

# Fuzzy Fault Currents: Theory and Applications

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**Abstract**— In this work, the concept of fuzzy fault currents (FFC) is introduced for modeling the uncertainties associated to conventional fault current analysis. In order to compute FFC, a general procedure for calculating functions of type-1 and type-2 fuzzy numbers is presented. This technique, which is based on extension principle and interval arithmetic, can be applied to several engineering problems where calculating functions of uncertain variables is required. The proposed methodology is used to compute FFC for a real distribution system. The results are shown and the advantages of the approach are discussed.

**Index Terms**— Type-2 fuzzy sets, extension principle.

## I. INTRODUCTION

IN many engineering problems, the calculation of reliable solutions depends on the availability of exact values for the variables of model equations. However, in daily practice, these precise values can not be obtained because the existing information usually is incomplete, imprecise, noisy, vague, qualitative or linguistic. Therefore, in this case the results obtained by deterministic approaches, which uses one specific crisp value as the most likely value for a variable, cannot be considered to be representative for the whole spectrum of possible results [1]. Consequently, it is necessary to introduce uncertain variables for modeling the available information and to implement procedures for calculating functions of these variables. To solve this problem, a common practice is to model the uncertain variables as type-1 fuzzy numbers, with their shapes derived from experimental data and/or expert knowledge [2], and to implement extension principle-based methods for evaluating the corresponding functions. Thus, the type-1 fuzzy results reflect the influence of the input uncertainties on the output function variables. If the type-1 fuzzy results are defuzzified, the most likely crisp values for the output variables can be obtained. The defuzzified values generally differ from the crisp results obtained by means of deterministic methods, since in the type-1 fuzzy approach the uncertainties have been included and processed through the calculation procedure.

In the type-1 fuzzy modeling, the values used in developing the membership functions of type-1 fuzzy numbers are often overly precise, because they require that each element of the

universe where the type-1 fuzzy number is defined be assigned a specific value of membership [3]. When the level of information is not adequate to specify membership functions with this precision, then it is necessary to use type-2 fuzzy numbers to represent the uncertainties of model variables [4], [5]. Nevertheless, the type-2 fuzzy modeling of uncertain variables implies calculating functions of type-2 fuzzy numbers. The computation of functions of type-1 fuzzy numbers is a well-known field of fuzzy set theory and has been documented in several publications [6]-[15]. However, less coverage has been given to the calculation of functions of type-2 fuzzy numbers [10], [14], [16]. In this work, type-2 fuzzy numbers are used to model the uncertainties associated to the most influential variables of conventional fault current calculation of power distribution systems [17], [18]. This modeling allows the introduction of the Fuzzy Fault Currents (FCC) concept.

## II. FUZZY FAULT CURRENTS

The normal operation of an electric distribution system may be disturbed or disrupted due to a system fault when abnormally high currents flow through an abnormal path as a result of the partial or complete failure of insulation at one or more points of the system. