement was assessed by testing the students after instruction" (Skinner, 1993, p. 3). Posttests were given after two weeks of instruction and after four weeks of instruction. The posttests included items that were both "skill-oriented" and "transfer of knowledge" items, defined as "those which tested content that was covered in instruction, but in situations different from those in instruction" (Skinner, 1993, p. 3). The results of this study revealed no statistical significance in the posttest scores of the two groups.

In a similar study, Lamm (1993) compared the effects of two different teaching strategies on developmental mathematics students enrolled in Developmental Business Mathematics by comparing test scores and final grades.

Like Skinner (1993), this was also a quantitative comparison study that used a pre-post-test design. The pretest and the posttest were modeled after the "Texas Academic Skills Program (TASP)" (Lamm, 1993, p. 1), a standardized test used by community colleges in Texas for placement and assessment of basic skills. The teaching strategy used for the experimental group was the "concept attainment method" (CAM), while, for the control group, the "traditional lecture/discussion method" was used (Lamm, 1993, p. 48). The CAM focused on "connecting activities" designed to build connections between the three ways of representing concepts: symbolic, verbal, and physical/visual (Lamm, 1993, p. 48). Group work, problem solving (by students), concept formulation (by students through activities), teacher demonstrations, and discussion (both students and teacher) were all included in the CAM. The traditional lecture/discussion method "involved the introduction of concepts through verbal/oral means. Example problems were worked for the subjects, and any questions they had were answered. Only the symbolic and verbal representations were used with the control group" (Lamm, 1993, pl. 48). The sample consisted of 29 community college students — 17 were in the control group and 12 were in the experimental group—enrolled in "two sections of a Developmental Business Mathematics course which had few business applications" (Lamm, 1993, p. 44). The two sections were randomly assigned to either the control group or the experimental group. One of the main results of this study was that "the students participating in the concept attainment method with group work did *not* on the average score higher on the posttest with the pretest as a covariant covering the TASP objectives than students who participated in the lecture/discussion method" (Lamm, 1993, p. 55 [italics added]).

Willis (1992) used a combination of quantitative and qualitative methods in her research. The results of Willis' (1992) study laid the foundation for this research on developmental mathematics students' beliefs. Her research focused "on the beliefs about mathematics" held by "students enrolled in community college mathematics courses" (Willis, 1992, p. 4). In fact, both Willis (1992) and I elected to use the same community college to conduct our research. For these reasons, I believe an in-depth description of her study and review of her findings are appropriate. The primary goal of her study was "investigate and describe beliefs about the nature, learning, and benefits of mathematics held by community college students who were enrolled in one of the four mathematics course levels of arithmetic, intermediate algebra, statistics, or differential equations" (Willis, 1992, p. 7). The result of the investigations and descriptions were to describe the "similarities and differences among the four levels of course enrollment" on three areas: "beliefs about the nature of mathematics as globally or rule oriented," students' ideas about how mathematics benefited them "as a tool and as a way of thinking," and "finding what strategies and activities" were used by the students for learning mathematics (Willis, 1992, p. 7). The sample for her study was 192 students enrolled in four courses: "33 in arithmetic, 76 in intermediate algebra, 71 in statistics, and 12 in differential equations" (Willis, 1992, p. 78). All of the students completed a three-page "survey packet concerning their beliefs about mathematics" and eight students (two from each course) were each interviewed for thirty minutes (Willis, 1992, p. 78).

Because Willis (1992) had eight different research questions guiding her study and the majority of her data came from the survey packet rather than from the interviews, the results are broad, general, and cover a wide range of topics. Because the focus was comparing similarities and differences between the four course levels, she expressed the results almost exclusively as comparison statements. Basically, she found the following:

1. Students in statistics responded significantly higher than students in intermediate algebra on the "mathematics benefits me as a tool" scale designed to measure students' beliefs about how mathematics can be used practically in real life situations (e.g. "decide on the better buy," help "earn a living" and "understand reports in my job") (p. 198).
2. Students in both differential equations and statistics responded significantly higher than students in intermediate algebra on the "mathematics benefits me as a way of thinking" scale designed to measure students' beliefs about how mathematics can be used in general reasoning and logic for solving "any type problem" (p. 198).
3. Responses to the survey question: How would you describe a mathematical way of thinking? "were categorized as logical/step-by-step,

numbers/formulas, complimentary/degretory, and miscellaneous. The largest category of answer for all course levels was the one named logical/step-by-step" (p. 199). Of the 33 arithmetic students, only 17 wrote a response to that question.

4. Seven of the eight interviewees "agreed with the statement that mathematics helped them think more systematically and logically," while six of the eight thought that, to some extent, they had gained something "from studying math that had helped them in solving non-mathematical problems" (p. 200). One interviewee "did not believe math would benefit her now or in the future" (p. 201). The other seven interviewees "spoke of math as being a benefit in the future but not necessarily to the exclusion of being a current benefit" (p. 201).

5. Survey responses indicated: (a) students in arithmetic scored highest in math anxiety followed by the students in intermediate algebra; (b) the lowest percentage of women students were found in the differential equations courses followed by the intermediate algebra courses; (c) that, regarding beliefs about the nature of math, intermediate algebra had the highest mean for rule orientation and the lowest mean for global orientation; (d) some of the most used strategies to learn math for intermediate algebra students were "attending math class each period, copying examples from the board, (and) paying attention to the math instructor's lecture;" and, (d) that, regarding benefits of

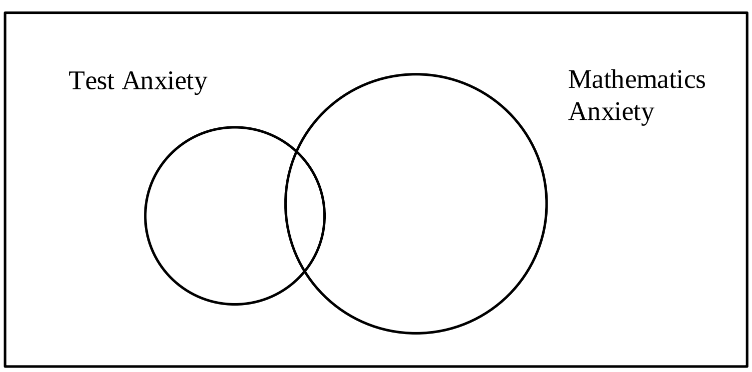
math (as a tool and way of thinking), "intermediate algebra had the lowest mean" of all the courses (p. 211).

Willis (1992) ascribes the limitations of her "investigatory study" to "certain confounding variables" like different instructors for the different levels that "were not controlled" (p. 238) and to the "breadth of the study" (p. 239). She lists several suggestions for future research to avoid similar limitations. One suggestion in particular is apropos for my study, it regards the use of surveys as a primary means of data collection. The researcher allows:

> Better information could be gained by focusing on a more specific aspect because, as noted by one of the arithmetic survey respondents, 'The questions on the survey are somewhat confusing because in some parts of math I'm fast and great, others slow and yet in another I'm terrible. It covers such a large difference in degree.' (p. 248).

Byrd's (1991) research, like mine, was qualitative in nature and based on the naturalistic paradigm. The main purpose of her study was "to begin to generate a theory of mathematics anxiety by describing its nature and antecedents" (Byrd, 1991, p. 71). She conducted "in-depth, semi-structured" interviews on six students enrolled in undergraduate developmental mathematics courses (Byrd, 1991, p. 75). The case studies generated by this research aided "in understanding the nature of mathematics anxiety, including manifestations of the phenomenon and whether it is state-like or trait-like" (p. 123). Respondents described anxiety in general terms with words like "tension, nervousness, stress, and worry," while the more specific terms implied either "emotionality" or "worry" (p. 123). Terms that implied emotionality included "a nervous stomach (also described as butterflies), sweating and sweaty palms, muscle contractions, a tightness in chest or throat, difficulty breathing, heart palpitations, tension headaches, restlessness, and feeling upset" (p. 123). Terms that implied worry included "fear of failure, feeling like a 'jerk,' and feelings of anticipation, disappointment in self and disapproval of self" (p. 123).

The question of whether mathematics anxiety was state-like or trait-like was tentatively answered when the researcher stated, "The fact that mathematics anxiety involves a specific stimulus implies it is more state-like than trait-like. However, the distinction is neither simple nor cut and dry" (p. 125). One of the "basic assumptions of the research," the idea "that mathematics anxiety and test anxiety are closely related," was supported by the data. The Venn diagram pictured below was used to illustrate the relationship between the two types of anxiety:



Related to the aspect of test anxiety was the attribute of "blanking out," a condition in which one participant "said she freezes up and cannot even think" (p. 131). The majority of the participants "perceived anxiety as debilitating" and that it interfered with test performance and learning (p. 133).

Byrd (1991) also discovered "a distinction between avoiding mathematics and simply choosing not to continue studying the subject" (p. 134). She received reasons other than avoidance from her respondents to explain why they "quit taking mathematics in high school" (p. 134). Some felt they had all the math they needed, others lacked the confidence that would allow them to believe they could pass another course, or they simply liked other courses better than they liked mathematics. Those respondents who truly avoided mathematics expressed "stronger negative feelings toward mathematics than simply choosing not to study the subject" (p. 135).

The final result of Byrd's (1991) study is the one most related to this study. It involved how her respondents perceived the nature of mathematics, and she explained the reasons and importance for this aspect of her study in this way:

> There must be something about the subject that causes people difficulty, that contributes to anxiety. In order to attempt to understand this aspect of the phenomenon of mathematics anxiety, the following question was asked, "Why math? How is it different from other subjects?" (p. 137).

Answers to that question "seemed tied to the personality of the respondent" (p. 137). One woman felt the difference was that in "courses like English or languages, there's more room for being creative and there aren't those rules" that exist in mathematics (p. 137). She went on to explain that although there were rules of a sort in something like art but that one didn't stop there, "once you learn those rules," the goal was then "to learn how to break them, overcome them" and she didn't see that in mathematics and felt "cheated" as a result (p. 137). Another theme that emerged was mathematics "cumulative nature" that many of the respondents had trouble with because they felt they "lost out on the basics and never seemed to be able to catch up or understand subsequent material" (p. 138). The final theme that resulted from Byrd's (1991) study and is related to my study is the idea of context. Most of her respondents felt that mathematics, unlike other subjects, had no context and therefore they could not get through it if they did not understand all of it in detail. One respondent likened it to reading a paragraph and still being able to get the "gist of it" even if she didn't understand "all the words in the sentence or the paragraph" because she could "get an idea from the other words" (p. 138).

Another qualitative study in developmental mathematics sought to find "the meanings successful females attached to their high school mathematics experiences" (Venters, 1992, p. 46). The researcher defined "successful" females as those "who had made the grade of A in the last remedial course offered by the community college" (p. 47). The other three criteria used to select the twelve participants were: they must be "first-year college students," they must be "females who had not been highly successful in mathematics in high school," and they must be of "traditional" age (18-20) (p. 47). Each of the twelve participants was interviewed once. All of the interviews "were conducted within a month's time and each interview lasted 45 - 75 minutes" (p. 53). The primary purpose of Venters' (1992) research was "to learn the meanings participants attached to their mathematics experiences in high school and, in doing this, to learn their perceptions of themselves as learners of mathematics" (p. 49). She focused on three categories: "perceptions of self as learner, influences on mathematics learning experiences, and the role of teachers in the participants' mathematics experiences" (p. 46).

The researcher presented the results of this study in six themes: internal belief system, role of the school, role of teachers, role of peers, role of family, and the lost year. The first major finding of this research was not related to the themes at all, however. Rather, it was related to the researcher's expectations:

> The participants were not the group of students that the researcher anticipated. They were not students who had experienced years of difficulty in mathematics in high school and had dropped out of the mathematics sequence. They were students who had taken three to four years of high school mathematics and, while not being among the top students, were nonetheless successful B and C students in high school mathematics. (Venters, 1992, p. 84).

Within the theme of internal beliefs, Venters (1992) revealed the following categories: gaining confidence in mathematical ability, learning to value time, and difficulties with visual and verbal mathematics. Her participants differed from previous research by contributing their success in mathematics to "new-found ability" rather than to effort (p. 86). Almost all of the participants expressed a connection between how much time they spent on learning mathematics out of class and success. Most made comments about realizing how important time was and how critical time management skills were to success. The difficulties in mathematics centered on "word problems, geometry, and graphing" for the majority of participants (p. 90).

Within the themes of the role of the school and teachers, Venters (1992) uncovered issues involving time in school, the teachers' time, guidance counselors' time, teachers' attitudes, and curriculum pace. The participants described "the scheduling of time in high school as incompatible with their needs" (p. 92). In contrast, they enjoyed the flexibility of scheduling time and classes in college as well as the freedom between classes. Almost all of the participants mentioned teacher's availability "as a sign of the teacher's willingness to help students" (p. 94). This availability referred to both the teacher's time during the class as well as time outside of class. According to the participants of this study, guidance counselors had little time for them and seemed to be concerned with only those students "who were going on to college" (p. 99). Since that was not a role any of them felt they fit into, "the role of guidance counselors in the high school experiences of participants of this study was negative or nonexistent" (p. 99). The participants viewed teacher attitudes as an important part of their experiences and preferred "teachers who were friendly, who showed concern, [and] who made learning fun" (p. 97). Regarding the pace of the curriculum, all of the participants acknowledged "repetition of courses as a possible method of success" and felt "learning algebra was much easier the second time around" (p. 101). Both peers and family members were found to be influential in participants' experiences and successes in mathematics. The final theme, the lost year, was created because "nine of the twelve participants lost a year of mathematics in some form during high school" (p. 108). Some reasons for the lost year were repetitions of failed courses, choosing not to take any more mathematics because they had met graduation requirements, and being advised not to take more mathematics because of low grades. All of the participants who had lost a year of mathematics mentioned peer influences as a major contributing factor.

Summary of Research in Developmental Education

Although much of the past research in developmental education has been quantitative in nature rather than qualitative, current trends are changing that focus. Likewise, developmental mathematics education (especially developmental mathematics students), which had not been the focus of much research in developmental education, is now becoming a topic that is receiving a lot of attention. As a result of these recent changes in the field, the research in this review is all from the 1990's. The studies in this review were selected because they are especially relevant to the research conducted in this study.

Both this study and Skinner's (1993) focus on developmental algebra students specifically. I believe my research builds on Skinner's (1993) by adding a qualitative element to the picture. She found no *quantitative* difference (as measured by pre- and posttests) in students who were taught by two different instructional methods but I believe there were differences—*affective differences*. My study attempts to confirm or deny that hypothesis by hearing, in the students' own words, descriptions of instructional methods they have experienced in mathematics courses.

My study, like Lamm's (1993), focuses on developmental mathematics students. Her study, like Skinner's (1993) showed no *quantitative* difference (as measured by pre- and posttests) in students taught by two different instructional methods. Again, I believe there were differences, the research design simply did not allow them to be found. I use a design in this research with the express purpose of discovering those differences.

Willis' (1992) research was crucial in the development of the research questions for this study. Her research also focused on developmental mathematics students (like Lamm and Skinner) but, unlike theirs, she was interested in students' beliefs not in comparing instructional methods. Willis (1992) research is like the rain forest out of which the particular tree of my study was selected. The focus of her study was very broad because it was an exploratory study. My research is very focused because it is a specific study of three individuals. Because her goal was of an investigatory nature, the research design she chose contained both quantitative and qualitative elements. As a result of her goals and her design, her results were very broad. My goal was to further her research by doing what she suggested, a more specific study without the limitations of having to quantify verbal statements made by participants.

I believe my study is based on hers in a variety of ways and goes beyond hers in significant areas. I replicate some of the questions she asked in her interviews in the interview guides for my study, others are not the same but more focused extensions of the ones she used. She also included participants from four different mathematics courses whereas I focus on only one, developmental algebra. The majority of data in Willis' (1992) research came from survey responses to questions. She admits there were concerns about this method because of different interpretations by participants and suggests future researchers address that concern. My study solves the dilemma by design: different interpretations become evident as the participant explains her/his responses during the in-depth interview process.

Byrd's (1991) study, like mine, is a highly focused qualitative study on developmental mathematics students. She was interested in a particular aspect of affect, anxiety. Her research is like a branch on the tree of my study. My study provides more context and a setting for interpretation of hers. My research is also a continuation of her findings on how developmental mathematics students perceive the nature of mathematics.

Venters' (1992) study was also qualitative in nature and, like mine, focused on developmental mathematics students' experiences in mathematics. Her research focused on the meaning successful females attached to those experiences while mine focuses on the effects those experiences have upon students' beliefs and attitudes. The researcher recommends further study "at a community college in another part of the country using a similar sample and population" to determine whether the themes that emerged were unique to the population she used (Venters, 1992, p. 127). I purposefully chose females for my study with the intent of extending Venters' findings. I purposefully chose an in-depth research design partly because of Venters' (1992) comments: "If the purpose of research is to identify and isolate factors within the educational experience which are impeding learning so that these factors can be removed or positively altered, then research must study these experiences from the perspectives of the learner, *in the learner's own words"* (Venters, 1992, p. 128, [italics added]).

CHAPTER FOUR: PROBLEM SOLVING AND AFFECTIVE ISSUES IN MATHEMATICS

Problem Solving

<u>The Lens: Schoenfeld's Problem-Solving Framework</u>

As I conducted my interviews, made field notes during observations, and watched problem-solving efforts, I viewed these activities primarily through the lens of Schoenfeld's (1992) problem-solving framework. This framework was also used as a field of view for data analysis. Because of the profound importance of Schoenfeld's (1992) problem-solving framework in my study, I will devote much of this chapter to a discussion of it. The first framework he designed had four aspects. I will discuss each of the four aspects in detail now, then include the fifth aspect from his newer work at the end. The reason for the two different frameworks and this division is the shift of focus that occurred in mathematics education research. One researcher described the shift in this way:

> At the forefront of mathematics education throughout the decade of the eighties was the concern for improvement of students' mathematical problem–solving capabilities. However, by the start of this decade, a shift in emphasis had occurred—not away from problem solving—but in the research

> orientation toward problem solving: a movement from the purely cognitive aspects of problem solving toward that of the affective aspects related to problem solving. With this shift in emphasis, came the study of beliefs or belief systems. (Willis, 1992, p. 2).

Schoenfeld's first framework reflected that original orientation toward cognitive issues while his later framework reflected the newer emphasis of affective issues in mathematical problem solving.

The first aspect of Schoenfeld's (1985) problem-solving framework, *cognitive resources* or, more simply, *resources*, center around the cognitive domain. Resources are defined as the "foundation of basic mathematical knowledge ... available to the individual" engaged in the problem-solving activity (Schoenfeld, 1985, p. 12). This knowledge may include "whatever mathematical information problem solvers understand *or misunderstand"* (Schoenfeld, 1985, p. 14, italics added). I think this is an important factor, *resources* include not only correct mathematical information, but also "flawed resources and consistent error patterns" (Schoenfeld, 1985, p. 46). In other words, the information in this category may be helpful *or harmful* to an individual engaged in problem solving. The following are a list of items that are included in the *resources* category: (a) informal and intuitive knowledge about the domain; (b) facts, definitions, and the like; (c) algorithmic procedures; (d) routine procedures; (e) relevant competencies; and (f) knowledge about the rules of discourse in the domain. (Schoenfeld, 1985, pp. 4-55).

Heuristics are the "general problem–solving techniques" one might use when trying to find a solution to a problem (Schoenfeld, 1985, p. 12). Once again, however, this may include "techniques they have (*or lack*) for making progress when things look bleak" (Schoenfeld, 1985, p. 14, italics added). Schoenfeld's (1985) list of *heuristics* look very much like Polya's (1985). He includes: "drawing figures; introducing suitable notation; exploiting related problems; reformulating problems; working backwards; testing and verification procedures" (Schoenfeld, 1985, p. 15). As he reveals later in the book, however, what may appear to be the best way to help students in learning heuristics, as well as how to use them, does not work. It would seem, Schoenfeld (1985) reasons, "Once the most important heuristic strategies have been discovered and elaborated," we "should provide direct instruction in these strategies, thereby saving students the trouble" of figuring them out on their own (p. 71). However, there *is* a problem: "It hasn't worked, and not for lack of trying" (Schoenfeld, 1985, p. 71). Basically, the conclusion is a constructivist one: Students must discover and develop their own problem–solving strategies by engaging in problem–solving situations.

Control is described as "how one selects and deploys the resources at one's disposal" (Schoenfeld, 1985, p. 13). This category, again, includes how, when solving a problem, one will "use, *or fail to use*, the information at their disposal" (Schoenfeld, 1985, p. 14, italics added). More broadly, Schoenfeld (1985) defines *control* as "global decisions regarding the selection and implementation of resources and strategies" which include "planning," "monitoring and assessment" (or self-reflection), "decision-making," and "conscious metacognitive acts" (p. 15). This is the first area in which problem solving is viewed in more than just a cognitive light. At first glance, one might think *resources* or *heuristics* to be the most important factor in Schoenfeld's (1985) problem–solving framework. I disagree; as does Schoenfeld (1985), when he states that control "is a major determinant of problem–solving success or failure" (p. 143). I think *control* has the most importance because one must decide on and implement the best heuristic, recognize and recover from mistakes (in resources or heuristics), and, in general, know which resources and heuristics to use when. Schoenfeld (1985) describes the idea of knowing what to use when in his description of "the cardinal rule of control in problem solving: Never implement difficult or time-consuming procedures unless you have checked to see whether other, far simpler procedures will work" (p. 101). In other words, "one needs to be efficient as well as resourceful" (Schoenfeld, 1985, p. 99).

Lastly, *belief systems* are the individual's "mathematical world view, which determines the ways that the knowledge in the first three categories is used" (Schoenfeld, 1985, p. 14). These may include beliefs "about self, about the environment, about the topic, (and) about mathematics" (Schoenfeld, 1985, p. 15). Because "humans are not the rational animals one would like to believe they are," the belief systems one holds have more influence on problem–solving behavior than the knowledge (facts) one knows (Schoenfeld, 1985, p. 148). Schoenfeld (1985) gives this example to illustrate that point: "People who believe that making a convincing case does not require carefully reasoned arguments are not likely to produce carefully reasoned arguments—or even think of producing them—despite the fact that they could do so without difficulty" (p. 152). Again, the importance of looking beyond the purely cognitive aspects of problem solving is brought to light.

Schoenfeld's (1985, 1989) problem–solving framework and later research on the role of context in problem solving led to an important discovery. He realized there was something else influencing problem–solving behavior, something that had been implied but not explicitly stated in his original framework of four categories. As a result, Schoenfeld (1992) revised his framework to include this component and expanded some of the categories to clarify what was involved. The components in his framework were changed to the following: "the knowledge base, problem-solving strategies, monitoring and control, beliefs and affects, (and) practices" (Schoenfeld, 1992, p. 348). Although the terms are a little different, the first three categories are basically the same: the knowledge base (previously resources), problem–solving strategies (previously heuristics), and monitoring and control (previously control). He expanded the fourth category to include affect, a vital factor only implied in the first framework. The final category, practices, is entirely new and demonstrates the importance of the classroom culture not only on the remaining areas of the framework (especially beliefs and affects), but also on mathematics learning and problem solving.

The Affective Domain

Affective Issues in Mathematics Education

Schoenfeld (1985) recognized an important factor about students' problem–solving success (and failure): "Students' problem-solving performance is not simply the product of what the students know; it is also a function of their perceptions of that knowledge ... that is, their beliefs about mathematics" (p. 14). Lester (1994) confirms that "the ability to solve mathematics problems develops slowly over a very long period of time because success depends on much more than mathematical content knowledge" (pp. 668-9). He continues by explaining that mathematical "problem-solving performance seems to be a function of several interdependent categories of factors (e.g. knowledge acquisition and utilization, control, beliefs, affects, and sociocultural contexts)" (Lester, 1994, p. 669). One of these categories, namely affect, received a great deal of attention in much of the mathematics education literature produced during the 1980's and 1990's (McLeod, 1994).

Although Schoenfeld (1985) recognized the importance of the affective component on problem solving in the 1980's, he did not explicitly include it in his problem–solving framework at that time. As previously stated, in his 1985 work, Schoenfeld had only four categories for his problem–solving framework, but, in 1992, he revised them to include "affect" specifically (Schoenfeld, 1992, p. 348). The new, revised framework consisted of the following categories: (a) "the knowledge base" (formerly *resources*), (b) "problem-solving strategies" (formerly *heuristics*), (c) "monitoring and control," (d) "beliefs and affects," and (e) "practices" (Schoenfeld, 1992, p. 348). It is the fourth category of this new framework, *beliefs and affects*, that will be discussed next because of its importance in what developmental students think and believe about the nature of mathematics.

Definition of terms. McLeod (1992) contends "that affect plays a significant role in mathematics learning and instruction" (p. 575). It is the opinion of many researchers and mathematics educators that the reason affective issues are important in mathematics learning and instruction is "that cognition and affect are interrelated" (Adams, 1989, p. 192; see also Mandler, 1989; Marshall, 1989; and McDonald, 1989). The terms *affect* and *affective domain* have been defined and used in different ways (Hart, 1989), however, I will use them interchangeably here. *Affect*, like *problem* and *problem solving*, is a difficult term to define because it "has meant many things to many people" (Mandler, 1989, p. 3). McLeod (1992) describes the "affective domain" as the "wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). The affective domain, a general term, can be divided into three specific categories of "affective responses to mathematics": (a) "beliefs, which may include beliefs about mathematics, self, mathematics teaching, and the social context"; (b) "attitudes"; and (c) "emotions" (McLeod, 1992, p. 578). Beliefs, attitudes, and emotions represent "increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (McLeod, 1992, p. 579). This is true because "beliefs are largely cognitive in nature" and tend to develop "over a relatively long period of time," while emotions "involve little cognitive appraisal and may appear and disappear rather quickly" (McLeod, 1992, p. 579).

Hart (1989), in discussing how "psychologists, mathematics educators interested in research on problem solving, and mathematics educators interested in research on attitudes toward mathematics" have different definitions about the terms belief, attitude, and emotion, gives excellent definitions for each of the three (p. 37). Hart (1989) uses *belief* "as it has been used by Thompson (1985), Silver (1985), and Schoenfeld (1985) — to reflect certain types of judgments about a set of concepts" (p. 44). *Attitude* toward an object is defined as the "emotional reactions to the object, behavior toward the object, and beliefs about the object" (Hart, 1989, p. 44). Lastly, *emotion* is defined "as in Mandler's book (1984), to represent a hot gut-level reaction" (Hart, 1989, p. 44). Because the literature dealing with affect in mathematical problem solving abounds with discussions about these three areas, I will discuss each of them in more detail.

Belief systems. There is a great deal of literature dealing with students' beliefs. This includes students' beliefs about the nature of mathematics, beliefs about themselves as mathematicians, and beliefs about what constitutes appropriate mathematical instruction. One reason for the amount of literature may be the notion that "students may not be able to become better problem solvers unless they change their beliefs about mathematics" (Frank, 1988, p. 32). Frank (1988), like other mathematics educators and researchers, realized how influential students' belief systems can be on problem–solving performance, or any mathematical behavior (Garofalo, 1987; Garofalo and Lester, 1985; Schoenfeld, 1987). Beliefs can have either positive or negative effect on students' problem–solving performance. Unhealthy student beliefs about mathematics have serious implications for problem solving (Garofalo, 1989a; Garofalo, 1989b; Lester, Garofalo, & Kroll, 1989; Frank, 1988; Adams, 1989; Schoenfeld, 1985).

For example Schoenfeld (1992), in discussing a common student belief that one should be able to "solve any assigned problem in five minutes or less," points out a "serious behavioral corollary" that has a negative effect; namely, that a student with that particular belief "will give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered" (p. 359). Likewise Frank (1988), in remarking about the common student belief that "the goal of doing mathematics" is only obtaining "right answers," revealed a corresponding behavior of focusing "almost entirely on answers and whether those answers were right or wrong" instead of engaging in true mathematical thinking (p. 33). If, however, a student holds a healthy belief about mathematics, for example, that "a demonstration of good reasoning should be rewarded even more than" one's "ability to find correct answers", then s/he will respond with positive behaviors such as explaining their methods of arriving at a particular solution (NCTM, 1989, p. 6).

Beliefs not only influence students' performance on mathematical problem solving, but also the mathematical instruction that takes place. Borasi (1990) gives "four key categories" of "dysfunctional" mathematical beliefs, a term used to describe beliefs that could adversely "affect mathematics instruction" (p. 176). In brief, these beliefs deal with the scope and nature of mathematical activity and the nature and origin of mathematical knowledge (Borasi, 1990). Examples include the belief that mathematical problems are "always well defined and have exact and predetermined solutions," that "there are no gray areas" in mathematics, and that "mathematics always existed as a finished product" (Borasi, 1990, p. 176). Obviously, "these beliefs reflect a limited appreciation of the nature of mathematics" (Borasi, 1990, p. 177). I think Borasi (1990) best describes the results of her study on dysfunctional beliefs in the conclusion:

> In this article I have examined the nature and consequences of students' holding a dualistic view of mathematics—that is, the belief that mathematics is a collection of disjoint, predetermined, and absolutely correct facts and procedures that are used to solve specific problems and that teachers are supposed to pass on such facts and procedures to students, who in turn will memorize them for later recall and application for the solution of given problems. It has been argued that students holding such a view are likely to be passive learners, ignorant of the need of making personal meaning of the material presented in class, lacking constructive strategies to deal with learning

> difficulties, and therefore liable to experience limited success in school mathematics. (p. 181)

Student beliefs about the scope and nature of mathematics and the scope and nature of mathematics instruction have been discussed; next, student beliefs about themselves as mathematicians will be addressed. One aspect of this type of belief, metacognition, has received a great deal of attention in the mathematics education literature. Because of this attention, this area will be examined in more detail in a later section of this chapter.

All affective issues are closely related, therefore it is impossible to separate them from each other completely. In other words, although beliefs about self, beliefs about the nature of mathematics, and beliefs about the nature of mathematics instruction have been treated as separate categories, they actually overlap. Schoenfeld (1987) illustrates this point in his definition of metacognition that includes "beliefs" as well as knowledge about your own thought processes" and "control, or self regulation" (p. 190).

Attitudes and emotions. The final two areas of the affective domain, *attitudes* and *emotions*, have received some attention in the mathematics education literature as well, though not as much as beliefs. These two areas, like the other, are closely related. Attitude, like affect, means different things to different people. Taylor (1993) illustrates this:

> Attitude is defined by Thorndike and Barnhart's (1968) as a "way of thinking, acting, or feeling." Funk and Wagnall (1969) define attitude as a "state of

> mind, behavior, or conduct regarding some matter as indicating opinion or purpose," while Random House (1984) defines attitude as a "manner, disposition, feeling, position, etc., toward a person or thing" (p. 6).

Attitude "encompass[es] affective, behavioral, and cognitive components"; and, because of this, "formation of an attitude is a complex process involving interaction among many factors including: culture, the family, socialization, schooling experiences, relationships with role models and mentors, etc" (Taylor, 1993, p. 7). Some "examples of attitudes toward mathematics would include liking geometry, disliking story problems, being curious about topology, and being bored by algebra" (McLeod, 1989, p. 249).

According to McLeod (1992), attitude can be defined as the collection of "affective responses that involve positive or negative feelings of moderate intensity and reasonable stability" (p. 581). He goes on to explain that attitudes toward mathematics generally develop in one of two ways: (a) as a result of "the automatizing of a repeated emotional reaction to mathematics," or (b) an "assignment of an already existing attitude to a new but related task" (McLeod, 1992, p. 581). Notice that emotion is included in the first description of the way attitudes develop. Emotions, typically, "are subjective reactions to specific situations" (Lester, Garofalo, & Kroll, 1989, p. 76). Obviously, emotions can be either positive or negative; however, as Lester, Garofalo, and Kroll (1989) point out, a positive emotion is not always helpful in (just as a negative emotion is not always harmful to) the individual's problem–solving efforts. McLeod (1989) gives an example of the difference between attitude and emotion, "Students who say that they dislike mathematics one day are likely to express the same attitude the next day; however, a student who is frustrated and upset when working on a nonroutine problem may express strong positive emotions just a few minutes later when the problem is solved" (p. 246).

The research on these areas of the affective domain has been quite limited. Attitudinal studies have focused mainly on preferences while studies dealing with emotion have received little attention at all (Lester, Garofalo, & Kroll, 1989, p. 76). As McLeod (1992) puts it, "The emotional reactions of students have not been major factors in research on affect in mathematics education" (p. 582). Perhaps this is because "it is generally accepted that preferences and attitudes are traits, albeit perhaps transient ones, of the individual, whereas emotions and moods are situation- or time-specific states"; and, it is easier to define and study individual traits rather than attempting to create a certain situation or time-specific state (Lester, Garofalo, & Kroll, 1989, p. 76). McLeod (1992) echoes this sentiment when he states that "most research in the past has looked at products, not at processes, and at beliefs and attitudes rather than emotions" (p. 582).

The affective domain, as it relates to mathematical problem solving and the category of *beliefs and affects,* improved the framework Schoenfeld (1992) developed to examine the problem–solving processes. In summary, affective issues are central to mathematics education. Especially in light of two of the five goals presented by the NCTM (1989), those of helping students gain confidence in themselves as do-ers of mathematics and of helping students understand and appreciate the value of mathematics, the importance of the affective domain must be recognized (McLeod, 1989).

Metacognition and Cognition

Like areas of the affective domain, metacognition and cognition are also closely related and difficult to separate. According to Garofalo (1987), "metacognition refers to the knowledge and control one has of one's cognitive functioning, that is, what one knows about one's cognitive performance and how one regulates one's cognitive actions during performance" (p. 22). He elaborates, "Knowledge about oneself as a mathematical performer includes knowing one's strengths and weaknesses, tendencies, and typical behaviors, combined with being aware of one's repertoire of tactics and strategies and how they can enhance or facilitate performance" (Garofalo, 1987, p. 22). The control (or self-regulation) aspect of metacognition, on the other hand, "has to do with the decisions one makes concerning *when*, *why*, and *how* one should explore a problem, plan courses of action, monitor one's actions, and evaluate one's progress, plans, actions, and results" (Garofalo, 1987, p. 22).

Mathematics educators generally define cognition as one's thinking and metacognition as thinking about one's thinking. Because it is sometimes difficult to distinguish what is metacognitive from what is cognitive, Garofalo and Lester (1985) suggest that "cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (p. 164). An important factor of metacognition is self-regulation, or control, as already mentioned. According to Schoenfeld (1987), "One way to characterize efficient self-regulation is to say that the people who are good at it are the people who are good at arguing with themselves" (p. 210). He goes on to discuss how to help students become better at arguing with themselves and reveals the classroom culture as an important factor (Schoenfeld, 1987).

Good metacognitive skills lead to good problem-solving performance. Campione, Brown, and Connell (1989) give this description of metacognition and its role in students' learning:

> Successful learners can reflect on their problem-solving activities, have available powerful strategies for dealing with novel problems, and oversee and regulate those strategies efficiently and effectively. They find learning challenging and see themselves as in control of their destiny. Weaker students, in contrast, acquire fewer problem-solving strategies, are less aware of the utility of those strategies, and do not use them flexibly in the service of new learning. They are not convinced that they can control their performance and tend to be relatively passive in learning situations. (p. 94).

Campione, Brown, and Connell (1989) also give an excellent account of metacognitive issues as related to mathematical instruction and assessment. They explain that "metaproblems" occur in students because "traditional educational practice rarely incorporates metacognitive and contextual factors in learning"

(Campione, Brown, & Connell, 1989, p. 96). Examples of some of these "metaproblems" fall under two main areas: (a) students tend to have "a distorted view of what the academic tasks are," for example, the belief that correct answers are math; and (a) "the knowledge and skills many students acquire tend to be encapsulated and inert, available when clearly marked by context (as they are, for example, on many standardized tests) but not serviceable in other circumstances as tools for learning" (Campione, Brown, & Connell, 1989, p. 100). The latter of the two metaproblems agrees with the assumption made by Lester, Garofalo, and Kroll (1989) that one reason for "an individual's failure to solve a problem successfully when the individual possesses the necessary knowledge stems from the presence of ... metacognitive factors that inhibit the appropriate utilization of this knowledge" (p. 75).

CHAPTER FIVE: RESEARCH DESIGN, METHODOLOGY, AND PROCEDURES

Introduction

The purpose of this study was to examine in detail the beliefs and affects three developmental college students have about the nature of mathematics and themselves as mathematicians and to determine what influence those beliefs and affects had upon them when doing mathematics. The research design included open-ended personal interviews (using an interview guide approach), task interviews (participant observations), and classroom observations (field observations).

The research question for this study focused on three categories: (a) students' beliefs and affects about the nature of mathematics and its teaching, including what mathematics is, what it means to "do" mathematics, and what constitutes a "math problem"; (b) students' beliefs and affects about themselves as mathematicians, including student problem–solving strategies and issues related to metacognition; and (c) how these beliefs and affects influence how mathematics is taught and how students do mathematics, including strategies students use to learn mathematics and teacher actions that help and hinder that learning process. This chapter presents the research plan and the methods and procedures used in this study.

The results from this study, as with any qualitative research, should not be generalized beyond the population used. It is also worth recalling that generalizability is not a goal of qualitative research as in quantitative research. However, because of the information-rich nature of the case studies, the findings from this study can be used to obtain a more focused and detailed picture of these three individual developmental mathematics students. In addition, the results that support or refute other research studies (whether quantitative or qualitative) provide a basis for continued research on developmental mathematics students' beliefs.

Qualitative Research

Naturalistic Inquiry

> *"Live among the peoples of the world as they live. Learn their language. Participate in their rituals and routines. Taste of the world. Smell it. Watch and listen. Touch and be touched. Write down what you see and hear how they think and how you feel. Enter into the world. Observe and wonder. Experience and reflect. To understand the world you must become part of that world while at the same time remaining separate, a part of and apart from" (*Patton, 1990, p. 199, from Halcolm's Methodological Chronicle).

The purpose of this study was to gain a deeper understanding of what developmental mathematics students believe about the nature of mathematics and its teaching. To gain that understanding in the best way possible, I chose to use the method of naturalistic inquiry. A recent review of research revealed that "quantitative research is much more prevalent" in the field of developmental education (MacDonald & O' Hear, 1995, p. 3) than the "qualitative educational anthropological research" (Venters, 1992, p. 38) that I believe is needed to adequately address affective issues. In a follow-up review to discuss flaws in the research, the authors of the developmental education research review commented that, although qualitative studies were "less likely to contain flaws than quantitative studies," in their review of those studies, "three (43%) were seriously flawed" (MacDonald & O' Hear, 1996, p. 8). In light of these rather alarming results, I made every effort to describe the details of my research design, methodology, and procedures and to illustrate that I am aware of the particular challenges involved with this type of inquiry and have taken every opportunity to ensure the quality of the research. I agree with the authors' assertion that "it is critically important that research informing the field be of high quality" (O'Hear & MacDonald, 1996, p. 8).

Methods Decisions

> *"When in doubt, observe and ask questions. When certain, observe at length and ask many more questions" (*Patton, 1990, p. 7, from Halcolm's Evaluation Laws).

Because of Patton's (1990) advice, I conducted qualitative research that produced thick descriptions of what three individual developmental mathematics students think about the nature of mathematics. There were several methods decisions I made:

1. Who is the information for and who will use the findings?
2. What kinds of information are needed?
3. How is the information to be used—what is the purpose?
4. When is the information needed?
5. What resources are available? (Patton, 1990, p. 12).

I will answer these in order. This study was primarily for my personal benefit and secondarily for the benefit of the developmental mathematics education community. Any person who desires to know more about what these particular developmental mathematics students think, know, and feel about the nature of mathematics may use the findings. I collected the information needed to produce this thick description by using: (a) "in-depth, open-ended interviews," (b) "direct observations," and (c) "written documents." (Patton, 1990, p. 10). The purpose of this research was to create three detailed case studies about these individuals experiences in, beliefs about, and attitudes toward mathematics. The information was needed in a timely manner; however, no distinct deadline exists. The resources available to me were: myself, my time, and a local community college that offers developmental mathematics courses and with which I had "prior ethnography" since I taught developmental mathematics there for two years (Corsaro, 1980, cited in Lincoln & Guba, 1985, p. 251).

Research Plan

Design Issues

"In qualitative inquiry, the problem of design poses a 'paradox.' The term *design* suggests a very specific blueprint, . . ." but in qualitative analysis, this is not very realistic. (Lincoln & Guba, 1985, p. 226). One may best understand this paradox when one realizes that, by definition, "the design of a naturalistic study *cannot* be given in advance; it must emerge, develop, unfold" (Lincoln & Guba, 1985, p. 225). Because of this lack of certainty, "qualitative inquiry seems to work best for people with a high tolerance for ambiguity" (Patton, 1990, p. 183). Although there are aspects of qualitative design that cannot be given *a priori*, some *can*. For example, the focus of the study, the paradigm that will be used to interpret the study, where and from whom the data will be collected, the phases of inquiry, types of instrumentation, data collection and analysis methods, and methods to achieve trustworthiness can all be defined and decided upon. (Lincoln & Guba, 1985, pp. 226 - 248). Although I present these aspects in a certain order, they should *not* be viewed in a linear fashion. Instead, "what is intended to be implied by this list is that all of its elements must somehow be dealt with" by the researcher (Lincoln & Guba, 1985, p. 226). Therefore, in the sections that follow, I present the ways and means I dealt with these elements. While every attempt has been made to ensure accuracy, it must be understood that the specifics of each aspect were subject to changes resulting from an emergent design.

Focus of the study. According to Selden & Selden (1993), "mathematics education research is a relatively young field" that has primarily focused on K-12 students, but that has recently begun to include information about "how college students learn, or do not learn, mathematics" (p. 431). At the conclusion of their article, the authors list several research questions that have had little or no investigation at the college level. One of these questions centers around what "college freshman believe about the nature of mathematics and its teaching" and what effect those beliefs have on the students learning of mathematics (Selden & Selden, 1993, p. 422). In essence, this question defined the focus of my study. I attempted to answer the following questions:

1. What are the beliefs and affects developmental college students have about the nature of mathematics and its teaching? This will include:
    a. What is mathematics?
    b. What does it mean to "do" mathematics?
    c. What constitutes a "math problem"?
2. What beliefs and affects do developmental college students have about themselves as do-ers of mathematics (including issues related to control and metacognition)?
3. What effect do these beliefs and affects have on students' doing mathematics and on their preferences for teaching methods, including:

> (a) student problem–solving strategies, (b) strategies students used to learn mathematics, and (c) teacher actions that help and hinder that learning process?

I wanted to know what developmental students believe about the nature of mathematics. This information has been requested from developmental mathematics instructors and educators, and, I believe, was a much-needed area of research (West, Garofalo, & Perdue, 1993).

Paradigm for interpretation of data. The naturalistic paradigm was employed for interpretation of data in this study. Because the data collected in this study came from three distinct sources, a variety of frameworks was employed in the interpretation phase. Although it was expected that these frameworks can, and probably will, overlap, I discuss each aspect of data collection separately. Task interviews, classroom observations, and open-ended personal interviews were the main sources for data collection.

I viewed information gathered from observation of the task interviews through the lens of Schoenfeld's (1992) problem-solving framework. I sought to discover information about the areas of control and beliefs and affect primarily. I attempted to identify when participants were engaged in different areas of Schoenfeld's (1992) problem-solving framework when they performed the tasks required. Tasks were similar to problems and procedures addressed in the participants' elementary algebra class at the time of the observations. The majority of the tasks presented during these observations were more like exercises or routine practice problems than like non-routine or applications-based problems. Some of the tasks were word problems however, and some were applications-based. The tasks are found in Appendix A. The follow-up questions I asked for each task are found in Appendix B.

I gathered data from observation of classroom behavior of students and teacher using a framework built on the standards set forth in the NCTM (1989) and AMATYC (1994, 1995) documents. These standards were used to create checklists to determine if student and teacher were playing the traditional roles or not. An example of the "traditional" role for the student might include passive, silent, "copying" behaviors while the teacher's role may involve her/him standing up at the chalkboard working examples for students to follow. The checklists also included items to determine if the students and teacher met the goals of the NCTM (1989) and AMATYC (1994, 1995). I also used various documents—class notes, examples of homework assignments, and class tests—to determine the nature of the mathematics being taught in the participants' current developmental mathematics class (see Appendices K and L for samples of notes, tests, and quizzes). I incorporated data from both types of observations into the interview phase as I discovered new areas that I believed to be germane to the goals of the study.

Answers to questions asked during interviews were analyzed from my perspective as a mathematics educator. Data was examined based on my experience and knowledge. Specifically, I sought to confirm or refute my own observations of developmental mathematics students and their beliefs. I attempted to discover if, for example, my descriptions of a *typical* developmental mathematics student (see Appendix J: Definitions) were as accurate and commonplace as I then believed them to be. I sought to find common themes that emerged from the data. These themes emerged from my own experiences as student, teacher, and educational researcher.

Location of participants. Participants were attending a local community college that, in order to maintain confidentiality, I assigned the pseudonym Mountain Top Community College (MTCC). I chose this two-year institution for a number of reasons. First, it is in the same geographical area as my work and residence, as well as the university. Aside from its convenient location, I also chose MTCC because of its course offerings. Based on the earlier definition of developmental mathematics, over 50% of the community college course offerings are developmental in nature. Finally, I chose MTCC because I am familiar with the campus, students, policies, and schedules since I was an adjunct professor there for two years and have conducted research there before this study. I believe my familiarity and prior ethnography with the field site is of utmost value. As Lincoln & Guba (1985) put it, "Time and resources will not always permit prior ethnography, but its utility is so great that we recommend it to anyone seriously interested in doing naturalistic inquiry" (p. 251).

I selected participants from an elementary algebra course conducted in the spring semester of 1996. This course is one of the developmental mathematics courses offered at MTCC. I chose to select participants from the elementary algebra course in general because I wanted a subject that fell roughly in the middle of the subjects included in the definition of developmental mathematics. For that reason, I did not want an arithmetic-based course or a pre-calculus course. I chose the specific Elementary Algebra course that I did (MTH 03-19) because it was offered at a time that was convenient for me and because it was a time I believed more *typical* developmental mathematics students would take a course. The course met in the evenings from 4:25 to 6:55 on Mondays and Wednesdays.

Participant selection. I produced extensive case studies from three willing participants enrolled in the previously discussed elementary algebra course at MTCC. Because I offered no tangible compensation for participation in my study, the main criterion for selection was, naturally, willingness. According to Patton (1990), "sample size depends on what you want to know, the purpose of the inquiry, what's at stake, what will be useful, what will have credibility, and what can be done with available time and resources" (p. 184). Because I wished to know about the beliefs of developmental mathematics students only, that was the first population I considered. Because of the amount of data I planned to collect from each participant (anticipated to be 200-500 pages before analysis), I felt three participants was a good number to be useful, credible, and still be realistic about time and resources. I selected three individuals based on the following criteria and for the following reasons.

Because I was interested in compiling detailed information about developmental mathematics students' beliefs that had the most trustworthiness, I sampled "purposefully;" in other words, I selected "*information-rich* cases for study in-depth" (Patton, 1990, p. 169). Patton (1990) defines information-rich cases as "those from which one can learn a great deal about issues of central importance to the purpose of the research" (p. 169). I used the strategy of "combination or mixed purposeful sampling" to select the participants for my study (Patton, 1990, p. 183). This type of sampling combines one or more of the more specific sampling strategies. I combined various aspects from the strategies of: "maximum variation sampling," "typical case sampling," and "criterion sampling" (Patton, 1990, p. 182-3).

First, from the strategy of "maximum variation sampling"—defined as that which "identifies important common patterns that cut across variations" (Patton, 1990, p. 182) — I selected participants that were widely diverse with regard to the aspect of being a *typical* or *atypical* developmental mathematics student. Since I was especially interested in the beliefs of the *typical* developmental mathematics student, I chose two students who fit that description/stereotype (described in a moment) and one student who did not. It was my opinion that this selection emphasized the area I was most interested in and achieved the maximum variation possible. Characteristics of the typical developmental mathematics student can include the following:

1. A student who has, in his/her own opinion, never done well in mathematics.
2. A student who has poor grades in K-12 mathematics, especially high school mathematics.

3. A student who did not take any more than the required mathematics courses for high school graduation.

4. A student whose attitudes toward mathematics include one or more of the following: hatred, fear, frustration, or avoidance.

5. A student whose beliefs about mathematics include one or more of the following:

    a. Mathematics is just a bunch of rules that are useless in real life.

    b. Only certain people can "do math" (and they do not believe they are one of those people).

    c. The rules that govern mathematics do not make sense.

    d. The "instant math" myth: the belief that if they look at a math problem and do not know instantly what to do to solve it, then they can not solve that problem.

I assessed these characteristics through informal questioning at the time the participant volunteered for the study. For the purposes of classification, I defined a *typical* developmental mathematics student as someone who exhibited three of more of the characteristics listed above. If a student exhibited less than three of the previous characteristics, then they were classified as an *atypical* developmental mathematics student. Definitions for these and other terms are given in Appendix J. By using "a small sample of great diversity," my design produced "two kinds of findings" about typical and atypical developmental mathematics students: (a) "high-quality, detailed descriptions of each case, which are useful for documenting uniqueness," and (b) "important shared patterns that cut across cases and derive their significance from having emerged out of heterogeneity" (Patton, 1990, p. 172).

Second, from the strategy of "typical case sampling,"—defined as that which "illustrates or highlights what is typical, normal, average"—I chose two female students and one male (Patton, 1990, p. 182). As discussed in Chapter Two, the majority of students in developmental mathematics programs are female. Choosing participants so the majority was female illustrated the typical case found in developmental mathematics classrooms. I also believed I would find some important distinctions and differences by selecting participants in this manner instead of using all females. Venters (1992) conducted qualitative research on twelve successful females in developmental mathematics. She recommended further qualitative research to gain a better understanding of females' perceptions of mathematics. I believed I could achieve this understanding through typical case sampling. To ensure maximum variation within the typical case structure, I sought one female participant who was a typical developmental mathematics student and one who was not. The male participant could have been either typical or atypical; as it turned out, he was typical.

Third, from the strategy of "criterion sampling"—defined as "picking all cases that meet some criterion" (Patton, 1990, p. 183) — I selected participants that meet the criteria of being developmental mathematics students at MTCC, currently enrolled in the course, Elementary Algebra (MTH 03-19), and willing to participate in my study. I discussed the reasons for those criteria earlier and, therefore, I will not repeat them here.

It must be noted that, although it was the least important, the factor of "convenience sampling" cannot be ignored (Patton, 1990, p. 180). According to Patton (1990), the strategy of choosing cases that are "fast and convenient" is "probably the most common" and "least desirable" of all the sampling strategies. In making my decisions about location and selection of participants, I used convenience sampling to a small degree. In addition, by making willingness of the participant the major criterion for participation, the sample chosen was, to a small degree, based on convenience. I believe these uses of this strategy were acceptable, however, and did not diminish the trustworthiness of the data.

When I asked for participants for this study, I gave an informal presentation to the class (see Appendix C, Oral presentation to students). Those students who were interested in participation were given an information sheet (see Appendix D, Information sheet for potential participants). The three students who were chosen for the study and agreed to participate also completed an informed consent form (see Appendix E, Informed consent form).

Methods and Procedures

Instrumentation

According to Lincoln & Guba (1985), there are two types of instrumentation in qualitative research, human and nonhuman. I used both types in my study. Obviously, the human instrument in this study was myself. The non-human instruments included various documents such as my field notes from observations and interviews and samples of participants' homework, tests, and class notes. The human is the "instrument of choice in naturalistic inquiry" because of many reasons, among them adaptability and responsiveness (Lincoln & Guba, 1985, p. 236). The human instrument has been used extensively in fields other than educational research as well. Anthropology and sociology, among others, used the human instrument almost exclusively. A human instrument, like any other, must be trustworthy to be useful. Determining the trustworthiness of a human instrument is similar to determining trustworthiness of any instrument, and, like other instruments, the human instrument can be refined (Lincoln & Guba, 1985).

Because I conducted qualitative research before this study, I felt I had been refined as an instrument. I also relied upon my Qualitative Advisor, a mentor in the field, to ensure that refinement continued throughout the process. I also conducted in-depth open-ended interviews with developmental mathematics students in my previous research. I believe this experience was very beneficial in refining my interview skills. I learned to curb the desire to lead the participants by my questions and I learned to recognize the various effects the interviewer can have on participants' responses. I believe my prior experiences in defining categories from transcribed data also assisted me in the analysis and interpretation phases of this research.

From my experiences with qualitative research, I understood the importance of being aware of my own expectations. In this study, I expected to find that the developmental mathematics students believe that mathematics is a collection of arbitrary and unrelated rules and procedures that make no sense either separately or collectively. I also expected to hear stories of math classes where the students sit passively observing the instructor doing mathematics at the front of the room. I expected these students to voice feelings of frustration, anxiety, and fear of the subject and to show evidence of avoiding it. From the females, I especially expected to hear beliefs stating that they just are not "math types."

Phases of Inquiry

According to Lincoln & Guba (1985), there are three phases of inquiry that a naturalistic researcher must address: phase I, orientation and overview; phase II, focused exploration; and phase III, member checking. The purpose of phase I is to "obtain sufficient information" to know "what is important enough to follow up" on in detail (Lincoln & Guba, 1985, p. 235). The primary purpose of phase II is to obtain detailed information about those elements (determined in phase I) deemed important. Phase III is designed as a check to confirm that the information obtained from phase II has been accurately and adequately captured. I addressed each phase of inquiry in each interview and observation with every participant. Generally speaking, phase I was determined by previous research in developmental mathematics education as well as from my own knowledge and experience. I addressed the goals for phase II during the interviews and observations. Phase III occurred through member checking both during the interviews (for example, I might comment, "You mentioned earlier . . ." during an interview to give a participant the opportunity to correct any misconceptions or misunderstandings I may have) and at the end of the data collection phase (the transcribed data were available for the participants to review, correct, and append). I also conducted a member check after the analysis phase with the two participants who completed the study. The case story written on each participant was available for that person to review, correct, and append.

Data Collection

I audiotaped the personal interviews that I conducted with each participant and later had them transcribed. I conducted three interviews with Cal. They were spaced so that the first occurred during the beginning of the semester, the second during the middle of the semester, and the last at the end of the semester. The goal of the first interview was to obtain his math history; the topic of the second interview was primarily a comparison of the current math class with previous classes; and, the third interview was his evaluation of the current class and the effects it had on his beliefs. Chris was the first participant I interviewed and I used the first interview with her as a "trial run." I conducted three more interviews with Chris after I revised the questions I used from the first one. The goals of the last three interviews were the same as those of Cal's. I conducted one interview with Joan that occurred just before mid-term. The focus of that interview was a math history and a comparison of her current class with others she had taken. She dropped the course soon after that interview and I was unable to schedule further meetings with her.

I began each interview by reading a short introductory statement (see Appendix F, Interview protocols). Questions followed the interview guide designed for that particular interview (see Appendix G, Interview guides). The first interview with Chris was used to create the guides for the other interviews. Modifications, additional questions, or other changes were made from the results of this initial interview. The interviews were progressive, that is, information from earlier interviews were used to address additional areas in later interviews.

The classroom observations were scheduled so some occurred before mid-term and others occurred after mid-term. I used observation checklists and kept field notes during the observations. The focus of the observations was on teacher actions like types of questions asked, nature of the mathematics being taught and emphasized, and nature of the assessments given, and on student behaviors such as taking notes, asking questions, and level of involvement in the class. I created the observation checklists from a variety of sources including the NCTM (1989) and AMATYC (1996) documents. I conducted a total of four classroom observations that lasted 90-120 minutes each.

I videotaped the task interviews. They were scheduled so one occurred before mid-term and the others occurred after mid-term. The focus was on the participant's performance on tasks similar to those presented in their Elementary Algebra class. I designed the tasks for the task interviews *after* classroom observations to ensure similarity of the tasks presented. The task interviews were progressive; that is, the tasks in the latter interview built on tasks from the previous interview. I conducted three task interviews with Cal, three with Chris, and one with Joan. The list of tasks given and follow-up questions asked are found in Appendices G and H, respectively.

It should be mentioned that the interview protocols, interview guides, observation checklists, and tasks (for the task interviews) were subject to review and revision by my peer debriefer, committee members, and qualitative advisor. The interview protocols, interview guides, tasks, and research plan were approved by the Committee for the Protection of Human Subjects at the University of Virginia.

Trustworthiness

Because of the differences in conducting quantitative and qualitative research, judging the results of these types of research is also very different. Naturalistic inquiry is not concerned with generalizability, reliability, or validity (internal or external) as its quantitative neighbor is. Rather, the issues for trustworthiness in qualitative research are confirmability, transferability, dependability, and credibility. Lincoln and Guba (1985) loosely translate confirmability, transferability, dependability, and credibility to mean neutrality, applicability, consistency, and truth value, respectively. I address each issue in the following sections as well as the methods I implemented in order to achieve the desired trait.

Confirmability. *Confirmability* is an issue that strives to measure the amount of contamination caused by researcher bias. I listed my expectations and assumptions in earlier sections of the study (before the data was collected) in an effort to control those effects. I also kept a reflexive journal—where I recorded personal reactions, reasons for decisions, perceptions, and comments—to ensure neutrality. This journal was used at the end of the study to ensure the results were justified by the data and were not influenced by my unconscious bias or other types of bias that I was unaware of during the research. The journal was part of the audit trail that contained "raw data" (recordings of the interviews & observations, transcriptions, etc.), "data reduction and analysis products" (summaries), "data reconstruction and synthesis products" (categories), "process notes", materials related to "intentions and dispositions" (predictions and expectations), and "instrument development information" (observation checklists) (Lincoln & Guba, 1985, pp. 319-320).

Transferability. *Transferability* does for qualitative research what generalization does for quantitative research. It gives the degree to which the results are applicable to other individuals, groups, or situations. I produced thick descriptions of each case study that included the data from the personal interviews, classroom observations, and task interviews as well as from the documents (like samples of homework and tests and my field notes and observations checklists). I also believe that my choice of combination or mixed purposeful sampling added to transferability because I chose participants with the purpose of revealing similarities and differences between variations as well as within types.

Dependability. *Dependability* of qualitative research relates to reliability of quantitative research. It is the degree to which someone else might expect to be able to replicate the results given similar circumstances and participants. The factor that affects dependability the most are the processes used to collect and analyze data. I gave detailed accounts of the research plan and methodology earlier in this chapter. I believe the reader can judge from these descriptions how the research was carried out and why the methods decisions were made as they were. In addition, it must be noted, the processes for data collection and analysis were subject to my doctoral committee's approval before the research proceeded. I believe that process and approval added to dependability of the results. The audit trail used to establish confirmability provided for dependability as well. The decisions made during the emergent design process were documented in the audit trail and methodological journal as well as the reasons for and justifications of those decisions.

Credibility. *Credibility* of qualitative research is the most important issue to address in trustworthiness. It is somewhat similar to validity in quantitative research. It addresses the degree to which one may believe the results to be true. Another way to view credibility is as a check to see how well the results of the research match the reality of the situation being studied. The data may be distorted from reality due to the researcher's presence, researcher's involvement with the participants, researcher's bias, participant's bias, or because of the manner in which the data was collected. I was aware that my presence in the classroom during the field observations affected what I observed. I took several steps to minimize that effect: I observed only (did not participate), I strove to be as unobtrusive as possible (for example, I sat in the back of the room), I established a certain familiarity with the students and the instructor (before the first observation, I had been in the class on two other occasions), and I did not attempt to hide why I was there and what I was doing. I did not know any of the students who volunteered to participate in my study so my only involvement with them occurred during data collection. My expectations as the researcher have already been addressed in the previous sections.

I addressed credibility by the methods of triangulation, peer debriefing, member checks, prolonged engagement, and persistent observation. There are several types of triangulation: method, researcher, and source. The basic idea behind triangulation is that if the results are similar from multiple sources gathered in multiple ways by multiple methods, the chance is much higher that those results match reality. I used multiple sources (e.g. participant's own words and actions, my observations, and information from various documents) gathered from multiple ways (e.g. audiotaped interviews, transcriptions, field notes, samples of participants' work, videotaped task interviews and classroom observations) by multiple methods (e.g. personal interviews, task interviews, and classroom observations as well as various documents) to ensure that credibility. I accounted for being the only researcher by constantly using member checking during the data collection processes to ensure that the information that I thought I was receiving was true and accurately reflected the reality of the situation. I also used member checking at the end of the data collection process by allowing each participant to view and make changes to the transcribed data.

My qualitative advisor helped to ensure that what I did was credible. The personal interview guides, tasks, and follow-up questions were all reviewed by my qualitative advisor. We discussed the instruments, made revisions, and came to consensus about their structure and content. I also met periodically with my qualitative advisor throughout the data collection and data analysis processes in order to gain additional insight, voice methodological decisions, and discuss various themes and ways of presentation as they emerged.

My peer debriefer played a crucial part in this qualitative study. We met at least once a week during the data collection and data analysis phases of the research. She reviewed the tapes and transcriptions from the interviews and produced a collection of themes that she felt emerged. We then compared the themes I had found with the ones she found and came to consensus about a strategy for categorizing the data. She read the case stories, gave feedback, and helped me to write the stories more clearly and accurately. I consulted my peer debriefer in all of the major methodological decisions and sought her advice throughout the process of writing the research results. In essence, she provided a much-needed "sounding board" for ideas, thoughts, problems, and issues that arose throughout the entire study.

Two factors that contribute to trustworthiness of a human instrument and to credibility of results are "prolonged engagement" and "persistent observation" (Lincoln & Guba, 1985, p. 192). Prolonged engagement can be viewed as "the investment of sufficient time to achieve certain purposes" (Lincoln & Guba, 1985, p. 301). Data collection took place for the duration of the Elementary Algebra class. I believe that was sufficient time "to learn the context, to minimize distortions, and to build trust" (Lincoln & Guba, 1985, p. 307). Persistent observation is observation of sufficient depth to "identify those characteristics and elements in the situation that are most relevant to the problem or issue being pursued and focusing on them in detail" (Lincoln & Guba, 1985, p. 304). I believe the length and number of interviews and observations allowed for persistent observation in this study. I was in the class for a total of four field observations. I observed the participants in problem-solving activities for a total of seven times. I believe that was sufficient time to identify and assess "salient factors and crucial atypical happenings" (Lincoln & Guba, 1985, p. 307). Combined with the various types of triangulation and the use of a peer debriefer and member checking, I believe I increased "the probability that credible findings will be produced" (Lincoln & Guba, 1985, p. 301).

Data Analysis

I used my previous experience in qualitative data analysis to create descriptive categories that emerged from the data obtained. After I consulted with my qualitative advisor and my peer debriefer, I used the agreed-upon categories as guides and created a series of three case stories to describe the beliefs and affects of these developmental mathematics students and to identify the effects of those beliefs and affects on their learning of mathematics. I wrote the case stories with the intent of giving a very detailed and descriptive picture of three individuals and their experiences in and beliefs about mathematics as well as the affective responses to mathematics that resulting from their beliefs.

The authors of *Crossroads in mathematics: Standards for introductory college mathematics before calculus* state "the ultimate goals of this document are to improve mathematics education and to encourage more students to study mathematics" (AMATYC, 1995, p. 1). They believe that, in order to improve mathematics education, certain standards (for intellectual development, for content, and for pedagogy) must be established and met (AMATYC, 1995). Their framework of these three types of standards "will enable *all* students to widen their views of the nature and value of mathematics and to become more productive citizens" (AMATYC, 1995, p. 9). Because the intent of this study was to discover more about what those views are of the nature of mathematics, I used the AMATYC (1994, 1995) documents as a framework for analysis of data, especially data collected from the classroom observations.

During the task interviews of this study, I focused primarily on cognitive and metacognitive issues related to problem solving. Specifically, I looked for signs of the two previously mentioned metaproblems (see Chapter Four) and for evidence that supports the claim that good metacognitive skills leads to good problem–solving performance. During the task interviews and the classroom observations, I focused on affective issues in mathematical performance. Specifically, I attempted to ascertain which areas of Schoenfeld's (1992) problem–solving framework were in use during various times in the observations. The goal of the majority of interview questions was to determine the individual's belief systems that were in place as well as the effect those systems had on the individual's learning of mathematics.

CHAPTER SIX: CAL'S CASE STORY

Personal Interviews

Personal Interview Data

Personal information. Cal is the youngest participant in my study. He turned 21 at the end of the semester about a week before finals. He is single with no children but he does have a girlfriend to whom he wrote love letters in his math notebook when he got bored in class (see Appendix H, Samples of Cal's notes, tests, & quizzes). He works full-time as a surveyor, a job he has held for almost 3 years, and is a part-time student. Most of his income is spent on his four-wheel-drive truck, boat, and motorcycle. He enjoys driving fast, playing softball, and making money.

Student information. According to the description of *typical* developmental mathematics students given in the previous chapter, I characterized Cal as a typical developmental math student because he met the following criteria:

1. He stopped taking mathematics in high school as soon as the graduation requirements were met. He took basic algebra in ninth grade, introduction to geometry in tenth grade, and algebra I in eleventh grade. Cal did not take any mathematics courses his senior year.

2. He truly believes school math is “a bunch of rules” with little or no application to his life or his work.

3. He does not believe he can do well in school mathematics. In his words, “Math isn’t one of my strong points ‘cause it takes too long. I don’t have a lot of patience, [math is] something I work on and try, but [I] never really picked up on math too good.” He has this belief despite earning average or above average grades in his mathematics classes in high school, “I did all right. I ended up getting like C’s and B’s, stuff like that.”

During the time of my study, he was enrolled in an elementary algebra course for the third time. The first time he attempted the class, he had just graduated high school and started taking “a bunch of classes with friends.” As Cal put it, the group “got wrapped up in the college scene” and were involved in “going out partying.” They would “go out and stay out until 4, 5 o’clock in the morning and try to get up at 8 and go to class and it didn’t work.” I got the impression that Cal was attending college classes because he believed society expected it, and it was what one did after high school graduation in order to get a good job. He said during one of the interviews that he was not mentally ready to commit to attending a four-year college and, as a compromise, he opted for community college:

> Cal: I still think eventually [four-year] college is for me, but not right now. I’ve got to get my head, if I were to go to [a four-year] college, I want to be like everything I have devoted to college and not waste my time. I mean, if

I'm going to go up there and waste my time, in a way I'd rather waste it making money.

Diana: Rather than spending money.

Cal: Right. If I'm going to go away to college, I would have my mind, body, and soul to getting perfect grades and right now I'm not like that. I want to make some money.

Diana: [Why are you taking the community college courses?]

Cal: Just kind of keep myself up-to-date, I guess. Keep myself up-to-date on what is going on and an expensive way to say 'hi' to friends that you don't get to see for awhile, and, I mean, just a way to keep my brain working a little bit as far as school goes.

His idea of keeping "up-to-date" and keeping his "brain working" seemed to involve simply being a student. He wanted to remain part of the educational system despite his dissatisfaction with that system. This was because he realized that the system "was not going to change anytime soon" and, as a result, he felt his only option was to remain part of it.

During his first attempt at college, he also had a conflict with a teacher in another class, English. Both the realization that he was not doing well in his classes and the conflict with the teacher caused him to withdraw from all his classes that semester and prompted him to join the Marine Reserves:

Cal: I just decided, all right, I was going to try to take a couple of classes and see how I did on that and then, I got some teachers that I probably shouldn't have got. Not the class, but, it's just, I didn't get along with, especially one of

> my teachers and I mean, I don't think I'm a hard person to get along with, but me and him didn't get along at all.
>
> Diana: Was this a math teacher?
>
> Cal: No, this was an English teacher, and we ended up doing views on certain things and he had different views than I did and he tried to use them against me. Because like, I'd write a report and then he'd come back and tell me that instead of just grading it on how it was written, he'd grade it on how he felt about what I wrote. Me and him didn't get along at all so I ended up getting into a lot of trouble in the school because me and him had a little conflict [laughs]. So it ended up where I got told I would have to leave but nothing happened to him, and so that just set me off and I left and joined the Marine Corps.

After his stint in the Reserves, he returned to the same community college and enrolled in an elementary algebra course for the second time. The second time, he made it partway through the course when a surveying job he "couldn't turn down" conflicted with the class so he made the decision to withdraw from the course and take the job.

> I left and then I joined the Marine Corps and was gone for about a year, joined the Reserves actually, and was gone for about a year with training and everything and came back and took a math class that, the same one I'm in now, but ended up conflicting with the job I wanted about half-way through, because I got the job and I couldn't turn down the job. So, I left half-way through that [class, the second time] and ended up taking it back again so this is actually the third time I've took the class.

The third time proved to be the charm for Cal; he passed the course on the third try. He explained to me that, although he had attempted the same class on three different occasions, it was "not for reasons that [he] failed it or something like that, but just conflicts with school, or just didn't want to be there or something like that." In other words, he wanted to make sure I understood that the repetition was not a function of his ability but rather it was a function of his choices. He indicated that he was taking the elementary algebra course because it was "transferable." He felt that one day in the future he would go to a four-year college so he wanted to "get all the core courses taken care of" at the community college.

Beliefs About Self as a Do-er of Mathematics

I can do it if I want to. Cal had complete confidence in his ability to do any type of mathematics with one condition: he must "want to do it." For Cal, wanting "to do it" required a belief, namely, that he "need[s] to know it." He stated, "When it's something in math, [it] doesn't really matter how complicated it is 'cause I can figure it out. I mean, if I need to know it or something like that, I can figure it out." His confidence in "doing math" seemed based in his ability to use technology. He believed, if allowed to use "computers and stuff like that," that he could "figure it out." His confidence is evident in his words:

> If I can sit down at the computer and [work], I mean, give me like a couple hours or something like that, then I'll try to figure it out. But if somebody gives me long enough on the computer, I'll figure out pretty much everything

> having to do with it. Without reading or anything, just go[ing] in [and] doing my thing.

I questioned Cal to determine what he thought were the reasons for when he had trouble in mathematics. His answers revealed more about his confidence in himself as a do-er of mathematics:

> Diana: And when you have trouble in math, what do you think is the reason?
>
> Cal: Just that I don't need to know how to do it.
>
> Diana: So, . . . if you take a test and you fail it, you don't attribute that to you not being able to do it?
>
> Cal: No. . . . I know I can do it. I mean, there's no question in my mind I can do it. It's just if I want to. It comes down to whether I want to do it or not and I guess that's why I don't do so good sometimes. . . . There's some stuff [in class] that, like I said, if I don't see the purpose in learning it, it doesn't stick with me. And there's other stuff [in class] that I've known since high school and I do it real quick and keep stuff in my head.

Beliefs About the Nature of Mathematics

Mathematics is cumulative. Cal believed mathematics was a series of steps or levels that must be attained in a given order. He believed these steps progressed linearly in that the prerequisite step must be mastered before continuing to the next level. The metaphor Cal used was that of a "ladder" in which the first rung consists of the "easy stuff" (e.g. "addition") and the subsequent rungs consist of "harder stuff" (e.g. "algebra" and "calculus"). Despite his stated belief about these steps in mathematics, in the task interviews he made several comments that indicated that he saw no connection between the steps (i.e. chapters in the elementary algebra textbook) taught in the elementary algebra course. For example, he explained, "We're on to something totally different now" as the reason for his trouble in remembering the procedure for factoring a binomial. The class topic that week was solving linear equations by graphing.

Conflicting beliefs about the nature of mathematics. A very interesting contradiction in beliefs, and resulting dichotomy of affects and behaviors, became evident after several interviews with Cal. The dichotomy involved his job and how he attempted to connect the mathematics he does "at work" with what he does "in school." I believe the frequency of the topic's occurrence in the interviews demonstrated the effort Cal made to connect those two pieces of his life and the little progress he made in the attempt. This struggle was heightened because of his conflicting beliefs. On the one hand, he had been told by teachers and, to some extent, believed himself that the mathematics he did in school and the mathematics he used at work were "the exact same thing." On the other hand, he was unable to see those connections for himself and therefore believed the two were "totally different." As is expected, this led to a lot of frustration for Cal. I think if he could have wished for one thing, it would be that he could see the direct connection between mathematics in school and mathematics "out in the field." His words illustrate the struggle:

> I know it's [algebra] kind of linked with surveying and stuff like that. . . . I know the purpose for it but the way I feel they teach it is pointless because when you get out [in the field], you don't, or at least I don't, relate x squared plus y plus z and all, whatever it is, to a turned angle. I know it's pretty close to some of the same stuff, but I don't relate the two at all. To me, it's in school and out of school and I can't put the two together. But I'm sure other people can. . . . I know it does [a relationship between in school and out of school mathematics exist] but I can't really; I mean, I know I use it every day. I know I use algebra every day at work, but, there's nothing I can really say, okay well this stands out and this stands out. It's all just thrown into a pile.

Teachers had told Cal over and over again that the mathematics he was learning in school was useful in everyday life. He told stories of teachers who, in response to the ever-asked "when am I ever going to use this?" replied "in surveying." Those responses disenchanted Cal because, as he put it:

> It was hard at first, the whole time I still can't figure out why you'd really have to know a lot of it, but I guess they [the teachers] say you need it for certain jobs and I've had almost half the jobs that they say you need it for and I've never used it. Everybody says you have to have algebra and stuff like that for surveying work. I've been surveying for three years. Everything's done. You don't have to really use it. Everything is done by either computer or you just put it in the machine and, I mean, I know it's nice to know it, but it's no need, to me it's just a waste of time. . . . It was good to know it [how to do something by hand] back in the day, but now you spend more of your time on

> a computer. Learn how to use a computer and you can do a whole lot more work.

This conflict between in-school and out-of-school mathematics also contributed to learning difficulties for Cal. Because he knew "better" ways existed, for example, using technology, he did not understand the reasoning for doing mathematical tasks the "old-fashioned way" that "took longer." He explained that if he did not understand the reasons for learning something, it was very difficult for him to learn and to remember:

> With me, personally, doing math, if I can get it in my head and figure out a reason why I need to know it, then I do fine with it. But stuff that I know I'll never use and I know it's pointless for me to learn it, then that doesn't stick with me good at all. I mean, if I know I'm going to use it, then it sticks real quick. Now if I don't, I have to learn it again even if I learned it before because it doesn't stay. If I need it though, I learn it fast, before most people, but I just have a hard time trying to figure out why I gotta learn something that I'll never use or why waste time learning it. . . . I don't like school and I never have but I know I gotta do it. But I'd rather be out working, doing something that I like to do, such as when I'm surveying like now, I like to do that. I mean, it has a lot of math in it but it's the math you do use, a lot of it is common-sense math, turning angles and stuff like that. And that stuff I have no problems with. I can figure that stuff out easy. And that's because, it comes somewhat from math class but mostly it because that's [in the field] hands on and you get out there and learn it and you know how to do it.

Cal's dual beliefs about the nature of mathematics resulted in dual affects. I coined the term *affective dualism* to describe the apparent contradiction of two coterminous affects that result from dichotomous beliefs. For Cal, these dual beliefs revolved about the nature of mathematics. As previously described, he saw the mathematics one experienced "in school" as both "totally different" and "exactly the same" as the mathematics one would use "out in the field." His dual beliefs resulted in conflicting affects and behaviors. If Cal viewed something as "school math," then he would act and feel differently than if he viewed it as an example of "real" mathematics that would be used "out in the field." Cal describes his affective dualism in this way:

> In school math, [it's] just somebody sticking a whole bunch of numbers together and saying, 'Figure it out.' I mean, it doesn't really have any meaning at all; but [instead] just saying, 'Okay, you gotta learn this and you'll use it someday and I just gotta teach it to you so hurry up and learn it.' And then, when you get out in the real world, math, it's [totally different]. I mean, usually you're going to buy something, I mean get a paycheck, ride down the road and figure out miles on the car, [things like that]. Lots of things have to do with it [real-life mathematics], but as far as associating that and the school [he can not]. I mean, I know it's exactly alike, I mean I know it is, but in my mind it's two totally different things from somebody saying, 'Here's a bunch of numbers, okay, figure it out' to going out and saying, 'Oh, I've got $20, what all can I buy?' or 'I've got $100, what all can I buy?' It's the same principle, but in one case you've got it in your hand and you're standing there doing it and in the other you're just sitting there and he's giving me some numbers, you know, what am I doing with them? . . . I mean I know it's the same stuff . . . but the reason for doing it is different. . . . If you put them

together [side by side], it's the exact same thing, but [real-life mathematics has] a meaning for doing it.

Beliefs About How Mathematics Should be Taught

Mathematics should be taught using a hands-on approach. Cal believed that mathematics classes should reflect the "common sense" mathematics he was experiencing at work, but they did not, and as a result, he was frustrated, bored, and annoyed:

> Diana: You mentioned surveying, did the geometry class you had in high school help when you started surveying? Did you see a connection between those two things?
>
> Cal: Yeah, in some ways. But in geometry, they'd just throw some numbers out, and put X's and stuff like that on it, that's all the stuff they'd use. Then, you get out there [in the field] and it's hard to relate what you learned in there [in the classroom] to the real life thing where you're actually turning angles and everything else. It's kind of hard to relate the two together because you're actually working with something physical instead of just writing something down on a piece of paper. It is for me. It was different for me to look at a book and see a bunch of numbers and figure it out, or to go out [in the field] and be looking at a machine and say, "All right, this is what I gotta do and this is how I gotta do it." It [classroom experience] helped, but 99% of the stuff I learned is probably from hands on, doing the job, asking questions while I'm there.
>
> Diana: So does that mean that maybe your geometry class would have gotten across to you better if it had been more hands on?

> Cal: Yeah, and instead of just sitting, sitting in a desk and watching somebody up in the front drawing things and telling us to do certain things, give us a reason why we needed to know it and saying, okay this is what, exactly what you would do and maybe they even take you out and do it and stuff like that. Then you'd know there is a reason why I gotta learn this. If I want to do this, there's a reason why I should know it. In high school they teach it, fly through it, next day go to this, next day go to that, and so I'm like, why do I need to know this? It'd be better if they spent time showing you what you needed it for. That's my attitude I guess. . . . I'd rather learn by just doing it instead of somebody standing up and showing me how.

Beliefs concerning students' and teachers' roles in the mathematics classroom. Cal's beliefs about what occurs in mathematics classrooms were similar to the other participants. These similarities in beliefs may be explained by the participants' similar experiences in mathematics classrooms. Because of his experiences, Cal thought mathematics classes were boring, consisted of all lecture, involved a lot of homework, were not connected to real life, and simply were a means to an end and just something that must be done. Although he said he did not like these things, he was not the least bit inclined to consider the possibility that they could change. His description of the [then current] elementary algebra class shows his beliefs:

> A lot of lecturing, a lot of lecturing, a lot of homework, if you have a question you ask it, and if not, it's assumed that you know it. . . . If it's something I need to know, I pick up on it real quick but if I feel I don't need to know it, then it's like I'll block it out. And when I do that, if I feel there's no need for

> me knowing it, then I go in and do something else while he's teaching that, balance my checkbook or something. . . . So my time isn't wasted. . . . In this class I'm learning the same stuff over again. Learning the same stuff I learned in high school and everything. Just all over again the same stuff I didn't need to know the first time, I'm learning again for no reason. . . . I just blow it off [the "stuff" he feels he does not need to learn]. I learn it for three months, make the teacher happy, whatever, then as soon as I'm out, it's gone. . . . I tolerate it because I have to; it's not like I can change it. It's not like anybody can change it. It's been that way for awhile.

Other Beliefs

The necessity of a college degree. Cal really enjoys his job as a surveyor. He feels he makes "good money" and is content, at least for the time being. He believes the job he has is comparable to, and even better than, the jobs he sees his friends doing. Because these are friends who have gone to a four-year college and graduated, this leads him to another belief: A college degree is not a requirement for a good job.

> I've done pretty good for myself. Got a job, pays real good, a career job that pays real good as a surveyor, and all kinds of other stuff, a truck, a boat. I think I'm doing all right for myself. I'm even going to college, I mean, not yet, but when I was growing up, everybody said, 'hey you gotta have that little piece of paper.' No you don't. As far as I know, unless I want to be a lawyer, which I don't want to be, or have to have a law degree of whatever it is, or a doctor, which I don't want to be, or some, one of those high money jobs where you make $300,000 a year. I don't need to make that kind of money. I

> mean, I wouldn't turn it down if someone threw it to me but I'm happy making what I make now and I can live off of it right now. In no time soon, what I make right now can support whatever I want to do, family or not. I'm throwing enough in the bank now; it's racking up quick. That is, I don't need a college education to tell me I'm doing fine right now. I'm ambitious. It [the job he has a surveyor] was a once in a lifetime opportunity. If I were in college I probably would have missed it because I have been there on and off about a year. In about another year, two years tops, just, I've known the guy, I've helped him out, and I mean, he's talking about like next year me going into a partnership with him and co-owners in the company and at 21-years-old, being a partner in a surveying [company], being a professional land surveyor or whatever, that, as soon as I get certified, to go into a partnership at 21-years old. I don't think that's a bad start. Everyone else I know doesn't think it's a bad start either. I'm making more money than a lot of people I know that are 30, 40-years-old and been somewhere for twenty years. Or a lot of my friends have gotten out of college with that piece of paper, they're having a hard time finding a job and the ones I do know, I can't think of any of that have graduated [high school] before I did and have come out of college. Three or four of them have come out of [names a prominent four-year-college] and that's supposed to be one of the best schools in the nation, and I'm making, not double what they make, but pretty close, and I've been doing it for awhile.

Affects

Dissatisfaction with curriculum and mathematics course content.

Another issue with Cal was his feeling that other people were "deciding things" about his education that he felt were his right to decide; for example, he thought he should have control over which courses he needed to take in order to earn a degree. He felt he should be able to take courses he was interested in and what he felt he needed, not what "somebody upstairs" decided was appropriate. He also felt that within a course he should be able to have some say in what he learned. He thought, for example, that all the time he spent in his current algebra class on factoring was "pretty much a waste" because he would "never use it." He said he was "doing it" simply "for a grade" and that it would serve no useful purpose now or in the future. That was probably his ultimate measuring device: if it served some useful (by his definition) purpose or if he could use it (implied at work).

One comparison Cal made during an interview revealed his views. He was explaining why he felt so much of school math was a "waste of time" because he would "never use it" and he said it was like "learning vocabulary words." As he put it during one of the task interviews, why should he have to "learn a word that is 14 letters long" if a word he already knows that is "4 letters long means the same thing?" He was positive he would "never use the 14 letter word" so the four-letter word would "do just fine." In a personal interview, Cal also mentioned this vocabulary-word comparison and stated again his discontent with mathematics curriculum:

> I mean, with me, I'm not going to be writing any books or anything like that, so as far as, like English, when I talk people can understand what I'm saying, so why do I need to know all these huge big words and know how to spell them because I'm not going to use them. The way I feel about it is, learn what

> you need to know. If you want to learn more, go and take the class and learn it. But if there's no need to know it, then why waste the time to learn it? You can be learning more of what you need to know instead of all this stuff around it. . . . And I don't see any point in it as far as like going to college and stuff like that. They make you take all these other classes that really have nothing to do with anything you want to learn. They say it's like the curriculum or something and I just don't agree. Why learn something that you're never going to use and never want to use? Some sciences and stuff like that, I'm interested in science, but I never saw the point in why I would have to learn how to, for instance, dissect a frog or the stuff they make you do. I mean, I hunt a lot but that's something different. . . . I don't hunt frogs. . . . As far as making me learn it just because somebody up above says that, 'Oh, I think they should learn it,' I don't like that. . . . Yeah [he feels frustrated because] I feel like it's been wasting my time. I mean, it's like, I pay money to learn this stuff. Why can't I learn what I want to learn and go in the directions I want to go instead of paying somebody all this money to teach me something that I'll never use?

Affective results from practices and mathematics teaching. Cal did have one story that was a very positive experience in school mathematics. He had a teacher in high school for eleventh grade algebra who was "a really good guy," and by the end of that class, Cal had a positive attitude toward math. Unfortunately, the next class he had was the first attempt at the elementary algebra class at the community college and it reversed any positive progress made before. As he described it, right after his high school class:

> Well yeah, I was into math. I mean I didn't want to go out and go off gung ho and sign up for 50 classes in it. I mean, after that class I was, 'This ain't bad. I learned a lot and didn't really know I was learning it and had a good time doing it.' Then I got to my first class up at [names the community college] and walked into class the first day and he [the teacher] said, 'This is what we're going to learn'—all right, lecture time. An hour and a half later I was out. I was catching z's.

The characteristics he seemed to like the most from that high school teacher was that he "knew" Cal; he recognized that Cal was not very interested in school and tried to make it fun. Cal also felt that teacher treated the class like equals instead of an "I'm-the-teacher-so-do-it-my-way" approach. He said the teacher allowed the students some freedom and "bending" of rules with the understanding that the work must be done afterwards. Cal described the teacher this way:

> Well, the teacher, me and him got along real good. I probably wasn't one of his best students 'cause it's high school and you're having fun, but me and him got along real good. I stayed after school and he helped me out. Pretty much guided my whole way through the class. I mean, I'd stay after school almost every day in his class, going over it with him, the same thing we learned that day. . . . So he worked with me real good with it, just learning everything, pretty much just learned it twice to get it stuck in my head.
>
> Diana: So, what would he do in class to teach? I mean, describe his actions [teaching methods].
>
> Cal: Lot of it was just work on the board and giving you problems and coming around and working with each person. . . I've had some class[es] where the guy just stands up there and does things on the board and does them so fast

> you can't figure it out. Then, that's when me and, I know a lot of other people in the class, would just say, 'All right, forget it. It's not that important anyway.' But he [the teacher from the high school class] would come around to each person and ask them, 'Do you get it?' Then he'd stay after school. He did whatever it took. And sometimes I'd even miss my next class 'cause I had like gym in my next class and I aced gym every time so it didn't matter whether I was there or not. The teachers, they didn't care you know. I got to stay after class and the next class he'd also have the same class, so I'd stay in there like half the class and go over the part I didn't understand and then I'd get up and leave. So, a lot of time he put into it was really what helped me out a lot and not letting me just say 'Forget it' because he wouldn't let me do [that]. I'd try [to give up] and then he'd grab me and come to my house and everything else. . . . That was probably my best math class ever 'cause he took the time to get to know me and he knew how I was. He was willing to help me out with it. . . . He made it fun. He worked with us. He worked with everybody. If everybody was rowdy one day, then he'd get rowdy with us, and then teach it a certain way. Then some days, he'd lock it down and [say] 'we're going to learn this today and everybody's going to be quiet.'. . . He told us [how to do something] when he was up at the board, [we would] just watch and try to get it, but when he comes around, that's when you'd ask your questions. He said, well one of the big things that stood out with me was that a lot of people had the same questions but he couldn't explain it to one person one way and have another person understand it. So each person he'd explain it for their own way.

Cal's most negative affects regarding mathematics teaching concerned his feeling that the methods used to teach mathematics were "the ancient ways of doing it." As mentioned in the previous section on beliefs, Cal believed current teaching methods were "behind the times" and, as a result, he had feelings of disgust, discouragement, and disillusionment associated with those methods:

> I know it's probably good to know it [how to do certain mathematical tasks by hand], but a lot of, I know me and I know a lot of other people just don't feel like we've got the time. I mean, if I can do something in five seconds, that will only take five seconds [to do with a calculator or computer] and the same thing is going to take me an hour [to do by hand], then I think it's pretty stupid to take an hour to do it. . . . Pretty much anywhere you go, if the computer shuts down or if the main computer shuts down, you're through for the day anyway. You're not going to do nothing, there's no need to do it by hand because you got to put it in the computer once it's done so [you should] just wait and do something else. . . . These days, I mean, for the time you spent, like at my job, if we sat there and calculated each point by hand, the time we spent doing that, we could have gone home or gone back to the office, picked up another job or picked up another calculator and made $700 in a day, that's running back and coming back out there I mean. . . . Yeah, when you're doing $300, $400 a job, it doesn't pay to sit there and work everything out there by hand and take 4 hours to do a job that should take 20 minutes or 30 minutes because, . . . that's a lot of math and a whole lot of calculations are done in a split second by a calculator and computer and stuff like that. But if you're going to work it out by hand, you're going to be there with about five trees of paper.

He described his feelings of current teaching practices as a ridiculous game, "I mean, you get up there and you do it. You know what Y is. You know what X is. And you know what C's going to be, or whatever, A, B, C, whatever. You know what they are.

Why play the game that you're not going to know?" He explained the choices available for teaching mathematics:

> You can do it on a calculator or you can do it by hand. I mean, I push a button, while they're still there writing their thing on a piece of paper, I'll be eating lunch! . . . I think math needs to catch up with the times. I mean, we're still learning stuff that, I mean it's good to learn it like I said before, but when you've got the world's moving about 6,000 miles per hour and you're still learning stuff about 3 miles per hour. I mean, when you got stuff that will do the work in a split second and you're still learning how to do it on paper, it's not a waste of time but it's not using your time wisely. I mean, the world is moving so fast [that] to me, you don't have time to sit around and you waste too much time just sitting around and figuring stuff out. I mean, to me, I'm only 20 years old and it's going by too quick for me to want to just sit around and do math or English or something like that when I know that there is no real reason for me to learn it if I can hit a button [on a calculator or computer]. . . . I just don't understand the whole thing why math hasn't caught up to the time period it's in. It's the computer age. . . . If you want to ride a horse to California, that's all on you. Go for it. There are still people around this town who ride a horse places, or ride a wagon places. I don't have time for it. I want to go. I want to get there, do the work and go somewhere else. I got a car with about 500 horses under the hood, let's go!

Observations

Classroom Observation Data

Observation data about the environment. The elementary algebra class met on Mondays and Wednesdays from 4:25 - 6:55 p.m. Class was held in a portable building with poor air circulation and poor artificial light. The temperature was rather warm and the air was stuffy. The twenty-three chairs were arranged in tight, very straight rows that faced the wall-length blackboard. The initial enrollment of the class was twenty-three students, that number dropped to twelve by mid-semester. The teacher used a small table, located in the front of the room, as a desk. In one corner of the room, there was a never-used TV and VCR on a cart that was chained and locked to the wall. The students' desks were very small, about one foot square. The room had three small windows and two large file cabinets.

Observation data about students' and teacher's behavior. Cal always sat in the last seat of the row closest to the window and across from the door. I never observed him taking notes on anything the teacher said; however, he did occasionally copy a problem that the teacher had worked on the board. I never observed him open his book during class. He often brought other things to do in class; for example, one night he worked on balancing his checkbook. He spoke up in class quite often, though many times it was just to making a joking comment. He referred to himself as "the class cutup." Cal missed approximately 15% of class meetings. His attendance was fair to good if compared to some of the other students in the class, poor if compared to the standard set forth in the course syllabus. He missed a few quizzes and one test, none of which were allowed to be "made up" by the teacher.

During an observation I made during the tenth week of class, the teacher began at 4:27 p.m. with seven students in the room. Cal was not one of those seven. He arrived about 15 minutes late. The class was reviewing a section that had been assigned for homework. Cal asked the teacher to work a problem: simplify $\sqrt{24x^5yz^3}$. The teacher worked the problem by breaking it into four square root problems that were multiplied together. The teacher asked one student about how the square root of 24 could be simplified, and then he asked Cal how the square root of x^5 could be simplified. Cal did not know the answer and asked, "Why can't you just leave it as the square root of x^5?" The teacher stated that the directions were to simplify and it was not simplified in the present form. As he continued working the problem, the teacher mentioned the assumption that the variables, x, y, and z, were greater than zero. Cal asked, "What if it was negative?" The teacher responded, "As far as we're concerned, we couldn't do it [if a variable was negative]." Cal's reaction to the teacher's answer was one of only partial belief. His expression seemed to indicate that he did not totally believe the explanation given, but he did not ask any further questions about it.

The teacher called on Cal again during that class period because he was talking to another student and not paying attention in class. The teacher told him, "I don't feel like I have your full attention." Cal did not know the answer to the question the teacher asked, Find the decimal approximation to the square root of 98 without using a calculator. The other student, the woman Cal had been talking to in class, was called on next and she answered, "2.64?" The teacher asked her, "Is that reasonable?" then went on to answer the question and find the approximation himself. Later in that class period, while the teacher was working more decimal approximation to square roots, she and Cal were reading the paper, the personals, and laughing and joking over the ads. The teacher went on to discuss the "quotient property of radicals" and began working example problems at the board. He found a final answer of $\frac{\sqrt{85}}{17}$ to one problem and asked Cal, "How do I know it's [the decimal approximation] zero point something?" Cal did not know nor did he seem to pay much attention when the teacher went on to explain it himself. Cal told me during a personal interview conducted after that class period that he thought it was "stupid" to do those types of problems without a calculator. As already discussed, if Cal did not think something was worthwhile, then he did not bother to learn it.

I observed Cal again during the last week of classes before final exams. It was "review day." Cal got his test results from the week before. The test covered three chapters of the book and was mostly on graphing linear equations, solving systems of equations by graphing and substitution, and using the Pythagorean theorem to find the missing side of a given right triangle. He scored a 59. Cal mentioned several times during interviews that he did well on quizzes but not on tests (e.g. "because my quiz averages right now are like 90-100 and my test averages are like 70-75") because the number of procedures he had to memorize was smaller for quizzes than for tests. He kept a record of his scores on tests and quizzes and it is obvious from that sheet that he spoke the truth (see Appendix H, Samples of Cal's notes, tests, & quizzes). He explained that he was "just not good at taking tests." When I asked why he could do well if it was a quiz and not do well if it was a test, he responded that it was because "not as much stuff was covered" on a quiz as a test and he could "remember it better." He also mentioned that he did not feel as much "pressure" with taking a quiz as he did when he had to take a test. This was true for other subjects as well, not just mathematics.

Task Interviews

Settings for the Task Interviews

One of Cal's task interviews was conducted at MTCC in an empty classroom. It occurred in the afternoon before his elementary algebra class met. The other two task interviews were conducted in an empty classroom of a local middle school. Both settings were somewhat stuffy and formal and not very home-like. The video camera was set up at an angle to the table or desk where Cal was seated both in order to prevent him from becoming preoccupied with being videotaped and to be less intrusive. I sat across from Cal out of view of the video camera.

Overview of Cal's Task Interview Data Using Schoenfeld's (1992) Framework

As stated in Chapter Five, I used Schoenfeld's (1992) problem–solving framework as a lens in analysis of the task interview data. The following overview of Cal's work is given to provide a framework for the reader to view the task interviews. This overview includes the categories of resources, heuristics, control and metacognition, and beliefs and affects. A general view of Cal's approach in solving tasks is given for each category.

Resources. At first glance, it appears that Cal possessed the resources necessary for successful completion of all tasks presented. Most of the tasks were very similar to ones he had experienced in his then-current elementary algebra class. He experienced these tasks by either working them himself (in homework, on quizzes, or on tests) or by watching the instructor work them at the board during class. However, on several occasions, his belief that if the task presented was an "in school" task, then it could not be solved using common-sense methods caused him to either not be able to solve the task at all or to take a much longer amount of time to solve the task. Likewise, if he viewed the task presented as an "out of school" task, he would not think of using a school-taught approach to solve it. Although these are actually control issues, I mention them here because Cal had two different *boxes* of resources. He had a box of resources labeled "in school" and another one labeled "out of school." The way he chose, implemented, and monitored these resources was the focus of the analysis.

Heuristics. Having two sets of resources for solving tasks that he deemed "in school" and those he labeled "out of school" was one way Cal responded to his affective dualism of the nature of mathematics. Another response involved the heuristics he employed in solving tasks. As Cal described it, he often had difficulty getting his "mind working right" in the beginning of the task interviews. I believe this struggle was a result of his belief that thinking logically was not a heuristic that was allowed in "school math" but rather that "out of school" math had exclusive rights to it. Because of his experience at work and his general personality, the trial-and-error and "thinking about it" heuristics were the strategies he preferred to use when first approaching a task. However, these heuristics were not very helpful to him when the task was an "in school" task and, as a result, he would next choose a strategy from the school-taught heuristics "pile."

To illustrate, I describe the general way he would approach the first task of an interview. Occasionally, the tasks I gave in the task interviews were similar to ones that had been covered in class several weeks earlier. When Cal approached a task of this type, he would try to use his preferred heuristic of "just thinking about it" in what he considered to be a common-sense or real-life-oriented approach. In other words, his common-sense approach meant applying strategies he learned from real life problem–solving experience as opposed to classroom-taught strategies. After he was unsuccessful using this approach, he would switch to a school-oriented approach and usually be able to solve the task. I noticed two exceptions to this general behavior. After the first two interviews, Cal had learned that many of the tasks given were "in school" tasks. He therefore started using heuristics that matched that type of task earlier in the interview. In addition, if the task given was one similar to the types given in class at that time, he would also go directly to his "in school" bag of heuristics and forego his preferred "thinking about it" strategy.

Control and metacognition. When given a task, Cal spent less than a minute looking at the problem or planning his approach before he began writing. Cal did not seem to think any of the algebra tasks presented to him (in class or during the interviews) were hard. If it had "been awhile" since he had done them, however, he often had trouble and did not get the correct answer. When I asked for an example of a problem that he did consider hard, he said he "always had a problem with the ones that [were] like a train leaves . . ." and the ones that had a "trick" or some "inside joke" to them. Other examples, like interest problems, were not hard but just "a pain," because he had to do them by hand (the instructor did not allow use of calculators) and it was tedious.

Beliefs and affects. Cal would often make comments that (a) reflected his lack of confidence in his ability to deal with "in school" tasks, (b) illustrated his beliefs that mathematics "in school and out of school are two different worlds, totally different," and (c) showed his displeasure at having to take courses that "somebody thinks [he] should know." One thing that kept recurring in the task interviews with Cal was the effect of the passage of time on his ability to remember what he had been taught. He commented that a task was either difficult because it had "been awhile" since he had worked tasks of that type or easy because it was similar to tasks he was presented in class at the time. Other participants made similar statements. Cal also referred to this phenomenon in his personal interviews in terms of how the passage of time affected his grades in class:

> If I can do something in one class and have a test on it the next class I'm fine. But when we go for long periods, learn a lot of stuff and then have a test on it, I don't do good. . . . Different chapters and different sections and stuff like that doesn't stick with me real long because so much of it, I don't need to know except for a grade and I've never really let grades run much of my life at all.

Cal, like the other participants, had most difficulty answering the follow-up questions: "What does your answer mean?" and "What do you think it might be used for?" [see Appendix B, Follow-up questions for the task interviews]. Often, he had no idea or believed it to be simply "something in school" rather than a useful application to something in life. He also thought other people might use algebra, like "scientists and doctors," but he would never need it. The only tasks he was able to give a meaning to were the word problems that he viewed as more "out of school" types of tasks. For "in school" tasks, Cal's control behaviors like "monitoring," "self-assessment" or self-reflection, and "decision-making" largely consisted of mimicking what he saw teachers doing.

The belief that math is a tedious, exact science with no room for creativity is one that came up in Cal's task interviews as well as some of the other participants (see Chapter 8, Chris' Case Story). Some resulting affects of this belief were frustration and impatience. He explained his frustration with math this way, "It has to be so exact . . . you can be off by decimals and it'll be totally wrong . . . [I] don't have the patience." This belief is connected to his affective dualism about the nature of mathematics. He thought that "in school" mathematics was tedious and exact, but viewed the mathematics he did "out in the field" as interesting and useful.

Task Interview Data

Cal's actual work on the task interviews can be viewed in Appendix I. The entire list of tasks given to participants during the task interviews is given in Appendix A. The follow-up questions asked during the task interviews is given in Appendix B.

Task interview #1. Because Cal approached tasks differently when he viewed them as "in school" rather than "real life", he would sometimes be unable to solve a task in the beginning of an interview, but successfully solve a similar task at the end of that same interview. In the first task interview, for example, he was unable to complete the first three tasks successfully but did successfully complete the last two.

While working on the very first task, factor completely: $5r^2 + 10rs - 20s$, he commented almost immediately, “I can’t get my mind working right.” He sat, alternating between staring at the task and into space, for almost a minute and seemed to be trying to retrieve a piece of information lodged somewhere in the corner of his mind. I asked him about the difficulty he was having. He commented that it had “been awhile . . . month, month and a half” since he had done problems of that type, and that he could not remember the procedure. I told him to take his time and he worked on the task about three minutes, arrived at an answer of $5(r + 6)(r - 4)$, then (correctly) announced, “I think it’s wrong.” I asked him why he thought it was wrong and he stated that the answer just did not “look right.” I asked him how he could check to see if his answer was correct, and he responded, “Go in reverse.” After I asked about checking procedures, he checked his answer, found it to be incorrect, then worked for about two more minutes before he gave up by announcing, “I don’t know right now.” He commented that when “this stuff [factoring trinomials]” was “fresh in [his] head,” he considered all problems of that type “easy” but that the “stuff we’re doing now [which, at that time in the class, was graphing linear equations] has no relation to this.”

The second task and the fifth task were almost identical; both dealt with factoring. The second task, factor completely: $16^2 - 25^2$, prompted the immediate question, "Can I use a calculator?" I told him he could. He worked only a minute or two, then announced an answer of -369. I asked him to explain how he got his answer: "I did 16^2 and got 256, then subtracted the 25^2, which was 625, and got -369." As he was talking, he entered the numbers in his calculator and read the resulting display to me. When he got to the answer of -369, he said, "Hold up, that's a negative so it'll be a positive," and then worked the problem treating the subtraction of 25^2 as $(-25)^2$. He entered the calculations into the calculator and got an answer of 881. As he did this, he made an offhand remark about "screwing up as usual" referring to his perceived error of not realizing that the squared term would result in a positive answer.

The third task was: The area of a rectangle is 21 square feet. What are its dimensions if the length is 5 feet less than four times the width? He worked about two minutes then seemed frustrated that he was not getting anywhere and commented, "This is even stuff I use at work and I can't get it." He continued to try to solve the problem, however, and made comments that demonstrated his understanding of the basic concepts of area. For example, he mentioned that it was "a small rectangle" because the area was "only 21 square feet." He also commented that he "knew" the answer was "3 and 7" but he could not "do it using the math." In other words, he could not use school-math (e.g. a formula or classroom-taught procedure) to solve the task, but he could solve it using "common-sense" (e.g. realizing that 3 and 7 were reasonable guesses since the product had to equal 21).

The fifth task, factor completely: $r^2 - t^2$, caused him to respond that it "can't be factored anymore" after he had looked at it approximately 30 seconds. I asked why he thought that and he answered, "Either way you're going to end up" then trailed off, looked at the problem another few seconds, then said, "Hold up" and wrote something down. He then announced the answer was $(r - t)(r + t)$. I asked him how he could check and see if his answer was correct and he told me he could "factor it back out" then correctly explained the method of checking using the FOIL method. Although he correctly explained the procedure to check his answer, he still asked me, "Is this right?" At this point in the task interview, my role changed from objective observer and questioner to teacher. I asked him to compare the second task and the fifth task. He noticed the similarity. I pointed out that the second task had one factor as an answer and the fifth task had two. He did not seem disturbed by this observation. After several minutes of prompting him and asking leading questions, he said he could have written the answer to the second task as (16 - 25)(16 + 25) but he didn't because it involved "two different ways of learning it, when you've got letters you do it this way [indicated parentheses], when you've got numbers you do it that way [pointed to the calculator]." I asked, "Even though the instructions on both tasks were 'Factor'?" He answered, "Yeah." He viewed the second task as something he could solve using "common sense" whereas the fifth task was something he did "in school" and therefore different rules governed the solution-finding process.

Task interview #2. Cal seemed more prepared in the second interview than the first. I felt this could either be attributed to learning that had taken place in the interim or to his switching from the "just thinking about it" common-sense strategies to the school-oriented strategies more quickly than he had done in the first interview. Although my role was that of objective observer in the beginning of the interview, I quickly assumed the role of tutor/teacher for the remainder of the interview.

The first task was to factor $5t^2 - 32t - 21$ completely. When he started working, he seemed to try to "figure it out" by just "thinking about it" in what he considered to be a common-sense way, but did not make much progress. When he realized this, he stated, "I can't think how to do it. I know how to do it but I can't figure it out." I probed the statement by asking what he meant. His reply indicated that what he could not "think how to do" was the procedure he had been taught in class. After working a minute or two more, he made the transition to using a school-taught heuristic, remembered the procedure, and correctly answered and checked $(5t + 3)(t - 7)$. When I asked what he thought the process of factoring was used for, he replied that it was "easier to use." When I probed further, however, he was unable to give an example of how it made it easier except to say, "We just do it that way in class."

The second task was: Evaluate for x = -9, $\frac{x+9}{x^2+2x-1}$. He stated his solution as $\frac{0}{62}$ which he said was "undefined." I asked him what the answer would be if it would have been $\frac{62}{0}$ and he responded, "It would just be 62." I asked him what the domain was and he said questioningly, "62?" When asked what the definition of domain was, he answered, "Something can't equal zero." At that point in the interview, my role changed to teacher and I lead him to the correct response of 0/62 being zero and 62/0 being undefined. Although he listened to me and responded to my questions, I felt like he was just humoring me and did not really care which quotient was zero and which was undefined.

The third task was to evaluate $\frac{2x^2}{x^2+9}$ for x = -3. He thought this problem was "easy" and correctly answered "18 over 18 or 1." When I asked what his answer meant, he replied, "If x = -3, that thing's equal to 1." I asked what that was "good for" and he said, "Make it easier." When I probed, I found he had no real life connection, only a vague idea that "scientists and doctors" would "use it."

I gave the fourth task with the intention of discovering if he would see a more definite use for factoring than just to "make it easier." He was asked to solve the equation $x^2 + 2x - 1=0$. Not only did he not see a real use for factoring but he also was unable to make any progress in solving the task. When he had stopped working, I asked what his answer was. He said, "It doesn't work." He had tried to factor it using the method that was taught in class (finding the sum and product of two integers, m and n, and rewriting the equation in terms of those variables) and was unable to find a pair of numbers that "worked." The strategy of completing the square or using the quadratic equation never crossed his mind. Cal seemed slightly upset that he was unable to complete the task. When I asked if the task was unfamiliar to him or if it was difficult, he responded in the negative. Instead, he justified his lack of success by commenting "After you take the test on it, it's gone." He said he had successfully solved tasks of that type in the past and he did not view that type of task as being especially hard, it was simply that too much time had passed and he no longer remembered the procedure. As he put it, he was doing it "all for a grade, not going to use it, just going to [need it] to pass the class." He also commented that he did not "like it" but that was just "the way it was." Because Cal mentioned "forget[ting]" and "not remember[ing]" so often, I asked if he had trouble with his memory in general or if it was specifically math-related topics. He said his memory was fine when it came to "stuff" he "needed" or "used all the time" and that it was just certain "school things" that he was unable to remember longer than a day or two, "long enough for the test." He went on to explain that he could remember very detailed procedures for work because "you know a reason for it." He also added that if it was something that "made sense," he could remember how to do it.

Cal proved his statement when he worked the last task. It was a word problem: Sarah is operating a machine that can produce 14 parts in 20 minutes. How long will it take her to produce 224 parts? Although he stated that he had not done word problems of that type in class in several weeks, he successfully set up, solved, and explained his solution to this task. When I pointed out the apparent contradiction, he explained by saying that he could work this problem by just "thinking about it" and he did not need to "remember anything." I believe he was referring to using common sense instead of an arbitrary algorithm he learned in a classroom. He said he thought there was "more of a real reason to know how to do this problem than the others." It was the only task in which he was able to answer the question, "What does your answer mean?" in any kind of meaningful way. He said because he knew the "rate" Sarah's machine made the parts, he could then find how long it would take to make any number of parts and that information would be very useful in a number of ways: "trying to meet a deadline," "finding out when you can leave [work]," and "when you're done."

When Cal explained how he solved this problem, it was evident he understood the mathematical relationships of ratios and proportions. However, he did not know the "right words" to explain them in correct mathematical terms. He said he worked the problem by first finding out how many "20 minute shifts" Sarah would have to "pull" in order to produce 224 parts. He found the number of shifts by dividing 224 by 14. When he found she needed 18 shifts, he then multiplied that number by 20 minutes to find the amount of time it would take to work those shifts. He found it would take 360 minutes. Dividing 360 minutes by 60 minutes, because "that's how many minutes are in an hour," he then converted 360 minutes to 6 hours. He said he did not check the problem because he "knew" it was right. He added that the only thing he could have "messed up on" was the operations of division and multiplication and he used a calculator for that so he was confident it was right. He also mentioned that he had already checked to make sure he entered the numbers correctly as he typed them in the calculator.

As a follow-up to his statement about "know[ing]" his answer was correct, I asked, "How do you know when something's wrong [with your answer]?" He said, "I just look at it, if I don't check it." He also said he had a "feeling" if his answer was wrong. He was unable to explain further about what his "feeling" was precisely or about how exactly he "knew" if something was right or wrong. I believe both the feeling and the knowing were based on past experience with tasks of that type or a related type. I asked the following metacognitive question, "How accurate is your feeling?" He responded that "most of the time" his feeling was pretty accurate. I also asked the following control-related question, "At what point [in the problem–solving process] do you try another approach?" "I get frustrated with math real quick" was his answer. My observations of Cal verified that response. He seemed to work on a problem only a few minutes before he either quit in frustration or asked for help. If he felt he was making progress, then he would work several minutes longer than if he felt "lost."

Several affective issues were discussed in the second task interview. Among them, his feeling that mathematics "needs to catch up." As he put it, "Everything else is getting updated and modernized but we're still learning stuff from back in the day." I believe this feeling was so strong in Cal because the instructor of his algebra class did not allow calculator usage except in a few limited situations (e.g. as a "check"). In his job, however, Cal saw the power of technology, used and appreciated that power, and could not understand why, "if you've got a match, you're still rubbing sticks together." He commented that, when he had a choice between "doing something" that would take "ten seconds" versus "ten hours," he would choose the faster method. I asked, "Have you learned anything in this algebra class that would allow you to do something you needed to do quicker?" Basically, his answer was no. Although he said he had learned a few "little tricks," he said it was "nothing a calculator couldn't do quicker." His general feeling was that "math needs to catch up." I asked questions about his attitudes toward his algebra class. His general attitude was negative though resigned. He thought it was a "waste of time, not to know it, but to have to do it [by hand]." He went on to explain that he felt working a "few problems" by hand was enough and that "hundreds of them" were not necessary when a calculator or computer could perform the tasks so much faster. He commented that his attitude would probably be more positive if the course approached mathematics from a more "practical" standpoint. As in the personal interviews, Cal commented on the discrepancy he saw between what teachers had told him about the practical use of mathematics and the actual use that he experienced at work. He remembered asking teachers on numerous occasions, "When am I ever going to use this?" He then recalled an answer a teacher had given him, "In architecture and surveying." Cal did not view these as real examples because he had worked in both of those fields and did not see the applications himself.

Task interview #3. The final task interview resulted in some interesting findings about how Cal's job as a surveyor caused him to have difficulty with graphing equations in class.

The first task was a story problem: An automobile uses 8 liters of gasoline to travel 84 km. How many liters are needed to travel 1,428 km? It took about five minutes for him to arrive at a solution for this task. He gave an answer of 136 liters. When I asked how he arrived at his answer, he said he "didn't use the problem" that he "just thought" about it and "worked it out." He found that "it took 17 trips of 84 to get 1,428" then "did 17 times 8 to get 136." When I asked how he found it took 17 trips he said he just "tried numbers" until one "worked." He did not check the problem until I asked how he could check it. He said he "didn't think it needed to be checked" because he was confident it was right since he used a calculator. He also said he did not consider this a "math problem" because it "used common sense." Like the story problem from the second task interview, he thought this problem had more "use" because he saw a real reason for knowing how to solve it: "so you won't run out of gas or something."

Even though he arrived at correct answers for all the tasks given in the third interview, he still was not confident in all of his solutions. He stated, "I'm not really positive it's right" when asked for his answer to the second task. The second task was also a story problem: Write an equation, using the variables S for the number of students and P for the number of professors, to represent the following statement: At this university, there are six times as many students as professors (Clement, 1982, p. 17). It did not take him long to solve this task, about one minute, and he considered it "easy." When he commented that he was not "really positive it's right," I asked him how he could check his answer to determine if it was right. He did not really know and said the only check he did was to "look at it." My role again changed to tutor as I led him to checking his equation, s = p * 6, with the original sentence by "supposing" certain values for s or p. After he was confident his equation was correct, I asked which version he felt was easier to use. He stated, "To read, the sentence" was easier but "to work it out, the equation" was easier. In other words, he felt that to get the idea of the relationship between the number of students and teachers, the sentence was better. If, on the other hand, one wanted to find actual values for the number of one variable given the other, the equation was better. After he told me this was an "easy" problem, I asked him for an example of a problem he thought was hard. He answered, "I always had problems with those ones like, like a train leaves, that had like a trick or inside joke to them." I asked if he had to solve story problems of that type in his algebra class. He said no, that there were "no word problems like that in class" but they did have to do "interest problems," which he said were "not hard, just a pain" because they were not allowed to use a calculator.

The last task, Graph: $5x + 2 = y$, took him about two minutes to do. He described the resulting graph as "a line going southeast" that was "a straight line, not horizontal or vertical." My role temporarily changed to tutor when I explained labeling quadrants as I, II, III, and IV instead of northeast, northwest, southeast, and southwest as he was used to from work. When asked how he knew the equation was going to result in a line, he answered, "Because it's all set to y." He said he thought this was an easy problem so I asked what he considered to be a hard problem. Although he said he did not really consider any of them "hard" he did "have trouble with ones like $x = 3$ or $y = 2$." I then asked him to graph $x = 5$. He plotted the point (5, 0) as his answer. I asked him to graph (5, 0) and he said, "It's the same thing." He commented that he had "got that kind of problem wrong" on a test and was upset because he thought it was right. My role once again changed to tutor as I asked leading questions about the difference between points like (5, 0) and (5, 2) and the line $x = 5$. Eventually, he revealed a belief that if there was "no y-coordinate" then "y's assumed to be zero" which he had formed from work. Apparently, when he entered coordinates into the portable computer he used in surveying, he only entered the x-coordinates because the y-coordinate was understood to be the line of sight, usually "set at 5000/5000."

Summary.

Cal demonstrated a remarkable consistency in beliefs and affects through all of his personal and task interviews. He approached mathematics from a very pragmatic point of view and was adamant in his beliefs that mathematics must be useful to merit the time necessary to learn it. His strongest beliefs revolved around what topics should be included in mathematics classes, how mathematics classes should be taught, and how important it is to incorporate technology into the mathematics classroom.

To summarize Cal's story, I describe him both as he was on the first day of the elementary algebra class and as he was at the end of the class. Here is the description of Cal on the first day of class:

Cal, a twenty-year-old single male whose primary goals in life are to have fun and make money, is taking elementary algebra for the third time because he dropped or withdrew the other two times he attempted the course. He realizes that he uses mathematics every day in his work as a surveyor, but can not relate that activity with anything that he has ever experienced in a mathematics classroom. He views the mathematics that he does at work as "common-sense" mathematics and "totally different" from "in-school" mathematics.

Here is the description of Cal at the end of the elementary algebra class:

Cal, having turned twenty-one and bought a new motorcycle, is even more convinced that, despite having passed the course, the mathematics taught in the classroom is useless in real life. He is now even more convicted that mathematics teaching is "behind the times." Having experienced a class in which technology was treated as cheating at worst and a method of checking at best, he now believes mathematics teaching is still "rubbing two sticks together" while ignoring the "match" that is available.

CHAPTER SEVEN: JOAN'S CASE STORY

Introduction

Joan participated in the study until roughly the middle of the semester then withdrew from the course and, subsequently, the study. Joan explained "failing grades" as the reason she withdrew from the course. Although she told me she would complete the study and hoped her withdrawal from the course would not affect my research, we were never able to meet to conduct the rest of the interviews. We scheduled several but she never attended. After several attempts, I concluded that, although she never notified me of her discontinuation, she had withdrawn from the study. Although Joan participated in one-third of the regular interviews and one-half of the task interviews, I feel the data added significantly to my study so I include her story with no reservations.

Personal Interviews

Personal Interview Data

Personal information. Joan is the oldest participant in my study (40-something). She had been out of school for almost twenty years and this algebra class was her first mathematics course since ninth grade. She is married with children and works at a local university in the Integrated Technology and Computing (ITC) department. She had been "taking classes" at the community college over the past five years or so but "had never declared any program of study, or anything, so they were just all these willy nilly credits all over the place." She explained, "Basically, when I left high school I went to work and I went to school a little bit too. I'd take a class here and there and [try] to decide what I really wanted to do." A few years ago, she met with a counselor from the community college and discussed the possibility of "at least do[ing] this Associate's thing." When she took the community college placement tests for entering the Associate Degree program, she found she "needed the developmental math" courses. "Somehow," she said, "I was able to take a lot of my other classes before I actually had to take this algebra class so I put it off."

Student information. According to the description of *typical* developmental mathematics students given in Chapter Five, I characterized Joan as a typical developmental mathematics student because she met the following criteria:

1. She felt she had never done well in mathematics.
2. She had poor grades in mathematics. She failed eighth grade mathematics and took it again in ninth grade.
3. She stopped taking mathematics in high school as soon as graduation requirements were met; in her case, this happened in ninth grade. Joan did take additional courses in "accounting and some of those business applications" in high school, but she did not consider them "formal math."

4. She had an attitude of fear associated with mathematics and a history of avoidance with the subject. Her recent history even included "putting off" the current elementary algebra class. She described her avoidance this way:

I put it off and put it off and put it off because I was really not looking forward to it and so I got down to statistics and economics and something else and I had to have algebra first [as a prerequisite] so I just had to bite the bullet and finally do it.

5. She held the following beliefs: (a) She believed that she was just not one of the people who could "do math," and (b) She believed that the rules that governed mathematics did not make sense.

Beliefs About Self as a Do-er of Mathematics

I'm not a math person. Joan believed she was not "a math person." One reason I got this impression was her uncanny ability to avoid taking mathematics in school. She took math in eighth grade, a class she called "a general math class," which she "flunked" and then repeated in ninth grade. After ninth grade, she took no more mathematics courses of any kind until I met her in the elementary algebra class for participation in my study. She explained, "Somehow I got all through high school without ever having algebra, that's why I'm taking it at [the community college] now." I asked about her avoidance:

Diana: I assume you had some control over what you could take, like in tenth grade and eleventh grade. Was there any particular reason why you didn't take math?

> Joan: I think just because I had such a hard time with it. I had such a hard time with math. I had a hard time with high school in general because it was kind of a culture shock thing. I went into high school the year before they had middle school and to go from an elementary school into a high school environment, I think I had culture shock for awhile and I was very timid at that time, and, you know, I'd never talk in class or anything, and lots of times I'd have trouble with my homework and didn't have anybody to help, and so, you know, once you get behind, you just kind of fall into a pattern.

She said she had a "hard time with high school in general" because she went from an elementary-style school, in which students stayed in the same room with the same teacher all day, to a high school-style, in which students had different teachers and went from room to room, when she went from seventh to eighth grade. She felt that change really rattled her and she never quite recovered.

Beliefs About the Nature of Mathematics

Mathematics is mysterious. The most outstanding result of the interview with Joan was her belief in a certain mystery surrounding mathematics and people who understood mathematics. She thought of mathematics as "something nobody would understand until they actually studied it." She, like some of the people Driver (1987) referred to in his article, seemed to believe the subject of mathematics fell from the sky in complete textbook-form rather than that (ordinary) people actually created mathematics. She commented that she felt mathematics was not a subject one could just "pick up and learn" because it was not attainable for the average person. She said, "I had no idea of what I was really doing until I actually had classes." Nothing about mathematics, including algebra, was obvious or intuitive to Joan. As she put it, "Sometimes I'll look at my algebra book and I think, 'Oh my god, what could that possibly mean?'" She described that, for her, "math has always been sort of like a code." I asked, "When you say code, can you elaborate? A code, like a secret code, some people know how to do it and some people don't?" She replied, "Exactly!"

Beliefs About How Mathematics Should be Taught

Mathematics teachers should be personally involved. Joan had a rather sad story about getting "lost" in her eighth grade class. After I heard her tell this story and observed her in the elementary algebra class at MTCC, I felt she also was "lost" in that class as well. She described herself in eighth grade as "spending lots of my days just kind of looking around and seeing who was there and trying to see how I fit into that whole situation." She said she "didn't particularly care for" her teacher that year because she did not make an effort to include her and "like I said, I just kind of got lost in the class." She said she enjoyed the ninth grade class much more because "that teacher really made you participate." She said, "You didn't get to sit in her class and not participate like I did in the first class [in 8^{th} grade] and get lost in all the people." Her memories sounded like she was one of the children who fell through the cracks of our educational system: "I can remember math was always a lot of homework and, you know, like I said before, sometimes I would just get stuck and not have any help and just get discouraged with it." I asked more about the teacher of the eighth grade class:

> Joan: I happened to have the same teacher for math and geography in the eighth grade and I flunked both of them. It was, I don't know what it was. I just, I just didn't particularly care for her [the teacher] and you know, like I said, I just kind of got lost in the class. She didn't pull you out and ask you questions. She didn't put you on the spot and I guess if I know I'm going to be under fire, I'm more prepared [for] that type of thing.
>
> Diana: So the eighth grade teacher, it wasn't that she necessarily did anything to discourage you?
>
> Joan: Right.
>
> Diana: She just did not hold you responsible for your work and you had the opportunity to slide by so you took it?
>
> Joan: Right. Well, you know, it was basically [that] she came in, she taught the class and we left. I mean, there was no, you know, in lots of classes you do all kinds of things: you can work problems on the board, you can get in little groups and work, you can work on a project. There are lots of things you can do. But this [class] was just basically come in, listen to lectures, turn in your homework, take a test. I'm sure I wasn't the only person who didn't really get pulled out and interested in math because of that technique. I just remember the classes being really long and no interaction.

Affects

Affective results from practices and mathematics teaching. Joan's description of what math classes were like sounded very similar to the descriptions given by the other two participants. She recalled feelings of discouragement, "I can remember math was always a lot of homework and, you know, like I said before, sometimes I would just get stuck and not have any help and just get discouraged with it." She described the routine of the class, "This was just basically come in, listen to lectures, turn in your homework, take a test." She recalled feeling rather bored and said, "I just remember the classes being really long and no interaction." The teacher's role, in Joan's words, was "Stand up, never smiled, lectured, you know, just go by the book." The students, on the other hand, were "really still, really quiet." As she put it, "I thought everybody sat in there like little statues" and "Nobody talked, it was just a real solemn little class."

The elementary algebra class was very difficult for Joan. She said:

> I'm not having an easy time with algebra, you know, as, I feel like the class is just going too fast, you know. By the time I really understand something, he's [the teacher] a chapter ahead of me and, you know, the tests are pretty much running that way too. We finish up a chapter and almost finish another chapter before we get our test on the previous chapter and, you know, just somehow that doesn't make sense to me. . . . I think it's just too much. I think you're learning so much that's so new, except half the people in the class, this is their second or third time in there, so you know, to slow down would hold up some people and, you know, I just don't think I could keep up

> at all if it went any faster. . . . But still, the pace, the whole pace is just a little too fast for me.

She discussed how disturbing it was to her that there were so many people in her class who had attempted the elementary algebra course two, three, or four times and her feelings of inadequacy compared to some of the younger students in the class:

> It was very discouraging the first night I went in there and this little girl sitting across the room from me said, 'Is this your first time?'. . . And I said, 'Yes, what does that mean?' And she said, 'This is my third time. I had to get permission from the Dean.' And, you know, it was very discouraging. That made it really kind of scary. . . . And there are a lot of people that have been out of school a lot less time than I have and we'll start working and doing things and, you know, they'll say, 'Oh, I remember this from school' you know, and I have no reference. You know, I have, this is just kind of all new and you know, [another student in class] and I, you know, we call each other and talk and decided we'd study together a little bit. But I think that most of the people in there are working adults and just don't live a life of leisure. They have other obligations and it's hard for adults to, you know, make even more time to study. But yeah, I'm thinking if the pace was just a little different I could handle it a little better. Then I look at it and think, 'Well, how come I can't speed up a little bit?' Maybe I should try to get ahead a little. Maybe I should go ahead and just get over this thing that I have about the test should be before we start the next chapter and just try to go with what's going on and not have this block about that.

Observations

Classroom Observation Data

Observation data about the environment. The elementary algebra class met on Mondays and Wednesdays from 4:25 - 6:55 p.m. Class was held in a portable building with poor air circulation and poor artificial light. The temperature was rather warm and the air was stuffy. The twenty-three chairs were arranged in tight, very straight rows that faced the wall-length blackboard. The initial enrollment of the class was twenty-three students, that number dropped to twelve by mid-semester. The teacher used a small table, located in the front of the room, as a desk. In one corner of the room, there was a never-used TV and VCR on a cart that was chained and locked to the wall. The students' desks were very small, about one foot square. The room had three small windows and two large file cabinets.

Observation data about students' and teacher's behavior. I observed Joan in class on two occasions. The first was near the beginning of the class and the second was near the middle, just a week or so before she withdrew from the course. She always sat in a middle seat in the row closest to the window and across from the door. Joan and Cal sat in the same row. Joan sat two seats ahead of Cal. Joan rarely spoke to the teacher or anyone else in the class if the teacher was speaking. She was too busy writing down every word that he said, every example he wrote, and every comment students made. She wrote down everything that she could, making no effort to filter out the important from the unimportant. She frantically copied all the examples he worked at the board but never made additional notes that would explain the procedure being followed or the reasoning behind the steps.

Occasionally, the teacher would distribute handouts with exercises for the students to work. He told them they could "work together" in completing these handouts. Joan always chose to work with the same two students who sat in the row next to hers. Both students were women, one had taken the class twice before and failed both times, while the other had just graduated high school. While working on the handouts, Joan talked a lot. She asked questions like, "Do either of you two know what's going on?" and "I don't have a clue about how to work this stuff, do you?" Most of the time, her classmates were no better off than she was in terms of mathematical understanding. Several times, I observed her simply copying their answers on her worksheet. When I asked her about this procedure during a break, she explained it was so she would have "something to go by" at home when she attempted the homework.

Joan's attendance was very good until approximately two weeks before she withdrew from the course. Apparently, she had made the decision to withdraw, and therefore stopped attending, some time before she actually filled out the paperwork and was removed from the roster. Joan arrived in class twenty to thirty minutes early the first four or five weeks of class in order to "have time to do some problems and get help" before class began. After that, however, she often came to class at exactly 4:25 p.m. or just a few minutes late.

Task Interviews

Settings for the Task Interview

The task interview with Joan was conducted on the campus of a local university. This location was chosen because it was close to Joan's place of employment and therefore convenient. We conducted the task interview in an empty classroom. The video camera was set up in a far corner of the room in order to view Joan at an angle. She seemed rather nervous during the task interview. She made comments like, "This won't take long, I don't know anything." I tried to put her at ease by talking about ordinary things (e.g. the weather, her job) for a few minutes before we began the actual interview.

Overview of Joan's Task Interview Data Using Schoenfeld's (1992) Framework

As stated in Chapter 5, Schoenfeld's (1992) problem–solving framework was used as a lens in analysis of the task interview data. The following overview of Joan's work is given to provide a framework for the reader to view the task interviews. This overview includes the categories of resources, heuristics, control and metacognition, and beliefs and affects. A general view of Joan's approach in solving tasks is given for each category.

Resources. Joan's resources were not adequate to allow her to solve the tasks presented. She did not have the mathematics background that would have allowed her to be successful (e.g. she had never taken any type of algebra in high school). Her resources primarily consisted of a mastery of arithmetic. She stated often during the interviews that she did not know what certain mathematical terms meant (e.g. factor), indicated that she had no experience with basic algebraic manipulations like solving for a particular variable, and demonstrated that she had no real understanding of the concepts or the procedures being taught in the elementary algebra class. She did understand some of the basic ideas of algebra however (e.g. using a variable to represent an unknown) and saw the some of the relationship between operations with numbers and operations with variables.

Heuristics. Like her resources, the strategies available to Joan were also very limited. Basically, if the task given in the interview was not exactly like one she had as a "worked example" from class, then she was unable to approach it at all. If the tasks were alike, then she could "follow the steps" and possibly arrive at a correct solution. I think Joan tried to reason her way through tasks but simply did not have the resources or experience that would allow her to succeed with that strategy.

Control and metacognition. My first impression of Joan when we were doing the task interview was that she did not have an understanding of algebraic operations like factoring or solving for a variable. She knew she did not know however, no secondary ignorance on her part, and blamed it on the way she had been taught. For example, when I asked her what her answer to a factoring task meant, she replied, "This doesn't mean anything to me. It means absolutely nothing." When I asked why she thought the process was called 'factoring,' she responded, "I have no idea." She then went on to say the process of factoring had "not been explained other than to show the operation." I felt she implied that since the meaning had not been taught, specifically or otherwise, she felt she should not be held responsible for not knowing it. She seemed to imply with her explanations an it's-not-my-fault attitude.

Beliefs and affects. Joan seemed rather anxious during her task interview. She made numerous comments that reflected her lack of confidence. She was unable to arrive at correct answers for any of the tasks, nor did she exhibit any real understanding of the concepts or procedures. One belief that became evident during the task interview was Joan's ideas of who does mathematics. She believed "other people" did mathematics, not her. One example of this belief occurred when she was explaining her thinking when she first looked at a task. She said her mental picture consisted of "that woman in the tapes working these problems at the board" referring to the tutoring tapes available at the community college learning lab. The other primary belief that was manifest was the idea that mathematics is an almost impenetrable mystery. She described her feelings about mathematics this way:

> I didn't really [know what to do]. We've worked with so many things. Logically, this doesn't equal anything so I just broke it down as much as I could… I tried to think back through things we've done and refer to any instructions. I don't always know what to do when I first look at a problem. I think, "OK, what do I do with this?" then I try to think back to something he's [the teacher] done, because this doesn't mean anything. It means absolutely nothing to me just looking at it. I just try to picture her [the woman in the tape] doing the problem and do the same thing.

Joan also mentioned that she "had never heard of factoring until a few weeks ago."

Task Interview Data

Joan's work on the task interview is found in Appendix K. The entire list of tasks given to participants during the task interviews is given in Appendix A. The follow-up questions for the task interviews are found in Appendix B.

Task interview #1. Although the interview consisted of three tasks, it lasted two hours. My role during the first task was that of objective observer but it changed when I realized she did not understand factoring and I felt I would not be able to gather useful information. Therefore, at the beginning of the second task, my role changed to tutor/teacher.

Joan gave an answer of $5(4x^2 - 3xy)$ to the first task I gave her: Factor $20x^2 - 15xy$ completely. When I asked her how she knew what to do when she first looked at this problem, she said she did not really know what to do with it when she first looked at it because "it doesn't equal anything." She was not confident her answer was correct because "It seems to easy, that's why I think I haven't completely factored it." She did say she could check her answer by "going backwards" but she did not check it because she was not sure she was finished. I offered to let her have more time to finish but she seemed unable to think of anything else to do to the problem so she said she was through working. When asked what she thought this task would be used for, she responded, "I can't imagine what any of this can be used for." When I asked her what her answer meant, she declared, "This doesn't mean anything to me. It means absolutely nothing." Although she said she had "no idea" why the process was called factoring, she defined factors as "ways to break down other numbers, smaller pieces."

The second task, factor completely: $2w^2 - w - 15$, was very difficult for her. She said, "I don't really know what to do with that. I'm not solving for anything and I don't understand factoring letters." I told her to set the expression equal to zero if she thought that would help her. She did that, seemed surprised that that could be done, and only spent 30 seconds or a minute on it before she announced that she "can't do it." She did not know how to check a problem of this type. She did not consider any of the problems of this type "easy" and she did not try to put the expression or equation into any kind of context to make it easier to understand. She did understand the idea of a letter standing for an unknown (like w for width). She got w = 15/2 as an answer. Joan said she would work about two minutes on a problem she was stuck on before she gave up if she was at home but only “a few seconds” if it was on a test.

Joan said the only two things she understood in algebra up to that point was finding a greatest common factor and factoring by grouping. She said she needed “four things” (terms) to group. Because of her belief, I gave her the task, factor completely: $3xy - 9 + 6x^2 - 3x$, as the final task of the interview. She did not consider changing the order of the terms as an option. She said $6x^2 - 3x$ had three as a common factor. When I asked if $(x^2 - x)$ had a common factor, she said it did, x, and then wrote $x(x - 0)$. I then acted as tutor and attempted to help Joan realize her error. I do not believe she understood my explanations, however, and I got the impression that she was just humoring me and pretending to understand so she could stop working on it. She never came to any type of solution for the final task.

Summary

It was obvious from her responses in the interviews that Joan held several unhealthy beliefs about the nature of mathematics and herself as a do-er of mathematics. As a result of these beliefs, Joan was very limited in the mathematics she would attempt to do. She was also limited in her level of success because of these beliefs. Primarily, Joan believed mathematics to be a type of “code” that other people knew how to decipher but that was a mystery to her.

To summarize Joan's story, I describe her both as she was on the first day of the elementary algebra class and as she was at the end of the class. Here is the description of Joan on the first day of class:

Joan, a forty-something married mom, is finally taking a math class after being enrolled in and taking classes at the community college for the last three years. This is her first mathematics class since ninth grade. She seems a little lost, dazed, and scared, especially when she overhears several of the other students mentioning that this was their second or third time taking the course. She believes mathematics to be a type of "code" that she has never been able to decipher.

Here is a description of Joan at the end of the elementary algebra class:

Joan is nowhere to be found. She failed the first few tests and quizzes miserably, despite attending class faithfully and even coming in to get extra help from the teacher. After the first several weeks of the semester, she started missing class and finally, at about the midpoint of the semester, she dropped the course. She left the course being even more convinced that she was right: she is not a math person.

CHAPTER EIGHT: CHRIS' CASE STORY

Personal Interviews

Personal Interview Data

Personal information. Chris is a thirty-something married female with three kids. Her participation in my study occurred during her very first semester in college. She had been out of school for over 10 years and had decided to start taking classes at the community college because she wanted to get a better job, possibly in elementary-school teaching. She was required to take the elementary algebra class because of her score on MTCC's entrance exam. She is working at a local restaurant as a cook. Chris had some of the same reservations Joan had about coming back to school facing "competition" from much younger students who were just "coming out of math classes and still remembered it."

Student information. According to the description of *atypical* developmental mathematics students found in Chapter Five, I characterized Chris as an atypical developmental mathematics student because she has the following beliefs and affects:

1. She recognizes her aptitude for mathematics.
2. She knows that she can "do" mathematics.

3. She believes that she can learn mathematics and be successful in it: "Math, it seems to come very easy to me."

4. She earned good grades in mathematics. She had a high A average in the elementary algebra course she was enrolled in during the study.

At the beginning of this study, Chris still had some typical beliefs and affects about mathematics however. For example, she did not like it. She said, "I'm not thrilled with math." One reason she gave for her dislike was that it made her "nervous doing" mathematics because she did not "want to be wrong." At the end of the study, however, she stated that she enjoyed math and was confident of her ability to get the correct answers if she had been shown a procedure. She, like the others in the study, had stopped taking mathematics in high school after meeting the requirements for graduation. For Chris, this happened in tenth grade. Interestingly, she did take "two years of accounting after that" but, like Joan, did not see those courses as having "anything to do with math" they were just "electives." Also like Joan, she repeated a mathematics course in high school. She took algebra in ninth grade and "that was a bust" because she "failed that one completely." She explained her failing in this way:

> It was driving me crazy. You had algebra problems up there and I'm like, 'Well, why would you do that?' I was constantly asking 'why.' I had to know why they had these certain ways of doing things. Why? And I found with algebra, you just do it. There's no, either the teacher didn't explain it to where there was a reason or whatever. [My question: "So it didn't make sense?"]

> No, it didn't make sense why I had to do certain things and the teacher either couldn't tell me or didn't.

In high school, her acceptance of just "doing" algebra and not asking why paid off. After the ninth grade algebra class she quit asking those questions, and, when she repeated the course the next year, she earned an A. In her words, "I just accepted it. I just figured, well, this is the way it is. . . . I just accepted it and said, this is how you do it, and this is the way you do it." She was still interested in knowing "why" after high school, but the focus shifted from "why do you do it that way" to a question she found herself asking a lot during the elementary algebra class, "What would you use this for?" Here is her description of what she wanted to know during the elementary algebra class at MTCC:

> I want to know where would this apply. It's not so much, well, why do I need this? It's more like, when will I need this? When will this problem come up? Does it come up in a nuclear physicist's life?

She explained that she enjoyed "having the knowledge now [in college] because" she could "do it" but she still wanted to know "How would [it] apply?"

Beliefs About Self as a Do-er of Mathematics

I can do this (but I still do not believe it). Chris stated that she enjoyed math but that she was "not real secure doing the math." During the interviews and the task interviews she made statements and displayed behavior that, at times, showed self-confidence and, at other times, showed no self-confidence. An example of a statement that showed she had little confidence in herself as a do-er of mathematics was: "I'm still always second guessing what I know and what I don't know and if it's right or if it's wrong." Later, in the same interview, she stated, "I feel like I can [do mathematics] and I can learn it and I can do quite well at it." When I asked if she has an attitude of confidence, she replied "Sure." I asked if she had that confidence all the time and she replied, "Oh yeah, as far as doing it. I'm pretty confident that anything he shows us, you know, that I can catch on to. By the end of class I'll know it and by the end of the homework, I'll remember it."

At first, it might appear that Chris exhibited contradictory behavior and made statements that were not in agreement. After all, in one part of an interview she said she "knows" she "can do it" and later, in the same interview, she admitted that she was "not real secure." When I asked her about these apparent contradictions, she explained what she meant. She said, "If someone shows me what to do then I am confident I can do it." If, on the other hand, she is left to "figure it out" by herself, then she is not confident that she can "get the right answer." She enjoys doing the procedural aspects of mathematics, feels she is successful in that, and is confident that her answers are correct. The aspects of mathematics that involve her deciding for herself if something is "done" (e.g. if an expression has been factored to its lowest form) make her "nervous" because she does not "trust" her judgment, and, as a result, is not confident of her answer. Likewise, the aspects of mathematics that involve deciding how to approach a problem that she has not been shown how to do also cause her to feel uncomfortable and ill at ease.

When asked how her attitudes toward math had changed after completing the elementary algebra course, Chris voiced the biggest change of all three participants. She said that, because she was "doing so well" in the elementary algebra class, it "made [her] feel like [she] can do it [math] and . . . do it well." She felt she had gained confidence during and after the elementary algebra class because she "was given math and [she] could do it." She felt math was "set aside from a lot of the other classes." The basis for her belief about the uniqueness of mathematics was a classification of mathematics being an either/or subject: "Either you were really good in math or you weren't really good in math at all" and, consequently, "you either are like 'Oh, I hate it' or 'I love it'." She said:

> I always had that belief that I just wasn't meant to do math. That I wasn't meant to, you know. But now this class has shown me that yeah, well, you do. I feel like I do have an aptitude for it.

Because she took the elementary algebra class during her very first semester in college, she also had gained a confidence that she could "do" college in general, in other words, that she could earn a degree. She explained the effect that the elementary algebra class had on her beliefs about herself as a do-er of mathematics and as a student in general:

> It has given me confidence and has shown me that I can, because I'll be honest with you, I have been thinking about it. I look at math and I look at

the rest of the courses and I'm thinking, 'If I can pass math, I think I can pass anything. That's pretty much the way I feel. If I can pass math, I can pass anything, because the rest of it's like, history, English might, I don't know, I'm not used to writing papers so we'll just hold on that one, but like history or science or something, it's in the book. If you know it, you can pass. If you look at it, you can pass. But math has always been, you got to know how to do it if you don't, I feel like I've gotten, this class has made me feel like I think I can handle these other classes. . . . This class has made me feel very well about myself. I thought at first, I was really nervous coming in the first day of class because I thought I was really going to be one of the slowest people in the class with all these kids coming out of high school. They're fresh. The math's fresh and I find out that I'm actually probably one of the quickest people to catch on, you know. I can usually figure it out before others in the class can, so that makes me feel really good. Makes me know that I'm not as dumb as I thought I was and that maybe, and I'm glad it happened now because I'm just starting into college and it makes me feel like I can, I can take, you know, all these classes over again and get a teaching degree myself. I have the ability and the brains to do it. . . . I think I can do it, plus it well, it has nothing to do with math, but it has exposed me to some of the students, . . . and I feel like, I can deal with these people, you know, I can compete with them. I can do it just as well as they can. . . . It has a lot to do with, I think, well I don't know, for me, my age. It has a lot to do with 'I can do this.' I'm thirty-some years old [laughs], 31. I keep forgetting if I'm 31 or 32, I'll be 32 in June, and I feel this is just, this is just me. I feel like I haven't accomplished anything in my life, you know. I work. I don't have a highly skilled job. Anybody can do my job with a month of training, you can do it if you have the aptitude to do it. But, you know, but I got married, had 3 kids and I feel like I've accomplished nothing in my life and here I come back to school and I

> get into a math class and I just feel like, I am not a math person at all, and I get into the class and I came in the class and people at work can tell you, I was so nervous that day thinking, 'I'm going to be the dumbest person in that class.' There were kids coming out of high school going into this class, you know. I thought, they're going to whiz by me. And here, you know, I'm at the head of the class. I'm getting high grades and I'm not having any problems. I'm not going home thinking, 'Oh God, I can't do this.' I don't study for tests because I know it so well. I've done enough to do it and that makes me feel good. Hey, I feel like maybe I can, you know, accomplish something.

After I heard her comment that she was "not a math person," I asked Chris for a description of a "math person." She told me that a math person was someone who "loves to do math" and "knows how to do it." I pointed out that by her own admission, she "enjoys math" and "can do it" yet she still stated that she was not a "math person." Chris described further that a "math person" is "passionate" about mathematics and enjoys the subject even when they do not know what to do, even when they have not been shown how to do something, and even when they get something wrong. That, she explained, was why she was "not a math person." She only enjoyed mathematics when she knew how to do it, got the right answer, and had a method of checking to see that her answer was correct.

Beliefs About the Nature of Mathematics

Mathematics is different from other subjects. Chris' definition of what mathematics is, and what a mathematics problem is, was one of the best given by the participants in this study. She defined math as "an answer finder" and stated that "you have to be fluent in it" in order "to go further in life." Mathematics is "a tool you need in life" according to Chris. She distinguished between a "problem" and an "exercise" in this way:

> When you say the word 'exercises' it makes me think there's a whole page. Like last night, I'm sitting there, there's a whole page of these things, you're doing the same thing over and over and over again. That's when you say 'exercises' that's what, oh Lord, I've got twenty problems just like the first one. . . . Drill, right, exactly. Just drumming it into your head. I think it's been drummed enough in my head. When you say 'problems' now, I just think of newer stuff, stuff that I don't know. When you say the word 'problem' that's what it means to me. He's [the teacher] giving us something new, something I have to figure out. But exercise, in my, to me, means just regular you got to get something, it's the same as #1, practicing over and over again. Beating a dead horse, basically.

She did say in a later interview, however, that this drill did not really bother her if it helped her remember what she needed to for a test. She thought that sometimes some of the students in her elementary algebra class asked for too much drill and repetition however; and, when that occurred, she felt "aggravated."

Not all of Chris' beliefs about the nature of mathematics were helpful for her. An example of an unhealthy belief that Chris held was the idea that mathematics, as a whole, did not make sense. She described herself in ninth grade algebra as "wanting to know why" certain procedures were the way they were in mathematics. She then described herself in tenth grade algebra as "just accepting it." I questioned her about the experience:

> Diana: Did you ever figure out why or did you just decide [Chris interrupts]
>
> Chris: I just accepted it. I just figured, well, this is the way it is. And I don't know that there is [a reason why], maybe I was just making it more complicated than it was. I don't know. I don't feel like I found out anything different than I knew before. I just accepted it and said, 'This is how you do it and this is the way you do it.'

Like Cal, Chris also held a belief that mathematics was somehow unique when compared to other subjects. She began revealing her belief during the conversation about her confidence level when she commented that "because it's math" she was successful in, she then concluded that she could "do anything." She also commented on the uniqueness of mathematics in terms of the reasons for doing it:

> Diana: Do you have any other classes where you're doing stuff and you don't know why? I mean, does it happen in any other class?
>
> Chris: No, no. . . . That's because in other classes it's quite obvious what it's for. Math is, to me, a more advanced thing. It's, I just, I don't know what it's for, I guess, I don't know, maybe if I thought about it hard enough I might figure it out but I don't [know what it is for]. The other classes, when you're

> doing something, well like the other class I'm taking is a computer class so it's quite obvious why I'm doing something. You know, it's just, there's just no reason to ask that question because it's obvious.

Another aspect of the uniqueness of the subject centers around how one learns mathematics. Like Cal and Joan, Chris believed mathematics to be a very linear subject with "certain things you have to know first" before other "things" can be learned. She defined "basic math" as "a lot of drill and knowing facts like what is 7 x 7" and "higher math," like algebra, as "dealing with unknowns." When I asked her about the connection between these two types of mathematics, "basic" and "higher," she replied:

> Well I mean, you have to know the first in order to do the second. I mean, you have to know how to multiply, how to divide, how to do basic math before you can even attempt to do algebra. . . . You have to know how to do those things.

Belief in the eventual usefulness of mathematics. One interesting point that came out in all the interviews with Chris was her "faith" that "eventually somebody is going to start applying it to something." She believed that, eventually, "it will all come together and there'll be a reason" for doing algebra. She held on to this faith despite her experiences in both high school and in the class at the community college that contradicted it. Here is part of the dialog that reveals her "faith"—she began by answering how she would begin teaching an algebra class to students:

Chris: "I would find a way to get across to them that this does pertain to your life. You can use it."

Diana: "I have to say that I find it interesting that even though you haven't heard anything yet to answer your question that this really does pertain to life, you're still convinced that it does."

Chris: "I just [laughs] I just can't imagine why I'm sitting in this class and I have to learn it and it, it doesn't, it has nothing to do with anything."

Diana: "So even though you don't have any specific examples, and no one has ever told you any real reasons, you still have faith."

Chris: "[laughs] I have faith!"

She finished that conversation by asserting "If I find out that nobody uses it, I'm going to get ugly!"

Chris' belief about the "eventual" use of mathematics was very deep-seated and, occasionally, caused her to behave in ways that were not useful to her. One example of a behavior resulting from this belief was that she had, at the time of the study, stopped asking certain questions about the nature of mathematics. Chris described herself as "always wanting to know why" in several of the interviews. She added, however, that this desire had changed because of her experiences in mathematics classes. In high school, she wanted to know why she needed to know a certain procedure or skill in mathematics. That desire changed after ninth grade because she decided there was no real reason why. As a result, she found herself in the elementary algebra class at MTCC wanting to know "where" it would apply yet never asking that question:

> I want to know, 'Where would this apply?' It's not so much, 'Why do I need this?' It's more like, 'When will I need this? When will this problem come up? Does it come up in a nuclear physicist's life?'. . . I find myself now asking, when he [the teacher] shows us problems and we do all the numerous math problems and homework and all that, I'm thinking, 'How would this apply? What would you use this for?' I find myself asking that a lot. 'Why, where would I get these figures, where would this come into my life? Where would it come into anyone's life?' I ask questions like that now. It's [having these questions] not hindering me because I figure, eventually somebody is going to start applying it to something. These formulas they give us and all that, I would like to know that.
>
> Diana: Have you gotten any answers to that question?
>
> Chris: I haven't asked. I haven't said, 'Well, where would that, where would I get that from?' I haven't asked that. I just figured it's going to come eventually.

Beliefs About How Mathematics Should be Taught

Beliefs about student's roles and teacher's roles in the mathematics classroom. Chris voiced some interesting ideas about what characteristics are found in a "good" teacher. She felt that the man who taught the elementary algebra class at the community college was a good teacher despite her admissions that the class was boring and consisted of a lot of drill and "drumming it into" her head. She liked the teacher because he was patient (a characteristic that Joan mentioned too, but that did not seem important to Cal). She liked the way he would keep "going over" something and doing it "step by step" so she could understand it. She also mentioned, though, that the students annoyed her sometimes because they would request more "going over" than she thought necessary.

Chris described the teacher's role in the elementary algebra class as one of leader, while her description of the student's role was much more passive. I asked, "What's the student's role?" Her reply was, "Well, some, like me, follow faithfully. Others, when you look at it, seem to stray. Some seem to prolong it. They [the students] try to draw it out, waste class time." I tried to clarify what she meant by asking, "But for the most part, the student's role is supposed to be to just listen?" Then, she responded with a comment that leads to another insight about her feelings concerning herself as a do-er of mathematics. She said the students' role was to "listen. For some to participate. I do participate. I will answer problems, but that is when I'm sure I know the answer." When I followed up with, "And you wouldn't volunteer something [referring to an answer] unless you were sure?" she stated, "Exactly."

Chris thought of the positions of student and teacher in the same type of linear or progressive levels manner as she did the levels of mathematics:

> 'Cause the teacher, I look at it as, he's up here [makes motion with hands to show height of teacher's level] and he's speaking mathematically and analytically and everything else and I'm still here [makes motion with hands to

> show her level, much lower than previous height]. . . . So I need someone on my level to maybe explain to me how she does it and how she thinks and then maybe I can do it.

Chris discussed the importance of good grades in her task interviews. Her method for getting those good grades, and her strategy for learning mathematics in general, seemed to revolve around "doing lots and lots of problems." She used the terms "conditioned" and "trained" to describe what had happened to her in math classes. This is a description of how she prepares to do a typical homework assignment:

> In order to know how to do the homework, I have to know how to do it. I pay attention to what he [the teacher] says and I do, I do the homework with like, I do the homework as soon as I can 'cause the knowledge is still fresh and I can still remember and I'm less likely to make mistakes and that gives it a chance and, you know, as I'm doing the homework, then it gets embedded, so I do it as soon after class as I can. I don't feel like I prepare to do the homework in any other, other than you know, pay attention to what he says and then do it. . . . I have my notebook which is a combination of homework and notes, you know, you just, if there is a problem I can just flip back because that's the most current thing as what he has just gone over. . . . I find myself flipping back saying, 'Now what was it we had to do here? What am I supposed to do now?' and I'll look back either at examples he had given us then I'll say, 'Okay, that's right.'

Because Chris believed, like Cal, that all students learn in different ways, she felt certain aspects of teaching were very important, for example, that the student is given a "reason" for doing the work:

> Everybody learns in different ways. Everybody acquires knowledge in different ways. You might not, you know, you might not have the same habits I do of learning things. . . . Give them a reason to do it. Give them a practical problem. . . . Something that pertains to them. Make it more personal. . . . As far as math goes, tell them why they'd be doing that. . . . I would [if I were teaching mathematics] find a way to get across to them [the students] that this does pertain to your life. You can use it.

Affects

Affective results from practices and mathematics teaching. Chris' affective results from practices and mathematics teaching underwent two changes. One change occurred during her ninth and tenth grades in high school. Another, more drastic, change occurred during the elementary algebra class at the community college.

Chris was given "a chance to start over" in high school when she moved at the end of her ninth grade year. She describes herself in ninth grade as unmotivated and uninterested:

> It was more like, when I was in ninth grade, I had my certain friends and all that. I probably wasn't in with the intellectual group and so, [in] class, when

> the teacher really [did not explain] or I didn't get the gist of why I was doing something, I just said 'Forget it, I'm not going to worry with it.'

After she moved and started tenth grade in a new environment, "it was a whole different set of circumstances" because she "didn't have all [those] friends" who were distracting her like before. The change was drastic:

> I applied myself more. I paid attention more because I didn't have the peers around me that I had before and I guess it was like, I don't, I'm not going to go out in the hallway smoking or whatever with them, so I'm going to sit here and I'm going to apply myself. So I did, and I did great.

Not all of Chris' experiences with practices and mathematics teaching were positive ones, however. She described one instance from high school in which the teacher called on her in a rapid-fire fashion and she "didn't have the foggiest clue" of what the answer was. Her clear account of the experience indicates the level of emotion associated with it:

> Oh, it [the feeling] was embarrassment. I felt stupid and I laid some of the blame on him [the teacher] because he was impatient. He rushed through things and he wasn't one that you would freely ask a question. You could tell what teachers will tolerate questions and [which ones] don't, and he was not one that would, you know, willingly, take a question. He'd do it, but it was like, 'I've done this a million times' or whatever. So it was, it was just embarrassment and the feeling of stupidity. [I thought] now everybody in the class knows I'm failing because obviously I don't know how to do this.

Holistically, she emerged from high school with little confidence in herself as a do-er of mathematics. She explained that, when she had to take the mathematics placement test for the community college entrance program, she "didn't even attempt it." She said she basically "went on and filled in my blanks" and did not even try to answer the problems in any kind of meaningful way. She explained her actions by saying, "I couldn't remember a thing I had learned before [in high school]. . . . I might have been able to [take the test], but I didn't even attempt it." Chris approached her college algebra class with this lack of confidence and the resignation that she would, more than likely, be unsuccessful again in mathematics.

From her college experience, Chris also had a variety of affective results from practices and mathematics teaching. She described the teacher in her college algebra course as being "big on reviewing and reviewing and reviewing and going over what you got wrong" which led to him "repeating what he said fifteen times before." That behavior "aggravated" Chris. As mentioned previously, one reason the excess review annoyed her was that she felt the students were simply "taking advantage" of the teacher's willingness to explain a topic. Another reason for her annoyance was a result of the teacher's actions. She felt that the teacher would assign "busywork" because "he [the teacher] feels like he has, two hours or however long the class is, he has this time to fill and he needs to fill it."

One issue that seemed to come up over and over again in the interviews with Chris as well as in her task interviews was that of not her "speaking out" in class. She told me about wanting to know the answer to her "why" questions but not actually ever asking the question in class. I asked for her reason for not asking and this was her response:

> I just, I do not speak out. That's all. You know, if I have a question pertaining to the class, you know, a problem or something, but I'm not, I just don't speak out. I figure he will tell me eventually so I'll just be quiet. [I ask, "What's the reason?" that she doesn't speak out] I don't want to draw attention to myself. That's pretty much it. I don't want to, you know, speak out. I just don't have that type of personality. I never have. [I ask if this is particular to math or if she is like that in other classes] Oh no. I wouldn't, no, I don't do it anywhere.

It was interesting that Joan made similar statements about not wanting to draw attention to herself and not wanting to speak out in class as well. Cal, on the other hand, seemed to relish speaking out and did so, often as a sort of comic relief.

Of all the participants, Chris' affect towards mathematics was the most changed. As a result of the elementary algebra class at the community college, Chris now has a very positive attitude towards mathematics. This positive affect is largely due to her realization that she can "do it" which, for her, meant, earn high grades. She explained, "I enjoy having the knowledge now because I can do it and it makes me feel better now that I can do it." Chris' story is yet another example of how success breeds confidence and enthusiasm:

> I have gotten to where I really enjoy the class. I look forward to the class. I like it because I can do it and I know I can do it. I enjoy being able to look at a problem now [and know how to work it]. . . . It feels great now. I enjoy it.

It feels great now. That's basically the only feelings I have. I enjoy being able to do it.

Observations

Classroom Observation Data

Observation data about the environment. The elementary algebra class met on Mondays and Wednesdays from 4:25 - 6:55 p.m. Class was held in a portable building with poor air circulation and poor artificial light. The temperature was rather warm and the air was stuffy. The twenty-three chairs were arranged in tight, very straight rows that faced the wall-length blackboard. The initial enrollment of the class was twenty-three students, that number dropped to twelve by mid-semester. The teacher used a small table, located in the front of the room, as a desk. In one corner of the room, there was a never-used TV and VCR on a cart that was chained and locked to the wall. The students' desks were very small, about one foot square. The room had three small windows and two large file cabinets.

Observation data about students' and teacher's behavior. Chris always sat in the very front of the room in the row closest to the teacher's desk. She took notes selectively and rarely spoke in class. Samples of Chris' homework, notes, tests, and quizzes are included in Appendix L. Chris was very attentive in class. She listened to the teacher, copied many of the problems he worked on the board, and, on most occasions, had her completed homework assignment to follow along when he spent class time reviewing previously taught material.

I conducted a classroom observation during the sixth week of the semester. During the entire 2 ½ hours of that class, Chris only spoke twice. Both times were when the teacher called on her to answer a question. Both times she answered the question correctly. The first question he asked was "How many terms are there?" as he pointed to the following problem written on the board: $3x(m + 2) – (m + 2)$. Chris answered, "Two." The other question involved the problem: $2ax^2 – bx^2 + 6a – 3b$. He asked, "What is the greatest common factor?" Chris answered that there was no factor common to all four terms. In another observation I conducted during the twelfth week of the semester, Chris did not speak during the entire class.

During class, Chris took notes occasionally. She did not copy everything that was written on the board. Instead, she seemed to sift through the information the teacher presented and write down only what she deemed important. Chris mentioned during a later personal interview that if she had had trouble with a homework problem, then she would often ask the teacher about it before class started rather than during class. She preferred asking, and answering, questions in a one-on-one setting rather than the classroom setting.

The teacher often distributed worksheets for the students to work on during class. He encouraged students to work in pairs or groups to complete the worksheets. In all of the classroom observations, I never observed Chris work with anyone to complete a worksheet; instead, she always chose to work alone. I asked her about this behavior. She told me that she chose to work alone because it was "quicker and easier" because she "knew how to do it" (she implied that the people seated around her did not know how to do it). As she put it, "It's hard to work in a group that's not at your level." She went on to say that she felt they (the people seated around her in class) "couldn't add" to her knowledge and since, she was "not a people person," she decided to work by herself. After conducting several personal interviews with Chris, I decided another reason for her choice may have been her belief that many of the other students in the class "didn't care" and she did not wish to associate with them. One way she determined the extent of this "caring" was if a student had done the homework assignment or not. Chris found after several weeks of class that she was in the minority as someone who consistently completed the homework assignments. As far as she was concerned, however, her efforts paid off in test and quiz grades. Samples of Chris' homework assignments, tests, and quizzes are found in Appendix L.

Task Interviews

Settings for the Task Interviews

All three of Chris' task interviews were conducted at MTCC. Two were in an empty classroom adjacent to the room where the elementary algebra class met and one was in an empty classroom in another building on campus. During the first task interview I conducted with Chris, she constantly looked into the camera and seemed distracted by its presence. In the remaining task interviews, I placed the camera at an angle to Chris in an attempt to minimize this effect.

Overview of Chris' Task Interview Data Using Schoenfeld's (1992) Framework

As stated in Chapter Five, I used Schoenfeld's (1992) problem–solving framework as a lens in analysis of the task interview data. The following overview of Chris' work is given to provide a framework for the reader to view the task interviews. This overview includes the categories of resources, heuristics, control and metacognition, and beliefs and affects. A general view of Chris' approach in solving tasks is given for each category.

Resources. Like Cal, Chris possessed the resources necessary for successful completion of all tasks presented. She possessed resources about how to follow the procedures to get an answer but she did not have the resources that would allow her to make mathematical connections, understand concepts, or discover the applications or uses for the procedures she knew. Unlike Cal, Chris had no real life experiences that allowed her to think about how the mathematics she was learning in class would relate to the mathematics required for various jobs or career fields.

Heuristics. Chris' problem–solving strategies consisted primarily of classroom-taught algorithms and procedures. She had little experience with nontraditional problems and therefore did not have many ideas of "how to set up [those types of problems]." Her primary heuristic consisted of matching the current task with one that was similar from her past experience, either in working herself or in watching the teacher work.

Control and metacognition. Like Cal, Chris mentioned time as a critical factor in remembering procedures. During the first task of the first interview (factoring a binomial), she commented, "We're on to something else now" when I asked if she considered the task easy or hard. She implied the task I gave her was harder at that point because the class time was being spent on different problems. I got the impression it was harder because she actually had to think and recall rather than just go on automatic pilot. Although she solved the task correctly, Chris did not know what the answer meant or what it could be used for. She had no mental picture associated with binomials or trinomials and did not know why the process was called 'factoring.' She defined a factor as "a number" and when asked to give some factors of 12 she guessed, "Numbers that multiply to be 12?"

Beliefs and affects. Chris made several comments, which led me to believe she had little confidence in her own ability to do mathematics despite statements she had made to the contrary during the regular interview sessions. For example, during the third task of the first task interview, she gave "ones that wouldn't factor" as an example of a task she considered hard. She stated that those types of tasks were hard for her because, "I don't trust my judgment." She went on to explain that, although she knew she could check by "multiplying it out" (and if it resulted in the original expression, it checked), that didn't help her to know if she had "gone as far" as she could go. In other words, she thought that expressions that factored into terms which still had a lot of "stuff" in them were hard because she didn't know if she was "finished" or not. This feeling came up in the second task interview as well. I asked her how she would check her answer (to a factoring task) and she replied, "Multiply it back out to see if what I've done is the same as what I originally started with . . . but whether it's correct or not, I don't know if I've factored it as far as I can." She also said she "wouldn't know how to figure that out" (whether it was "done" or not). At first glance, her confidence level seems to vary. Both in the regular interviews and in the task interviews she made statements that seem mutually exclusive (i.e. "I feel like I can do math" and "I'm not real secure doing the math. I'm still always second guessing what I know and what I don't know and if it's right or if it's wrong."). As explained in the personal interview section, her confidence level depended upon the type of mathematical activity she was required to do. If the mathematical activity consisted of simply following a procedure, then her confidence level was high. If, on the other hand, the mathematical activity required her to solve a problem that she had no previous experience with, then her confidence level was rather low.

Chris admitted to being very grade driven. In one of the task interviews I asked her for a choice if given a hypothetical situation in which she could take a math class and be guaranteed an A but would never learn the concepts (the why stuff) only the operations and a class where she would be taught the procedures as well as the reasons for those procedures but no guarantee of a good grade. At first she responded "I don't know" but then, after thinking about it awhile, chose the second route. She said she would feel she had wasted her time otherwise. She mentioned the importance of motivation in retaining information and said, "If math was an obsession, then I'd retain it." I asked, "Are you driven to learn math?" She responded, "No, I wouldn't say that [I'm driven] at all." I asked, "Are you driven to get through this course?" She replied, "Sure" then added, "And I'm driven to get good grades." When I asked about the motivation for getting good grades, Chris told me she needed to "prove" to herself that she could do it.

Task Interview Data

Chris' work on the task interviews is found in Appendix M. The entire list of tasks given to participants during the task interviews is given in Appendix A. Follow-up questions to the task interviews are found in Appendix B.

Task interview #1. In the first task interview, it was obvious that Chris was influenced by the presence of the video camera. She spent a great deal of that interview looking at the camera and acting very self-conscious. The tasks given in the first interview were very easy for her, so I used that interview as a way of finding out the extent of her algebra knowledge. Although she answered all three tasks correctly and experienced no difficulty whatsoever, I did gain insight into her lack of conceptual knowledge and inability to connect her experiences in the algebra classroom with anything in real life. Like Cal, Chris also commented numerous times about how important the passage of time was on her ability to remember how to solve a given task.

In the first task, Factor completely: $20x^2 – 15xy$, Chris mentioned almost immediately as she began to work, "Let me think. We're on to something else now." After she gave her answer, $5x(4x – 3y)$, I asked her how she could check and see if her answer was correct. She replied that she could "multiply it back out" to check her answer but indicated that she did not do it because "this isn't a test." She said she viewed the process of factoring as "You give me the answer, I come up with the problem." When I asked "How did you know what to do when you first looked at this problem?", she responded, "That thing [names the instructor of the algebra class] beat into our heads: Is there a greatest common factor?" She told me she thought this problem was "easy" because she had "done so many" of that type of problem and because "the numbers were easy [i.e. small]."

I asked a series of questions to determine Chris' metacognitive awareness and the extent of her conceptual understanding:

Diana: What does your answer mean?

Chris: What does it mean? I don't know what it means.

Diana: What do you think it [the factored form] could be used for?

Chris: I haven't the foggiest [idea] what this may be used for. I've been asking myself that since we've been doing these [in class].

Diana: Why do you think this process is called 'factoring?'

Chris: I have no idea.

Diana: What is a factor?

Chris: I don't know, a number? I guess a number.

Diana: So what would the factors of 12 be?

Chris: Numbers that multiplied to be 12?

I questioned Chris about any "mental pictures" associated with trinomials, factoring, or binomials and she replied that she did not have any, "No, absolutely none." I asked if she thought it would help if she did have that type of association and she replied, "Well, I don't know. Factoring is easy [now], it might confuse me if you told me what it was for or if you said 'oh, it's this triangle' or something." She commented that it "didn't bother" her that she had no mental pictures because she was "doing well in it [class]" but said that, if she was "lost," it "might help."

The second task was Factor completely: $2w^2 – w – 15$. Her first comment after I gave the task to her was "You just had to throw 'w' in there." After she completed the task, I probed about that comment and she explained, "If it's not an 'x' or a 'y' it throws me off for a minute. It just makes me wonder why they picked that letter." Chris only spent about 30 seconds looking at this task before she started writing. She only took about one minute after that to arrive at an answer of (2w +

5)(w – 3). She told me that she did not check her answer because she felt it "took too long to get the 6 and 5." This idea of "taking a long time" came up in her regular interviews as well. She had a real fear of "being the last one in the room."

Chris also felt this was an "easy problem" because "they're small numbers, it had to multiply to –30 and add to –1." She said she did not think of any factoring problems as actually "hard" but that "ones with big numbers, like multiply to 320 and add to 78," made her "grab for the calculator." I asked her for the typical amount of time she would spend on a problem before going on to something else. She said, "if it's a test, I'll fiddle with it for only a few minutes, maybe 5, then go on and come back to it at the end. If it's homework or something, I'll only spend a minute or two then just skip it and ask it in class." She revealed that "this section [on factoring] is the first section that [she] didn't go back and look at the back of the book" to check if her answers were right or not. She thought the reason for this was that "with factoring, it goes down in a pattern so I'm pretty certain [if it follows the pattern] that my answers are right."

The final task, Factor completely: $y^2 – 25$, was the "easiest" of all the tasks according to Chris. She only worked on this task for about 10 seconds before she announced the answer (y – 5)(y + 5). She said this task was so easy because it was "the difference of two squares" and recognizing that "type of problem" had been "drilled" so much she "saw it" immediately. She commented that this task was also easy because she "knew [she] was finished." When I asked what she meant, she said the first task was “harder because [she] looked at it [the answer] and thinks ‘there’s too much stuff in there, can I factor it more?’ That 4x – 3y bothers me, it seems like it should break down more.” She told me “ones that don’t factor” really “irritated” her and that she felt they were “hard” because “[she] didn’t trust [her] judgement.” This lack of trust in her own judgement seemed to result from past experiences in which she “made silly mistakes” and “got stuff wrong that [she] shouldn’t have.” She mentioned several times in task interviews that she did not “want to make another silly mistake.”

Task interview #2. The second task interview consisted of tasks that were more difficult for Chris. She was able to solve all of the tasks given and several interesting beliefs arose from this interview. Many previously stated beliefs and affects were reaffirmed during this interview as well. For example, I confirmed her convictions of the eventual use of mathematics and the importance of the passage of time in regards to her remembering procedures or algorithms.

The first task was Factor completely: $5r^2 + 10rs - 20s$. She worked about one minute on this task then commented, “It’s been awhile since we’ve done these, I’ve got to think.” About one minute later she asked, “You said minus 20s right?” I answered in the affirmative and, after approximately 30 more seconds, she stated, “I’m blocking it, I can’t do it.” I then questioned her about checking, her confidence, and strategies for solving this task:

Diana: So you’re done?

Chris: I suppose.

Diana: How can you check to see if your answer is right?

Chris: I can multiply it back out to see if what I've done is the same as what I started with, but when you say whether it's correct or not, I don't know. I mean, I don't know if I've factored it as far as I can. And I wouldn't know how to figure that out [if she had factored it completely].

Diana: Did you check your answer?

Chris: Yes.

Diana: And you checked it the way you described [by multiplying it back out]?

Chris: Yes.

Diana: Did it check?

Chris: Yes.

Diana: How did you know what to do when you first looked at this problem?

Chris: Just what I've been conditioned to do, the first thing I look for is the greatest common factor. I pulled that out and I don't see where I can factor it any further.

Diana: What was the greatest common factor in this problem?

Chris: Five.

Diana: Do you think this problem was easy?

Chris: Well, the basics of doing it was easy but, I mean, what I <u>did</u> was easy, but when I'm given a problem that either doesn't factor or I can't see where it

> factors, it bothers me. It [also] bothers me when I can't fully factor it. Because I don't know if I'm overlooking something or it just doesn't factor.

The second task was also a factoring task, Factor completely: $t^2 - r^2$. Chris arrived at her answer, $(t - r)(t + r)$, very quickly, about one minute. When I asked how she knew what to do when she first looked at this problem, she replied that, when there is no common factor, "It's automatic for me to look, when there's no middle term, for the difference of two squares." She told me this task was easy because she had "done so many" that she could "recognize it" immediately. Because Chris had commented on being "drilled" and "conditioned" in mathematics in previous interviews, I probed her about this belief:

> Diana: You've mentioned that you feel like you've been drilled in mathematics, do you feel that way about other subjects as well?
>
> Chris: No, [other subjects are] not as repetitive as in math, no where near it.
>
> Diana: How does this repetition affect the way you feel about mathematics, if at all?
>
> Chris: I don't really mind if it helps. If it gets me to remember or gets me to automatically recognize something like that [difference of two perfect squares], then it makes my life easier. If there was something else that would put it in my head and keep it there, then that's fine, I'll do it.

Chris seemed to imply that motivation to learn mathematics was not usually present so I questioned her to clarify this hypothesis:

Diana: You've said you have trouble remembering things in mathematics. Do you have trouble remembering other things as well, or is it just math?

Chris: Well, that's just the way I learn.

Diana: So, when you first learned to iron, you needed someone to show it to you many different times before you picked up on it?

Chris: Well, no, not with ironing.

Diana: Or what about when you first learned to drive, did someone have to tell you several times which pedal was which so you would remember it.

Chris: No, I guess if you're motivated or driven to learn something, it'll stick. I mean, if math was an obsession, then I'd retain it [the first time].

Diana: So, would you say you're driven to learn math?

Chris: No, I wouldn't say that at all.

Diana: Are you driven to get through this course [the elementary algebra course]?

Chris: Sure, yes. And I'm driven to get good grades.

Diana: You're driven to get good grades but not necessarily learn math?

Chris: Right!

Another interesting point that arose during this task interview was Chris' complete inability to connect the mathematics she learned in school to anything outside of the classroom:

Diana: What does your answer mean?

Chris: What does it mean? Absolutely nothing. It doesn't mean a thing.

Diana: What do you think this [the factored form] may be used for?

Chris: You got me. I couldn't imagine what I would do this for. I don't know who would use this or what they would use it for. Maybe if it were put into more common terms I could come up with something. But I don't know where something like $t^2 - 4$ would come from.

I gave the third task in order to determine Chris' ability to relate numbers to variables. It was Factor completely: $16^2 - 25^2$. Like the previous task, she worked this one very quickly only taking about one minute before announcing an answer of $(16 - 25)(16 + 25)$. She told me she could, "Basically, just look at it" to determine if her answer was correct. She also said, "If I did it correctly, it's easy" and that she had "never seen one like this before." At this point in the task interview, my role changes to a less objective one and I ask leading questions to determine what connections she noticed between the second and third tasks. I asked her to compare the second and third tasks and she told me, "They [the variables] were switched to numbers, there's basically no difference [between the two tasks]." She also told me the "t would be 16 and the r would be 25." At this point, she realized she could factor the third task further, "Wait, it's not factored completely, I'm not done." I gave her time to finish then asked what she did. She responded that she "picked the (16 – 25) term" to factor more "because of the sign" and that she "had to have the signs different so the middle term [would] cancel out." Her final answer was $(4 - 5)(4 + 5)(16 + 25)$.

I questioned Chris about the three different forms she had for this expression: $(16^2 - 25^2)$, $(16 - 25)(16 + 25)$, and $(4 - 5)(4 + 5)(16 + 25)$:

> Diana: If you had a calculator and you worked all three forms, what would happen?
>
> Chris: [Long pause] I'd get an answer.
>
> Diana: A number?
>
> Chris: Uh huh.
>
> Diana: Three different numbers?
>
> Chris: No, I suppose I'd get one number. [Hesitates a few seconds] It should be, if I did it correctly, it should be [one number for all three].

As in the last task, when I questioned Chris about the meaning of her answer and what it might be used for, she had no idea. I started to ask, "Do you often run into doing things in math class that" and she finished the sentence, "I don't know where it would apply to me? Yes." I asked how that made her feel about mathematics or about her class. She responded that it "didn't really bother" her or make her feel negatively because she likes the class and is "sure" that "one day" she will be told "what it's all [used] for." I asked if she thought that perhaps the reason she was "ok with it" was because she was doing well in the class. She said, "Maybe" and that, if she knew "a useful application," it would definitely help her learn if she was having trouble. She said if she were teaching an algebra class, she would try to "give it a reason" so her students would "understand the process of doing it" and that she would show them "what it's for, how it pertains to something." I have her a hypothetical situation in which she could choose between taking a class that only taught her procedures and did not give any applications but guaranteed her an A and a class that would teach both procedures and give applications but no grade guarantee. She thought about her choices for a minute or two then answered, "I don't know. If I would never learn why in the first class, I guess I would choose the non-guarantee because I'd feel like I wasted my time otherwise." I asked if she felt her time was wasted on the current elementary algebra class and she said no because she had "three more math classes to take" and felt that she would learn the applications "eventually."

The fourth and final task was a story problem: The area of a rectangle is 21 square feet. Find the dimensions if the length is 5 feet less than four times the width. Before I gave her the fourth task, Chris asked me what type of problem it was. When I told her it was a story problem, she exclaimed, "Oh, god no!" Almost immediately she was given this task, she commented, "Area of a rectangle, oh my god." She spent about one minute looking at the problem then stated, "It's been awhile since I've done one of these." After about another minute or two she murmured to herself, "Hmm, that's not right" and erased some of her work. She worked on the problem about three more minutes then looked up and asked, "Wait a minute, what do you want me to do?" I restated the problem and she asked herself, "What am I doing here [referring to "here" in the problem not "here" in the room]?"

This problem took Chris a long time to solve, approximately 15 minutes total. While working, she showed signs of frustration: took her glasses off, put her pencil down, sighed heavily, held her head in her hands, etc. After she had been working about ten minutes, I interrupted to ask, “If you were taking a test right now, would you continue to work on this problem or would you leave it and come back to it later?” She said she would probably leave it and come back later but then added, “If it were a test, this wouldn’t be a problem because it would be stuff I’d done recently and I’d know what to do.”

Chris gave her answer as “a width of 3 feet and length of 7 feet.” I questioned her about the heuristics she used in solving this task:

Diana: How did you know what to do when you first looked at this problem?

Chris: I didn’t know what to do when I first looked at this problem.

Diana: What did you do when you didn’t know what to do?

Chris: I started messing around with it to see if I saw something else [that was] familiar. Something that would tell me how to set it up.

Diana: What did you do?

Chris: Well, I tried to solve for ‘w’ right off the bat but it wasn’t coming out right. So then, I knew I needed ‘w’ to find ‘l’ and that the length equaled four times the width minus 5. . . . I got $21 = w(4w – 5)$ and distributed the w and then it looked a lot like something I needed to factor.

Diana: What did you get after you factored it?

Chris: I got $(4w + 7)$ and $(w – 3)$ and then I found w equaled three.

Diana: Did you do anything with the other term [(4w + 7)]?

Chris: No, I didn't 'cause I saw it was going to be a fraction and I ain't messing with no fractions.

Diana: So is it possible there is another solution to this problem?

Chris: Sure, it's very likely. [Works for a minute or two], No, it can't be negative so this won't work.

Diana: When you looked at this did it cross your mind to just say, 'Oh 21 square feet, 3 and 7?'

Chris: Sure but I thought it was cheating.

Chris admitted that this problem suggested a valid use for factoring but only partially because she did not think it likely that "You would know the area and that the length was five feet less than four times the width but not know the actual length and width." Despite how long it took her to arrive at an answer, Chris felt this was an "easy" problem and said that it only gave her difficulty because she "hadn't done any like that in awhile."

Task interview #3. The third task interview yielded data that confirmed several of my hypotheses about Chris, namely her lack of conceptual understanding, her limited number of heuristics, and her reliance on memorization in "doing" mathematics. The first task was a story problem and the only task Chris did not solve correctly. The story problem was: Sarah is operating a machine that can produce 14 parts in 20 minutes. How long will it take her to produce 224 parts? She worked on this problem about one minute, then removed her calculator from her backpack and worked about four minutes longer before giving her answer of "about 3 hours 14 minutes." As we discussed her approach to this task, she discovered she had used 220 instead of 224 as the problem stated so she worked it again. When I asked what her new answer was, Chris said [while looking at her calculator display], "320, uh, 3 hours 20 minutes, no, 320 minutes. I don't know, I can't think! I don't like word problems." I probed, "What is it about word problems that you don't like?" She answered, "They're not set up for you. You have to set it up." She told me she did not check her answer because she could "get through the agony quicker if [she] just say[s] this is the answer." I questioned Chris about how she solved this problem:

Diana: How did you work this problem?

Chris: Well, I kept thinking I needed to find the number of parts per minute.

Diana: How did you find that?

Chris: I divided 20 by 14, yeah, I divided 20 into 14, 20 minutes and 14 parts.

Diana: Did you do 20 divided by 14 or 14 divided by 20?

Chris: Uh, I did 20 divided by 14.

Diana: OK, what did you do with that number?

Chris: I multiplied it by the number of minutes.

After I questioned her more about her answer, Chris told me she would need to "divide 320 by 60, since there's 60 minutes in an hour, to get the number of hours she worked." I asked the next series of questions to determine Chris' confidence and the meaning she attached to the answer:

Diana: Are you sure your answer is correct?

Chris: No, not really.

Diana: Is there something that could happen for you to be sure?

Chris: Look at the answer in the back of the book! (laughs)

Diana: Anything other than that?

Chris: If I spent a lot of time with it I could be sure but I don't want to do that. I don't want to spend forever sitting here thinking about this in front of the camera [referring to the video camera].

Diana: But if you did spend that time, you could reach a level of confidence equal to what you had on that other problem [the factoring task she was "sure" of]?

Chris: Oh, sure.

Diana: OK. What does your answer mean?

Chris: Other than the obvious I don't know. It tells you how long it'll take to make those parts.

Diana: What do you think it might be used for?

Chris: Production work I guess.

Diana: Well, you know I always ask you what your answer means and what it may be used for but you don't always know. In this problem though, it does mean something and you do see a use?

Chris: Yeah, sure.

The second task was Graph: 3y – 9x = -6. I told her to draw the x- and y-axes on her paper because I did not have graph paper for her to use. She worked on this problem about three minutes. When I asked what her graph looked like, she did not give a very detailed description. She said it was "a line going up" and only said it was "kind of steep" after I probed her about it. I asked about the process she went through to graph this line and she told me she "got 3 points instead of just 2 to make sure." She said generally she picked values "like 0 and 1" and "if [she] get[s] a fraction, [she] just forget[s] it and go[es] on." When I asked why she responded, "I don't like fractions. Plus, they're hard to graph." When asked about checking her answer, Chris told me since "all three points were in a straight line" she knew it was correct. She thought this was an "easy" problem because she had "recently done it in class." The most interesting findings from this task arose when I asked her about what the graph meant:

Diana: What does this graph mean?

Chris: Doesn't mean a thing to me.

Diana: Do you see the two things as connected [graph and equation]?

Chris: No, I don't. There probably is though, but I don't [see it].

Diana: What do you think the graph might be used for?

Chris: I don't know. I don't have any idea what it's for.

Diana: There will be questions like this on your test today?

Chris: Oh yeah.

Diana: But no questions where you have to say what it means or what it's used for?

Chris: Right! That won't be on there.

Diana: How did you know what to do when you first started to graph this?

Chris: Just as a matter of memory I knew.

The third and final task was the story problem: Write an equation, using the variables S for the number of students and P for the number of professors, to represent the following statement: At this university, there are six times as many students as professors (Clement, 1982, p. 17). She only worked about one minute before she gave an answer of "p = 6 * s." I asked about checking her answer and she replied, "I don't know how I can check to see if the sentence matches the equation. I guess if I knew the numbers I could check." My role then changed to tutor and I asked leading questions like, "What if there were 6 professors, how many students would there be?" Quickly she realized her equation did not match the sentence and she revised her answer to "6 * p = s or s/6 = p" depending on whether she was given the number of students or the number of professors. She commented that this task would have given her problems if she had encountered it on a test. She felt both the sentence form and the equation were "equally easy" in use. Because of her responses in the graphing task, I asked, "What do you think the graph of students verses professors would look like?" She answered, "I haven't the foggiest." She then commented, "I don't know what a graph would show 'cause you're comparing two different things but here you only have one equation and it would be one line." I asked why she thought it would be a line. Her response was, "Well, that's all I've ever done is lines and I just assumed you wouldn't give me something I hadn't had before." I asked, "If you did graph this [her student-professor equation] and you got a line, would that line have any more meaning than the other line you got [in the previous task]?" She answered, "No, it wouldn't have any meaning." I asked what she thought "something like $x^2 + 2x = 6$ would look like if graphed" and she said she "didn't have any idea." She said she had "done stuff with those kinds of equations but it wasn't graphing."

Summary

Chris illustrates the dramatic change in attitude that can occur in a student when she experiences success in a mathematics classroom and realizes her aptitude for the subject. After successfully completing her first mathematics class at the community college, Chris then believed she could "do" college in general.

To summarize Chris' story, I describe her both as she was on the first day of the elementary algebra class and as she was at the end of the class. Here is the description of Chris on the first day of class:

Chris, a thirty-something married mother of three, is in her first semester of college, has been out of school for over ten years, and is scared to death that she will be the stupidest person in her math class. She does not consider herself a "math person" and is very concerned that the other students in the class will understand more quickly than she will. She is very afraid of being "left behind."

Here is the description of Chris at the end of the elementary algebra class:

Chris, having breezed through the class earning A's on all of her tests and quizzes, is enjoying her new-found sense of being able to "do math" and even "do college" in general. Because of her views that mathematics is "a more advanced thing," she now believes she can be successful in college because she was "given math" and could do it.

CHAPTER NINE: CONCLUSIONS

Research Questions

The purpose of this study was to investigate and describe three developmental mathematics students' beliefs and affects concerning mathematics and themselves as do-ers of mathematics. The questions that guided this research were:

1. What are the beliefs and affects developmental college students have about the nature of mathematics and its teaching? This question includes:
   a. What is mathematics?
   b. What does it mean to "do" mathematics?
   c. What constitutes a "math problem"?
2. What beliefs and affects do developmental college students have about themselves as do-ers of mathematics (including issues related to control and metacognition)?
3. What effects do these beliefs and affects have on students' doing mathematics and on their preferences for teaching methods, including: (a) student problem–solving strategies, (b) strategies students used to learn mathematics, and (c) teacher actions that help and hinder that learning process.

The three case stories generated by this research investigate and describe many beliefs about the nature of mathematics, beliefs about how mathematics should be taught, beliefs about how these three developmental mathematics students view themselves as do-ers of mathematics, and affective responses connected with students' beliefs. In this chapter, I discuss the findings, relate the results of this study to other researchers' findings, and give recommendations on how the mathematics education community may use the results of this study.

Discussion of Findings

Developmental mathematics students have many beliefs and affects about the nature of mathematics and themselves as do-ers of mathematics. The beliefs and affects of developmental mathematics students affect both their learning of mathematics and their opinions about the teaching of mathematics. I answer the research questions by recounting the three individuals' case stories that address each question of the study. In order to give the reader an overall impression of each of the three participants, I give the following descriptive phrase for each person:

- Cal is the *pragmatic user*,
- Joan is the *lost student*, and
- Chris is the *faithful follower*.

I classified Cal as the *pragmatic user* because he was only interested in the practical application and utility of mathematics. I classified Joan as the *lost student* because she viewed mathematics as a world in which she did not belong and, as a result, she often felt "lost" in class. Finally, I classified Chris as the *faithful follower* because, despite having no idea about how the mathematics she was learning in class related to anything in real life, she was content to follow along faithfully doing the procedures in class because she believed that "eventually" someone would tell her "where it applies."

Developmental Mathematics Students' Beliefs and Affects about the Nature of Mathematics and its Teaching

Although all three participants had differing beliefs and affects about certain aspects of the nature of mathematics, they shared some common beliefs about the way mathematics should be taught and some common beliefs about what it means to "do math". The major findings for this research question are:

- Students did not see how the mathematics they experienced in class related to real life.
- Students believed "doing" mathematics in class involved getting the right answer, not understanding a process.
- Students felt that mathematics teaching should involve them "doing" rather than "watching" mathematics.

Next, I discuss each of the three participants' beliefs and affects that reveal these findings.

Cal. Cal's beliefs about the nature of mathematics were dualistic. He did not see how the mathematics he was asked to do in class related to the mathematics he was required to do as a surveyor. As a result, he saw mathematics as either "in school" or "out in the field" and believed that the two were "totally different" in nature. To some extent, Joan and Chris shared this belief; however, they did not view the distinction as extremely as Cal did. Cal's beliefs about the nature of mathematics resulted in an affective dualism. Cal's affective response to a mathematical task relied upon whether he viewed the task as being a classroom task or a real-life task. If he viewed the task as an "in-school" task, then his affective responses consisted of feelings like boredom, lack of interest, and dislike and beliefs about the worthlessness of the task. If he viewed the task as an "in-the-field" task, then his affective responses consisted of feelings like enthusiasm, interest, and enjoyment and beliefs about the value of the task.

Cal believed that "doing math" in real life involved a practical application that had a known meaning and reason (e.g. a surveying job). On the other hand, he believed "doing math" in class was basically a type of ridiculous game because "you already know [what the answer will be]." He asked, "Why play the game [pretending] that you're not going to know?" As a result of this experience in class, he viewed "doing math" in class as simply a process or series of steps that one went through in order to get an answer, usually an answer that had no meaning and held no interest. Needless to say, this game was not one Cal enjoyed playing, which was why he made comments like "I'd rather just learn by just doing it" indicating both his preference for "doing math out in the field" rather than in the classroom and his beliefs about the dualistic nature of mathematics itself.

Cal's beliefs about how mathematics should be taught were very strong. He believed that current and past mathematics teaching consisted of dwelling on the "ancient ways" to solve a problem and that, in the future, it needed to "catch up" by using the technology that is available. He believed that the tasks presented in mathematics classes did not reflect those that one would be expected to do in "real life." He believed that mathematics content in mathematics classes was largely "stuff that was useless to learn." As a result of these beliefs and the associated affects, Cal's behaviors in class were often negative. Because he was bored and uninterested, Cal would sometimes skip class, come in late, or do other things (e.g. read the personals section of the paper or balance his checkbook) during class lectures.

Joan. Many of Joan's beliefs about the nature of mathematics were unhealthy. Her primary belief about the nature of mathematics was that it was a mysterious "code" that only certain people know how to decipher. Joan believed the "code" of mathematics was not intuitive or obvious and required one to devote much time to studying it before it could be understood. Joan's affective responses from this belief consisted of feelings like despair, frustration, fear, and hopelessness. Her affects resulted in behaviors like avoidance of the subject and lack of persistence when attempting to solve a mathematical task.

One of the reasons Joan held the belief that mathematics was a mysterious "code" that "no one would understand" was her inability to see how the mathematics she was experiencing in class related to anything she experienced outside of the classroom. She commented that she "knew" there were "people who use[d] it," referring to mathematics, but she "couldn't imagine" in what ways they used it. Her beliefs about this were so strong that she felt mathematics was like "a whole new world," a world that she was not, nor ever would be, a part of. Recall that I classified Joan as the *lost student*. I did this because on numerous occasions in interviews that was how she referred to herself (e.g. "I just kind of got lost in the class" and "[I got] lost in all the people"). I think one reason for her *lostness* was because of her belief about who does mathematics; namely, other people do mathematics, not her. I think this belief stemmed from an inability to see how the mathematics in class related to the mathematics of her life.

For Joan, nothing about the elementary algebra class made any sense or was intuitive. She made several comments about not "understanding," not "seeing," and not "getting" what was "going on" in class. Because none of the topics or procedures made sense, she resorted to the common strategy of just memorizing steps to get an answer. As a result, she, like the others, viewed the mathematics she did in class as steps to getting answer not a thinking or reasoning process. Obviously, it is difficult to think about or reason about something that makes no sense, has no meaning, and has no application.

Joan's beliefs about how mathematics should be taught were largely based on her K-12 educational experience. She believed mathematics teachers should be personally involved with their students, that students should be actively involved in class, and that tasks should be ones that the students can relate to. Joan's most positive experiences with mathematics teachers were when the teacher showed an interest in her as an individual and made an effort to "draw [her] out" and participate in class. I think Joan's desire to be "draw[n] out" reflected the belief that mathematics teaching should involve students "doing" rather than "watching" mathematics. She also believed the pace of most mathematics classes that she had experienced (including the elementary algebra class at MTCC) was too fast. She had no ideas about how the mathematical tasks she was asked to do in class related to anything in the real world. Because of this, Joan's affective response consisted of feeling that mathematics was an unknown "different world" that she could not relate to.

Chris. Many of Chris' beliefs about the nature of mathematics were healthy and accurately reflected some mathematicians' and mathematics educators' descriptions of the nature of mathematics. She believed mathematics was a "tool" that was useful in life and that it was a way of thinking that allowed one to figure out how to solve problems. Although she believed mathematics was useful in real life, she was unable to give any examples of how the mathematics she was doing in class was useful. Like Cal and Joan, she did not see how the tasks given in her elementary algebra class would relate to anything in the real world. As a result, one of her affective responses was curiosity. She wanted to know "where it would apply" to either her life or "anyone's life." A related belief was that, "eventually," she would find out how mathematics was used in real life.

Chris believed "doing math" in class involved getting the right answer (or, similarly, getting a high grade), rather than understanding a process. She admitted that her confidence in her answer depended upon whether or not she had been "shown" a process or not. If she had been shown a procedure, then she was confident she could "do it"; if she had not, then she was not confident. She knew that there were aspects of mathematics that she was not experiencing in class however, because she mentioned "researchers and scientists" and that they "really knew math," meaning that they could get an answer without being shown a procedure first. I think Chris' beliefs about what it means to "do math" reflected what she had experienced in mathematics assessments. She mentioned several times during task interviews that the types of questions I was asking (e.g. What could your answer be used for? What does your answer mean? Why did you choose the method you did?) would "not be on the test" implying that consequently, they were not important nor even part of what she was expected to do in class. She went on to say that the important aspect of the mathematics (and the questions that would be on the test) focused on getting the right answer, not explaining anything about the process of obtaining that answer.

Chris' beliefs about how mathematics should be taught reflected those of the other participants. She believed mathematics in the classroom should be presented in such a way that the students would see that it "pertains to them." She believed that mathematics teachers should realize that "everyone learns in different ways" and explain the mathematics being taught in more than one way. Like Cal and Joan, Chris believed mathematics teachers should spend more time explaining the "reason for doing it [mathematics]." Since Chris, like Cal and Joan, could not relate any of the mathematical tasks she was asked to do in class to anything in real life, she believed that either there was no relationship (between class tasks and real life activities) or that "the teacher either couldn't tell [her] or didn't."

Developmental Mathematics Students' Beliefs and Affects About Themselves as Do-ers of Mathematics

Although all three participants had some differing beliefs and affects about themselves as do-ers of mathematics, they also shared some common beliefs about individuals they called *math people*. The major findings for this research question are:

- Students believed that an individual was either a "math person" or not.
- Students did not believe they were *math people*.
- Students' confidence in their ability to do mathematics depended upon their individual assessments about the type of mathematics that was involved.

- Students did not believe reflective thinking and metacognition were important to the problem solving process.

Next, I discuss each of the three participants' beliefs and affects that reveal these findings.

Cal. Cal did not think of himself as a "math person". He had this belief despite passing the elementary algebra class and earning passing grades in mathematics classes in high school. Cal's beliefs about himself as a do-er of mathematics were dependent upon his belief dualism about the nature of mathematics. Like his affective responses, Cal's beliefs about himself, especially self-confidence, were completely different if he viewed the mathematical task as "in school" than if he viewed the task as "real life." If he viewed the task as an "in school" task, then he did not always believe he could solve the task and was only confident of his solution if the task was similar to one he had done recently. If it had "been awhile" since he had worked a task of that type, then he would not be confident that he had worked it correctly or arrived at the correct answer. If he viewed the task as a "real life" task and if he had access to technology, then he believed he could "do anything" and had utmost confidence in his ability to "figure it out."

During the task interviews, it became apparent that Cal did not view reflective thinking and metacognition as important areas of the problem solving process. He admitted that he rarely checked his work or checked his answer for reasonableness when doing tasks in class. He confirmed that with his actions in the task interviews as well. He had little metacognitive awareness. Most of the time, he was unable to answer questions about the nature of his thinking process and, moreover, was unaware of any real reason why he should be able to answer those types of questions or why the answers might be beneficial to him.

Joan. Joan's beliefs about herself as a do-er of mathematics were very powerful. She strongly believed that she was not "a math person." Because of this belief, she did not have confidence in her ability to do any type of mathematics beyond arithmetic. She was convinced that only other people could "do math" not her. Her affective responses and behaviors primarily consisted of lack of confidence, avoidance of mathematical tasks, and fear of the subject because she did not understand it. Joan exhibited some metacognitive awareness during the task interviews; for example, she knew she did not know the underlying concepts of the procedures she was trying to mimic (no secondary ignorance on her part). She did not view reflective thinking as being possible since it is impossible to think, reflectively or otherwise, about something that makes no sense.

Chris. Chris' beliefs about herself as a do-er of mathematics underwent a drastic change from the beginning of the elementary algebra course to the end of the course. Like Joan, Chris strongly believed, at the beginning of the elementary algebra course, that she was not "a math person." By the end of the course, however, Chris had discovered that she had an "aptitude" for mathematics and that she could "do it and do well in it." Even after successfully completing the elementary algebra course however, she still did not consider herself a math person. She defined a math person as someone who "enjoys it even when they don't know how to do it." Because she admitted to enjoying mathematics only when she could "get the right answer," Chris did not consider herself to be a math person. Like Cal, Chris' affective responses, especially self-confidence, depended upon what type of mathematical task she was asked to solve. If the task was one that could be solved by using a standard, classroom-taught procedure, then Chris believed she could solve it successfully and was confident her answer was correct. If the task was one that required more subjective thought (e.g. determining if an expression had been factored to lowest terms), then Chris did not always believe she could solve it and was not confident her answer was correct. Chris had little metacognitive awareness, and, like Cal, saw little value in it because metacognitive questions were not on the tests given in class, metacognitive issues were not addressed in class, and metacognitive processes were not discussed or demonstrated by the instructor of the elementary algebra class. Chris exhibited some reflective thinking during the task interviews; however, it was limited to the procedures and algorithms and did not include connections to real life or to other areas of mathematics, questions about reasonableness or applicability, or other control issues.

The Effects of Developmental Mathematics Students' Beliefs and Affects on Student Learning and on Students' Preferences for Teaching Methods

Many of the participants' beliefs and affects had harmful effects on their learning of mathematics, their problem-solving performance, and their desire to learn more mathematics. The major findings for this research question are:

- Students' in-class behaviors reflected their beliefs and affects about the nature of mathematics and themselves as do-ers of mathematics.
- Students did not expect to understand why a mathematics procedure was done the way it was in the classroom or where it could be applied in real life.
- Students were accustomed to the transmission method of teaching but did not prefer it; instead, they preferred teachers who actively involved them in the learning process and facilitated tasks designed to improve mathematical understanding, applications, and connections.
- The most commonly used strategy to solve tasks was matching the task given to one previously solved (usually by the instructor not the student).
- Students spent little time on reflective thinking and metacognition during the task interviews.

Next, I discuss each of the three participants' beliefs and affects that reveal these findings.

Cal. Many of Cal's beliefs and affects had harmful effects on his classroom performance and behaviors. The most obvious effect was the inability to remember a procedure or skill because he viewed it as "something [he'd] never use." He explained the effects quite eloquently, "If I don't think I'll need it or ever use it, I have to learn it over again because I won't remember it." As described in the heuristics section of Cal's case story, he used very different problem-solving approaches if he saw the task as being "in school" than if he saw the task as one from "real life." Because he believed the tasks from real life were useful and made sense, he was able to remember procedures or skills for solving those tasks.

Cal's beliefs and affects about the nature of classroom mathematics were reflected in his behaviors. Since he did not view the common behaviors of doing homework, copying the examples from the board, taking notes, and reviewing for tests as useful for learning mathematics, he did not do them. Instead, his behavior reflected his belief about it being a "game" – he simply memorized the rules and played by them when asked to on tests or quizzes.

Because of Cal's dualistic beliefs about the nature of mathematics, he did not believe the mathematics taught in class were useful or applicable to real life; therefore, he did not expect to see usefulness or application when he was shown a procedure in class. During task interviews, Cal's primary heuristic was "just thinking about it" in what he termed a "common sense" kind of way if the task was one he viewed as real life. For tasks he classified as "in school" tasks, however, Cal used the strategy of matching the task given to one previously solved. It was for this reason, he explained, that he would have "trouble" with tasks if it had been a long time since he had done tasks of that type. It was hard to match to a task done months ago but it was easy to match to a task done a day ago or a week ago. Cal attributed his low test scores and high quiz scores to the effect of the passage of time. Because of Cal's beliefs described in the previous section – namely, that reflective thinking and metacognition are not important areas of the problem solving process in classroom mathematics – Cal did not spend much time on either of those two activities during the task interviews.

Joan. Joan's beliefs about the nature of mathematics and herself as a do-er of mathematics had devastating effects on her performance on mathematical tasks. Her beliefs were so strong and her affective responses so debilitating that Joan was rarely able to even begin to try to solve the mathematical tasks presented to her. One particularly harmful effect that resulted from Joan's beliefs and affects was that she dropped the elementary algebra course and thereby continued her pattern of avoidance of the subject. Because Joan believed mathematics was a subject that was unattainable for the average person, she did not expect to understand the procedures she was taught or be able to see the connections to anything in her life.

During the task interviews, Joan's exclusively used the matching heuristic. I believe it was the only problem solving strategy at her disposal. Most of the time, the mental picture she had associated with the task involved someone else solving the problem, not her. This excerpt from Joan's case story describes her strategy and illustrates her *lostness:*

> I didn't really [know what to do]. We've worked with so many things. Logically, this doesn't equal anything so I just broke it down as much as I could… I tried to think back through things we've done and refer to any instructions. I don't always know what to do when I first look at a problem. I think, "OK, what do I do with this?" then I try to think back to something he's [the teacher] done, because this doesn't mean anything. It means absolutely nothing to me just looking at it. I just try to picture her [the woman in the tape] doing the problem and do the same thing.

Chris. Chris' beliefs and affects had both helpful and harmful effects on her learning mathematics. Her new-found belief that she could "do math" and "do [it] well" had a positive effect on her confidence in her ability to successfully achieve her goal of going to college. Another effect of this belief and confidence was an increase in enthusiasm for her next mathematics class. One possible harmful effect is that Chris may begin to believe that correctly getting an answer, accurately performing a procedure that she has been taught, and applying a given algorithm are the only activities that constitute "doing math." Chris' behaviors reflected her beliefs about how she learned mathematics. She repeatedly stated that her success in the elementary algebra class was due to her "paying attention" and "doing the homework." She proudly displayed pages and pages of homework problems that she had worked and examples that she had copied from class. Basically, Cal and Chris had completely opposite in-class behaviors and strategies for learning math, but they both were based on their individual beliefs. Interestingly, despite their using completely opposite strategies, they both passed the course (i.e. obtained the same end result using completely different means).

During task interviews Chris spent little time on reflective thinking or metacognition because she did not believe it was important since she knew "it won't be on the test." She used the strategy of matching the task given to one previously solved almost exclusively during the task interviews. It makes perfect sense that she did, however, since this practice had allowed her to score very high on tests and quizzes given in class. Chris was the one exception to the finding about student expectations of the applicability of mathematics. Although she had not experienced any connection between the mathematics in class and the real world, Chris still expected to "find out eventually." She explained that she could "not imagine" why she would be asked to do mathematics that was not used so she believed that there must be a use (even though she did not know what it was) and that she would be told some time in the future what that connection is. She did not expect to understand why a procedure was done the way it was however, because she believed there was no reason for it, that it "was just the way it was."

Connections to the Literature

This study is important because it connects three areas that have not been connected in many other research efforts. The three areas this study connects are: 1) developmental mathematics students, 2) qualitative research in mathematics, and 3) affective issues in mathematics. The next three sections discuss the connections to these three areas.

Connections to Developmental Mathematics Research

Willis (1992) found that the intermediate algebra students in her study scored highest on math anxiety. Regarding the nature of mathematics, she found that the intermediate algebra students believed in a rule-orientation rather than a global orientation, and that they "had the lowest mean" of all the groups about the benefits of math as a tool and a way of thinking (Willis, 1992, p. 211). Finally, she also discovered that the strategies intermediate algebra students used most to learn mathematics were "attending math class each period, copying examples from the board, (and) paying attention to the math instructor's lecture" (Willis, 1992, p. 211). My findings support her research and give additional depth to the findings.

The students who participated in my study would have been categorized with the intermediate algebra students from Willis' (1992) study since they were enrolled in an elementary algebra class. Joan and Chris displayed signs of mathematics anxiety but Cal did not. Their case stories serve as a deep well from which more information about this affective response can be drawn. For example, it becomes evident after reading Chris' story that her mathematics anxiety is only high when she is asked to solve a task that is unfamiliar to her or that involves inventing a new strategy. If the task is one that is familiar to her, that she has previous experience with, and that she has been shown a procedure for, then her anxiety level is very low. After reading Joan's story, it becomes evident that her anxiety stems from her deep-seated beliefs about the nature of mathematics and about herself as a do-er of mathematics. This anxiety is so high in Joan that it has a destructive effect on her learning of mathematics. Although Cal did not display any outward signs of mathematics anxiety, after reading his story, it becomes evident that the reason for this is his definition of mathematics. He was not anxious about mathematics that he could not do in class because it was not important to him. He did not see "in school" mathematics as having any practical purpose and did not "care much for grades"; therefore, he had no anxiety associated with school mathematics. If he had been given a mathematical task "out in the field" that he could not do, however, then he would have felt quite anxious about it. The case stories generated by my research give additional insights and explanations for the findings Willis (1992) reported in her research on mathematics anxiety.

The participants in my study confirmed Willis' (1992) finding that developmental algebra students believed the nature of mathematics to be more rule-oriented than global-oriented. Cal, Chris, and Joan made comments that revealed their beliefs that mathematics consisted of steps, procedures, methods, and rules. However, Willis' (1992) research implies that developmental algebra students did not believe mathematics benefited them as a tool and as a way of thinking as much as the other groups in her study. My findings indicate that perhaps this implication is unclear. All three of the participants in my study believed aspects of mathematics benefited them as a tool and as a way of thinking. I believe the discrepancy may be because Willis did not realize the dualistic view of mathematics that these students have. Although Cal, Chris, and Joan believed that aspects of *mathematics* benefited them as a tool and as a way of thinking, they were not able to see how *school mathematics* benefited them in any way except earning a grade. In other words, perhaps Willis' (1992) results reflected what students believed about the mathematics they were learning in school, not what they believed about mathematics in general.

The three case stories from my study also help to explain more clearly Willis' (1992) findings about the strategies students used to learn mathematics. First, students use the strategies of attending class and copying examples because they can be successful in class using them. Second, students do not use those strategies to learn mathematics in general; they use those strategies to pass mathematics courses. The difference between the two findings is significant because it indicates that there may be additional strategies students use to learn mathematics that are not used when they are trying to pass a mathematics course. Cal's story gives a few examples of what some of the different strategies may be. He indicates that when he really wants or needs to learn mathematics that he does so by actively "doing it" himself (referring to his work), not simply passively "watching someone at the front" doing it (referring to in class). In fact, his story indicates that the strategies of "attending math class each period, copying examples from the board, [and] paying attention to the math instructor's lecture" (Willis, 1992, p. 211) were not useful to him at all in actually learning mathematics; therefore, he did not often do them.

Byrd's (1991) research found that her participants believed in the "cumulative nature" of mathematics, which caused many of the respondents trouble because they felt they "lost out on the basics and never seemed to be able to catch up or understand subsequent material" (p. 138). All three of the participants in my study also believed mathematics was cumulative in nature. Both Cal and Chris likened the subject to a "ladder" in which one had to progress from rung to rung in order to move forward. Joan's story serves as a heart-breaking example of someone who felt she had "lost out on the basics and never seemed to be able to catch up" (Byrd, 1991, p. 138).

Another theme that resulted from Byrd's (1991) study is the context of mathematics. Most of her respondents felt that mathematics, unlike other subjects, had no context and therefore they could not work through a task if they did not understand the entire task in detail. Because of this lack of context, Byrd's (1991) participants believed mathematics was different from other subjects. The results of my study confirmed Byrd's (1991) findings. Chris and Joan viewed mathematics as a unique subject. They both felt that mathematics was different from other subjects because it was not "obvious" why a procedure was done the way it was, it was not "intuitive," and because it often "did not make sense." Cal especially related to

Byrd's (1991) findings about the context of mathematics since he had a completely different context for mathematics that he considered "in school" verses mathematics he deemed "out in the field." He viewed in school mathematics as a "game" that one played which involved feigned ignorance while he viewed the mathematics he did at work (i.e. "in the field") as practical, useful, and reasonable.

The results of my research confirm many of the findings from Venters' (1992) study. Venters' (1992) participants differed from those in previous research on confidence in mathematical ability because they contributed their success in mathematics to "new-found ability" – something that was dormant and newly-discovered – rather than effort (p. 86). Chris' story confirms this finding. Chris, at the end of the elementary algebra class, would have been classified as a successful female by Venters' (1992) criteria because she had an A average. Like the participants from Venters' (1992) study, Chris also attributed her success to a new "aptitude" for mathematics that she did not realize she had before the class. Chris' story adds depth to Venters' (1992) results by giving a detailed account of how she discovered her aptitude, what it means to her that she has it, and what effects it has on her current mathematical performance and future academic plans.

Almost all of the participants from Venters' (1992) study expressed a positive correlation between how much time they spent on learning mathematics out of class and successful performance in class. Chris' story echoes that sentiment. Chris stated several times that her good grades were directly related to her "spending time doing the homework." The difficulties in mathematics that were encountered by the majority of participants from Venters' (1992) study centered on "word problems, geometry, and graphing" (Venters, 1992, p. 90). In Chris' task interviews, she also had difficulties with tasks that involved graphing and story problems. In addition, she made several comments that indicated she was not confident in her ability to solve story problems.

According to the participants of Venters' (1992) study, guidance counselors had little time for them and seemed to be concerned with only those students "who were going on to college" (p. 99). Because none of them felt that was a role they fit into, "the role of guidance counselors in the high school experiences of participants of this study was negative or nonexistent" (Venters, 1992, p. 99). Both of the females in my study expressed similar experiences and beliefs. Joan explicitly stated that she did not think of herself as "one of the students who was going to college" and that she believed the guidance counselor at her school believed the same thing. As a result, she received little guidance from the guidance counselor. Chris also stated that the guidance counselor in her school "was just there" and did not offer much support or guidance.

The participants in Venters' (1992) study viewed teacher attitudes as an important part of their experiences and preferred "teachers who were friendly, who showed concern, [and] who made learning fun" (p. 97). Again, both females in my study confirmed this finding. Cal also made statements that reflected the importance of teacher attitudes, however, which may indicate that this preference is not exclusive to female students.

All three of the participants in my study shared the same "lost year" experience that Venters' (1992) participants did (p. 108). Some of the reasons given for the lost year of her participants were repetitions of failed courses, choosing not to take any more mathematics because they had met graduation requirements, and being advised not to take more mathematics because of low grades. All three of my participants lost a year (or more) of mathematics either because of a failed course or because they had met graduation requirements. This indicates that the theme of a lost year may be a theme for developmental mathematics students in general not just successful female developmental mathematics students.

Connections to Qualitative Research

Research in mathematics education and especially research in developmental mathematics education is a young field (Silver and Kilpatrick, 1994) that is "not yet well established" (Gravenmeijer, 1994, p. 443). McLeod (1994) summarizes the changes that have taken place in a specific aspect of mathematics education, the area of affective issues:

> The vast majority of studies of affective issues have involved the use of questionnaires and quantitative methods. Such methods have provided useful information, but the field needs to complement the data that are currently

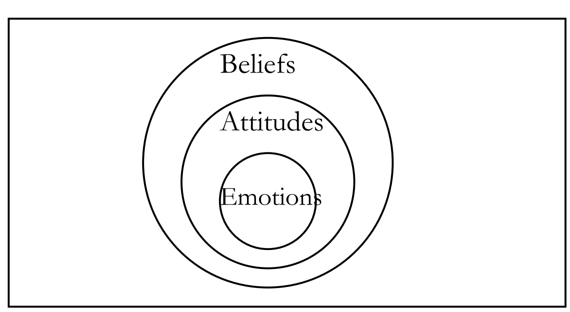
> available with new insights and new conceptions of what research on affective issues can be. (p. 643)

Gravenmeijer (1994) agrees by describing the shift occurring in "research paradigms" (p. 443). This shift is away from "large survey studies and studies that focus on general trends" and towards "interpretive case stories" (Gravenmeijer, 1994, p. 443). The case stories presented in the findings of my study are representative of this shift in paradigms. I believe one reason this shift is occurring is because mathematics researchers and mathematics educators are beginning to realize the importance of "build[ing] . . . on each other's work" (Silver and Kilpatrick, 1994, p. 738). Conducting "research that would reveal while attempting to duplicate" (Silver and Kilpatrick, 1994, p. 738) existing studies is exactly what naturalistic inquiry allows researchers to do. Qualitative research in developmental mathematics education, especially on the topic of affective issues of developmental mathematics students, allows researchers to "dig deeper" (Silver and Kilpatrick, 1994, p. 738) into the results of previous quantitative research and discover new information while building on known information. Because my study is based on several other studies on developmental mathematics students, the case stories and findings build on previous research, give new depth to existing findings, and add new reasons for and examples of students' behaviors resulting from their beliefs and affects.

"Qualitative research, also known as naturalistic inquiry, is an investigative approach which does not formulate questions or hypothesis but searches for 'truth' through information that 'emerges' from the study" (Silverman, 1983, p. 16). Often, because of the type of information that emerges from these truth searches, it is difficult to report the findings in a simple, concise fashion like a table of scores or a certain significance value for alpha. Instead, the results are unwieldy, convoluted, and anything but neat. As Gravenmeijer (1994) points out, both qualitative research and developmental research share this complexity of information and interpretation. Gravenmeijer (1994) contributes the "complex process," which includes student learning, student affective responses, and how researchers interpret that data, as "one of the main problems in reporting on developmental research." In this qualitative study on developmental mathematics students, I strive to present the truths that emerged in a way that illustrates "how certain elements of the results will apply to other situations." In the last section of this chapter, I describe how the case stories that resulted from this study apply to recommendations for change in mathematics teaching methods.

<u>Connections to Literature on Affective Issues</u>

Generally speaking, the affective domain can be divided into three categories of "affective responses to mathematics" (McLeod, 1992, p. 578) which include beliefs, attitudes, and emotions. These three categories can be viewed as a Venn diagram like the following:



Beliefs emerge from and are built on one's attitudes and emotions. Attitudes form when certain emotions are felt repeatedly (McLeod, 1992). The three categories build on each other and reflect varying levels of cognition and affective involvement. For example, Joan's emotional reaction when faced with an algebra task involved feeling fearful and worried that she would not be able to solve the task. After many instances where this emotional reaction was felt, Joan formed an attitude towards mathematics, specifically, that she did not like it, that included the behavior of avoiding the subject. Her attitude, combined with other attitudes about herself as a do-er of mathematics, led to the belief that she was not "a math person." Her attitude, combined with other attitudes about the nature of mathematics and combined with the previously stated belief, led to another belief that mathematics was a "code" that she could not decipher. All three of the participants experienced emotional "hot gut-level reaction[s]" (Mandler, 1984, p. 44) which, in turn, combined with behaviors to form attitudes, and, in turn, combined with other attitudes and previously-held beliefs to form new beliefs. The case stories presented in this study give much-needed insight into how beliefs are formed by allowing the reader to hear, in the participant's own words, the past experiences that each person brought into the classroom with them and how those experiences influenced their current experiences and expectations.

It is imperative that mathematics educators understand how students' beliefs are formed in order that the mathematical instruction given will positively influence that process. For example, Chris' belief that the goal of mathematics done in class is to obtain the correct answer is a common belief (Frank, 1988). Because of her belief, Chris was rarely able to give an acceptable answer to the follow-up question of "what does your answer mean?" during the task interviews. Instead, she focused "almost entirely on answers and whether those answers were right or wrong" (Frank, 1988, p. 33) not on the actual process of problem solving or critical reasoning. The reasons behind her belief and her behavior are found in the mathematics instruction she had received in high school and was currently receiving in college. The mathematics instruction given during the elementary algebra class emphasized obtaining the right answer and rarely illustrated reflective thinking, metacognitive issues, or control-related strategies.

Schoenfeld's (1992) discussion of the common student belief that I term the *instant math myth* – the belief that one should be able to "solve any assigned problem in five minutes or less" (p. 359) – includes the resulting behavior of lack of perseverance in problem solving. All three of the participants in this study held that belief when the assigned problem was an in-class task but did not believe it when the problem was a real life task. For example, when given a typical factoring task (e.g. Factor completely: $5t^2 – 32t – 21$), Cal would spent only one or two minutes before giving up or asking for help. When given a task that he perceived as more "real life" however (e.g. a story problem), Cal spent over ten minutes and persevered until he completed the task. Chris demonstrated similar difference in her beliefs during the task interviews by the amount of time she spent on various tasks as well.

As stated in Chapter Four, students' beliefs influence not only their problem solving efforts but also the mathematical instruction that takes place in the classroom. Borasi (1990) states the belief that "there are no gray areas" (p. 176) in mathematics and Chris' story adds depth and breadth to that belief and its resulting effects on instruction. Chris described how when the instructor "went step by step" through a procedure it helped her to "learn" the mathematics being taught. She gave examples of student behaviors (resulting from student beliefs) that influenced the mathematics instruction taking place in the classroom. For example, she recalled instances when a student would "keep asking" for a procedure to be explained (reinforcing the belief about mathematics being well-defined) and the instructor would "keep going over it and over it." Borasi (1990) describes how students who hold this belief about the nature of mathematics are "likely to be passive learners, ignorant of the need of making personal meaning of the material presented in class, lacking constructive strategies to deal with learning difficulties, and therefore liable to experience limited success in school mathematics" (p. 181). I believe Chris' story both support and refutes those claims. It is true that Chris was a passive learner and that she did not construct personal connections to the mathematics she was taught in class. However, she *was* aware of the need to construct that meaning and even had a very strong desire to make the mathematics meaningful; she was unable to make the connections because of her lack of experience with mathematics instruction that reflected the importance of those connections. In addition, Chris' story clearly illustrates that, contrary to Borasi's (1990) prediction, students may hold unhealthy beliefs about the nature of mathematics and what constitutes appropriate mathematics instruction and *still be very successful* in mathematics courses.

Taylor (1993) discusses the importance of classroom experiences in the formation of attitudes toward mathematics. As already discussed, attitudes are instrumental in the formation of beliefs. Taken together, this leads to the conclusion that classroom experiences influence the formation of one's beliefs toward a subject, beliefs about oneself as a do-er of that subject, and beliefs about how the instruction of that subject should take place. In addition, researchers have shown that the development of good metacognitive skills are also influenced by the classroom culture (Schoenfeld, 1987; Campione, Brown, & Connell, 1989). The classroom culture is largely shaped by the teacher – teacher actions, teacher attitudes, and teacher beliefs. It is for this reason that the recommendations that arose from this study are directed to mathematics teachers. In the next section I discuss the implications of the findings of this study and offer recommendations on how mathematics teachers can assist their students in developing healthy beliefs and affects.

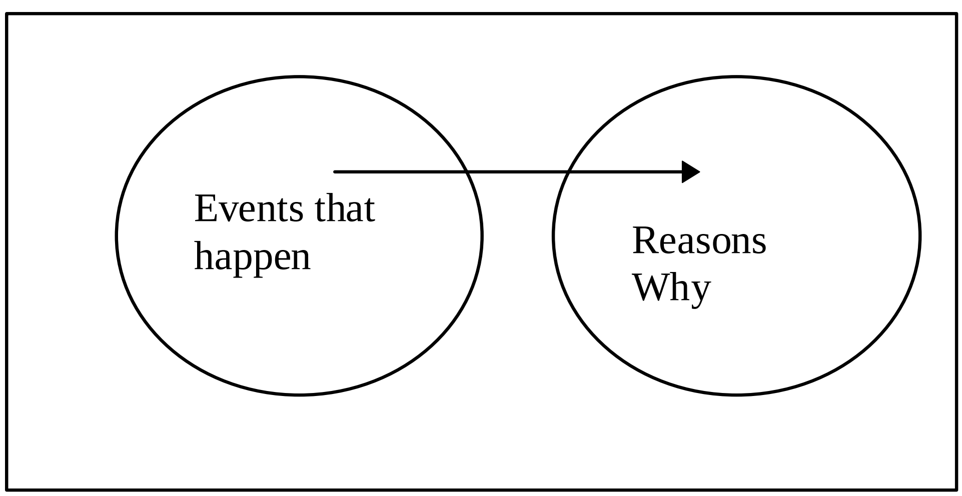
Implications and Recommendations

Implications

What occurred with Cal, Chris, and Joan during that elementary algebra class was much more than their learning or not learning algebra, it was deep and lasting changes in their beliefs and affects. The changes in these students' beliefs and affects will have more affect on their future learning or not learning of mathematics than the algebra that they learned or did not learn in that class. The crux of the findings of this study rest on that realization. Because students' "attitudes and beliefs about the subject. . . influence their orientation toward future activities" (Hiebert et al., 1996, p. 17), it is imperative that mathematics educators gain a deeper understanding of students' affective responses to mathematics.

It is clear that students' beliefs and affects about mathematics are strongly influenced by their classroom experiences, which are strongly influenced by the classroom teacher. Considering the goals of the NCTM *Standards* (1989) and the AMATYC *Standards* (1994) for students to understand and appreciate the value of mathematics, become confident in themselves as do-ers of mathematics, develop their mathematical power, and experience mathematics that is both meaningful and relevant, the importance of understanding students' beliefs and affects, and how teachers influence those beliefs and affects, cannot be overemphasized.

The three case stories from this research give clear indications that these students know what they want from mathematics classroom instruction, that they do not believe they are getting what they want and need, and that they want significant changes to happen in both the ways in which mathematics is taught and in the types of mathematics that is taught. I believe these stories also indicate an overall pattern for the development of students' affective responses to mathematics. These affective responses stem from the students' attempts to connect events that happen in class with reasons why they happened. To explain this behavior, I use the mapping metaphor for affective responses. A student's affective responses to various things that happen in a mathematics classroom results from a mapping from the set of "events that happen" to the set of "reasons why." The diagram below illustrates the mapping:



Some examples of this mapping may include:

Events that Happen Set		Reasons Why Set
I got an A on the test! →		I knew the subject matter.
I failed the test. →		I did not study.
I can't get the answer the teacher wants →		I am stupid.
	OR	The teacher is not good.
	OR	There is no answer.
I do not understand. →		I am not a math person.
	OR	It makes no sense.
	OR	The teacher did not explain it well.

As in a mathematical mapping, in the affective mapping an item from the "Events that Happen" Set may be mapped to one or more items in the "Reasons Why" Set. From the case stories generated by this research, it seemed to me that, for these students, when doing mathematics in the classroom, they attempted to connect the events that happened in class with the reasons why they happened. If they did not know the reason, then they either created a reason or created a mapping to a reason that already existed. The results of the mappings they created are their affective responses to mathematics. For example, if a student created a mapping from *I got a high grade on this test* to *I memorized the formulas* then that student is likely to create an affective response to test taking that includes the belief that mathematics is memorization and that learning is remembering. Resulting behaviors will include heuristics that allow for quick memorization, little time spent on planning or reflection, and control strategies that do not involve monitoring or metacognition. The teacher, or things controlled by the teacher, dictated how the students in this study mapped many of the items from one set to the other.

Recommendations

Because of the teacher's tremendous influence, it is logical that the recommendations of this research would address teacher actions. The major recommendations resulting from this study are:

- Teachers should incorporate tasks that clearly illustrate how various mathematical skills, concepts, and procedures taught in class relate to the outside world.
- Teachers must incorporate mathematical tasks that will illustrate more clearly what it means to "do math" including the struggle, the doubt, and the gray areas of the problem solving process.

It is apparent from these three case stories that these students wanted to know how the mathematics they were learning in class related to the world outside of class. Teachers should illustrate clearly how various mathematical skills, concepts, and procedures taught in class relate to the outside world. Darken (1995) states that "what [students] are doing in the real world doesn't resemble what they had in school, so they think it is not math" (p. 26). As Cal's story so clearly illustrates, it is not so much that he thinks it is not math, it is rather that he sees it as a different type of math. Teachers must find ways to help students like Cal realize the error of that viewpoint. Not surprisingly, the recommendation to make mathematics meaningful to the student has been made by various organizations (e.g. NCTM, AMATYC) concerned with mathematics education reform. NCTM and AMATYC also recommend that teachers use mathematical tasks that connect various areas of mathematics to each other and connect mathematics to other subjects. Teachers must recognize that "the abstract approach is not working" because "it is not meaningful" to the students (Darken, 1995, p. 24). The three case stories generated by my research help explain why it is not meaningful. The participants in my study described in detail that mathematics in the classroom was not meaningful because they could not relate it to their lives or to anything in the world outside of the classroom. As Darken (1995) recommends, "the intersection between classroom mathematics and the real world needs to be increased" (p. 22).

Teachers must also incorporate mathematical tasks that will illustrate more clearly what it means to "do math" including the struggle, the doubt, and the gray areas of the problem solving process. The "shared expectations of the teacher and students" creates the classroom "culture" (Hiebert et al., 1996, p. 16). In turn, "the culture of the classroom will determine how [mathematical] tasks are treated by students" (Hiebert et al., 1996, p. 16). If the goal of mathematics instruction is "understanding" as assumed by Hiebert et al. (1996, p. 15), then mathematics teachers must allow the student to experience the full range of affective responses involved in the journey to understanding. Current researchers admit that "genuine thinking is too often absent from classrooms" (Hiebert et al., 1996, p. 18). In addition, they believe that "the source of the problem is not so much the tasks themselves as the way in which students are expected and allowed to treat them" (Hiebert et al., 1996, p. 18). Because those expectations and allowances originate with the teacher, it is recommended that teachers become adept at facilitating students' progress in the struggle and help them to enjoy the doubt that arises when one is truly engaged in the problem solving process. The case stories from my research show that not only do these students not have experience in that struggle but also that they do not view it as a valid part of the problem solving process. Teachers must assist students in becoming math people as defined by Chris. According to Chris, a "math person" was someone who "enjoy[ed] it even when [they didn't] know what to do." Teachers must help students to learn to enjoy the doubt. The experiences related by the participants in this study confirmed what Pintrich, Marx, and Boyle (1993) stated: "Classroom organization and the nature of many classroom academic tasks may encourage students to get it done, not think it through" (p. 181).

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APPENDIX A: MATHEMATICAL TASKS

1. FACTOR COMPLETELY: $5r^2 + 10rs - 20s$

2. FACTOR COMPLETELY:

   a. $16^2 - 25^2$
   b. $r^2 - t^2$
   c. $t^2 - r^2$

3. FACTOR COMPLETELY: $5t^2 - 32t - 21$

4. The area of a rectangle is 21 sq. ft. What are its dimensions if the length is 5 ft. less than four times the width?

5. EVALUATE: for $x = -9$ $\quad \dfrac{x + 9}{x^2 + 2x - 1}$

6. EVALUATE: for $x = -3$ $\quad \dfrac{2x^2}{x^2 + 9}$

7. An automobile uses 8 liters of gasoline to travel 84 km. How many liters are needed to travel 1,428 km?

8. Sarah is operating a machine that can produce 14 parts in 20 minutes. How long will it take her to produce 224 parts?

9. GRAPH:
   a. $5x + 2 = y$
   b. $3y - 9x = -6$

10. Graph (-2,4) on the X,Y plane.

11. Write an equation, using the variables S and P to represent the following statement: "At this university, there are six times as many students as professors." Use S for the number of students and P for the number of professors (Clement, 1982, p. 17).

12. SOLVE: $x^2 + 2x - 1 = 0$

13. FACTOR COMPLETELY: $20x^2 - 15xy$

14. FACTOR COMPLETELY: $2w^2 - w - 15$

15. FACTOR COMPLETELY: $y^2 - 25$

16. FACTOR COMPLETELY: $3xy - 9 + 6x^2 - 3x$

APPENDIX B: FOLLOW-UP QUESTIONS FOR THE TASK INTERVIEWS

1. How can you check to see if your answer is right?
2. How did you know what to do when you first looked at this problem?
3. When you got your answer, did you check it for accuracy? If yes, how? If no, why not?
4. Do you think this problem was easy? If yes, why. If no, what would be an example of an easy problem?
5. What does your answer mean? What do you think it might be used for?
6. What do you think of when you first look at problems of this type?
7. [For factoring tasks only] This process is called "factoring"; why do you think it is called that?
8. [For factoring tasks only] What is a factor? Give an example.
9. [If opportunity arises] How do you know if something is wrong with your answer? At what point in the process do you stop and try something else?

APPENDIX C: ORAL PRESENTATION TO STUDENTS

My name is Diana Perdue and I am currently conducting research for my doctoral dissertation in mathematics education at the University of Virginia. I am here seeking willing participants who are presently enrolled in a developmental mathematics course at a community college. I chose PVCC for my study because I taught here for several semesters while I completed my coursework at U.Va. The purpose of my study is to find out what developmental mathematics students believe about the nature of mathematics and its teaching. I am not interested in your grades, scores, speed of your multiplication skills, or anything of that nature. I am interested in you telling your math story: your experiences in high school and college mathematics courses, your views of mathematics teaching and teachers, your beliefs about what mathematics is, and your opinion of yourself as a mathematician.

Perhaps you are thinking, at this moment, that you might be interested in helping me out but you want to know what you have to do. I have a typed information sheet which I hope will answer all your questions. If you are interested in knowing more about participation in this project, please raise your hand and I will give you this sheet.

Thank you very much for your time and attention. If you do choose to participate, you will have my eternal gratitude as well as an acknowledgment in my dissertation when it is completed.

APPENDIX D: INFORMATION SHEET FOR POTENTIAL PARTICIPANTS

I am currently involved in a research project as part of my doctoral dissertation in mathematics education at the University of Virginia. My research participants will be students currently enrolled in a community college developmental mathematics course. Through interviews and observations, I hope to learn their beliefs about the nature of mathematics and its teaching as well as obtain a clear view of their mathematics experiences in high school and college. Although many studies have been done to evaluate scores, grades, and test results, my project will focus on the individual's "story" of mathematics (experiences in high school and college, thoughts about teachers and teaching styles, beliefs about what math is, etc.).

You have been identified as a possible participant for this study. If you agree to participate in this study, the following would be required of you:

1. Sign a consent form to participate.

2. Respond to a series of questions in three (3) interviews.

3. Work a few problems in two (2) observations.

4. Allow me to observe you in class on two (2) occasions.

The interviews will last between 45 - 60 minutes each and will be audiotaped and later transcribed. The problem-solving observations will last approximately 30 - 45 minutes and will be videotaped. The problems you will be asked to solve will relate to material currently being covered in your class. The classroom observations will be the length of your class session and will not require you to do anything other than what you usually do in class. I will keep field notes (may include topics discussed, problems worked, questions asked, etc.) during the classroom observations. You may select a pseudonym at the beginning of the project which will be your identification throughout the study. Only you and I will know your pseudonym and you will not be identified at any time during the project. All information from the interviews and observations will be strictly confidential and kept secure in my home. At the completion of this project, a summary of the findings will be made available to you if desired.

Participation in this study is entirely voluntary. You are free to withdraw at any time and/or refuse to answer any questions without incurring any penalty or prejudice. I do stress, however, that because of the in-depth nature of this project, you realize the importance you have as a participant and the impact you would have if you did not complete the project as agreed. If, after reading this information sheet, you do elect to participate, you will be asked to sign a consent form that acknowledges your understanding of this project, its purpose, and your role in it.

If you have any questions about this study, please do not hesitate to contact me. I feel very strongly that the detailed case studies which will result from this research will be very useful to developmental mathematics students and educators. The process of describing your beliefs about the nature of mathematics as well as reflecting on your mathematics experiences may benefit you and give you insights about yourself and your abilities as a mathematician.

Diana S. Perdue (dsp4e@curry.edschool.virginia.edu)

1713 Owensfield Drive

Charlottesville, VA 22901

(804) 977-3340

APPENDIX E: INFORMED CONSENT AGREEMENT

University of Virginia

A Descriptive Study of Developmental Mathematics Students' Beliefs and Affects

Please read this consent agreement carefully before you decide to participate in this study.

Purpose of this study: I am conducting research as part of my doctoral dissertation requirements at the University of Virginia. The focus of the study is developmental mathematics students' beliefs about the nature of mathematics and themselves as mathematicians.

Because you are currently enrolled in a developmental mathematics course and have expressed a willingness to participate, you have been asked to give your informed consent by reading and signing this form.

What you will do: Your responses to the interviews will provide the majority of data for this study. Additional data may include samples of your notes, homework assignments, and tests from your current developmental mathematics class. You will be asked to participate in three (3) regular interviews which will be audiotaped, two (2) task interviews which will be video- and/or audiotaped, and to make samples of your homework assignments, class notes, and tests available for possible inclusion in the data.

Time required: A total of about six (6) hours is required for this study.

Risks: There are no risks associated with participation in this study.

Benefits: There are no direct benefits to you for participating in this study.

Confidentiality: The information that you give in this study will be handled confidentially. You and your schools will not be identified in this study. Anyone you mention in the interviews will not be identified in this study. The only person, other than myself, who will have access to the tapes from the interviews will be the person who transcribes them. The tapes and other documents from this study will be kept secure.

INFORMED CONSENT AGREEMENT

University of Virginia

A Descriptive Study of Developmental Mathematics Students' Beliefs and Affects

Voluntary participation: Participation in this study is entirely voluntary.

Right to withdraw from the study: You may withdraw from this study at any time and/or refuse to answer any questions without incurring any penalty or prejudice.

How to withdraw from the study: If you want to withdraw from this study at any time, inform me of your decision (you may do this in person, over the phone, or in writing).

Payment: You will receive no monetary payment for participating in this study. When the study is completed, a summary of findings will be available to you if desired.

Who to contact if you have questions about this study: If you have any questions, contact me, Diana S. Perdue, by telephone at 977-3340, by e-mail at dsp4e@curry.edschool.virginia.edu or by regular mail at: 1713 Owensfield Drive, Charlottesville.

Who to contact if you have questions about your rights in this study: Dr. Jerry Short, Chairman, Committee for the Protection of Human Participants, 287 Ruffner Hall, University of Virginia, Charlottesville, VA 22903. Telephone: (804) 924-7471.

Agreement to participate: I agree to participate in the research study described above.

Agreement to allow data generated to be used in publications, presentations, or conferences: I have read and understand the above explanation of the study. I give my informed consent to Diana S. Perdue to use any data I have provided during the course of this research project. I understand video clips of my task interviews may be shown to others including members of the research committee or members of the education community at large. I give my permission for the regular interviews to be audiotaped. I give my permission for the task interviews to be audio- and/or videotaped. I understand that samples of my work and excerpts from my interviews may be used in publications or in conferences.

Signature: ______________________ Date: _________________

You will receive a copy of this form for your records.

APPENDIX F: INTERVIEW PROTOCOLS

INTRODUCTORY STATEMENTS

Interview Protocol #1. This interview will focus on finding out about your past experiences in mathematics classes in high school and/or college. I am interested in your beliefs, attitudes, and opinions related to those experiences. If you are not certain about what I am asking, if you could view the question in more than one way, or if you need me to repeat a question, please tell me.

I want to tape record this interview so I may have it transcribed later and not have to rely on my memory or notes to recall your responses. I may take notes while we are talking to remind myself of additional questions I want to ask you or to mark questions you have answered.

Before we start the interview and before I turn on the tape recorder, do you have any questions about the interview process?

Interview Protocol #2. This interview will focus on your experiences in and beliefs about your current developmental mathematics class, elementary algebra. I am interested in any comparisons you can make between this class and other mathematics classes you've taken.

I want to tape record this interview so I may have it transcribed later and not have to rely on my memory or notes to recall your responses. I may take notes while we are talking to remind myself of additional questions I want to ask you or to mark questions you have answered.

Before we start the interview and before I turn on the tape recorder, do you have any questions about the interview process?

Interview Protocol #3. This interview will focus on ways your attitudes and beliefs about the nature of mathematics and yourself as a mathematician may have changed because of this mathematics class or others.

I want to tape record this interview so I may have it transcribed later and not have to rely on my memory or notes to recall your responses. I may take notes while we are talking to remind myself of additional questions I want to ask you or to mark questions you have answered.

Before we start the interview and before I turn on the tape recorder, do you have any questions about the interview process?

APPENDIX G: INTERVIEW GUIDES

INTERVIEW GUIDE #1

I. Summarize your mathematics education from 8^{th} grade until now.

II. Tell me about a math class that stands out for you.
   a. Describe what the teacher did.
   b. Describe what the students did.
   c. Describe what you learned / did not learn.

III. How did you come to be in the developmental mathematics program here (the elementary algebra class)? Describe how this class matches your ability level in mathematics.

IV. Describe your feelings about mathematics.

V. What is mathematics?
   a. What is a "math problem"?
   b. What is an "exercise"?

INTERVIEW GUIDE #2

I. Describe this mathematics course (elementary algebra).
   a. How does this class compare with previous classes you've had? (in content, teacher's style / method, and amount of math learned)

II. Describe the nature of the mathematics in this course.

III. Describe how you feel about yourself as a learner of mathematics.
   a. When you do well, how do you explain it?
   b. When you have trouble, what do you think is the reason?
   c. Has this class affected how you feel about yourself as a learner of math?
      If so, how?

IV. Describe the strategies you use to learn math.
   a. How do you prepare & do a typical homework assignment?
   b. How do you prepare for a test?

c. What are some things you would tell another student to avoid when trying to learn math?

d. Did your instructor suggest activities / strategies which could be used in learning math?
If yes, what? If no, do you think it would have been useful?

INTERVIEW GUIDE #3

I. Describe your attitudes toward mathematics, towards yourself, towards your instructor.

II. Have any of your thoughts, beliefs, or attitudes changed as you have had different mathematics courses?
If so, how?

III. Have any of your thoughts, beliefs, or attitudes changed as you have progressed through this course?
If so, how?

IV. How has taking this course benefited you?
If response is it has not, have other mathematics courses benefited you?
If future benefits are mentioned, are there any immediate benefits?

V. How do you use math from your course in your life outside of school?
If response is do not use, how do you use any math in your life outside of school?

APPENDIX H: SAMPLES OF CAL'S NOTES, TESTS, & QUIZZES

APPENDIX I: CAL'S WORK ON THE TASK INTERVIEWS

APPENDIX J: DEFINITIONS

Developmental Mathematics: College (two-year or four-year) mathematics courses below the level of calculus that are designed to (a) give students a second chance to learn high-school mathematics, (b) prepare students for calculus, (c) prepare students for other college mathematics courses, or (d) prepare students for the job market.

Developmental Mathematics Student: A student who is judged by their college's criteria (usually a standardized entrance exam) to be underprepared for college-level mathematics course.

Typical Developmental Mathematics Student: A developmental mathematics students who exhibits three or more of the following characteristics:

1. A student who has, in his/her own opinion, never done well in mathematics.
2. A student who has poor grades in K-12 mathematics, especially high school mathematics.
3. A student who did not take any more than the required mathematics courses for high school graduation.
4. A student whose attitudes toward mathematics include one or more of the following: hatred, fear, frustration, or avoidance.

5. A student whose beliefs about mathematics include one or more of the following:

   a. Mathematics is just a bunch of rules that are useless in real life.

   b. Only certain people can "do math" (and they do not believe they are one of those people).

   c. The rules that govern mathematics do not make sense.

   d. The "instant math" myth: the belief that if they can't look at a math problem and instantly know what to do to solve it, then they can't solve that problem.

APPENDIX K: JOAN'S WORK ON THE TASK INTERVIEWS

APPENDIX L: SAMPLES OF CHRIS' NOTES, TESTS, & QUIZZES

APPENDIX M: CHRIS' WORK ON THE TASK INTERVIEWS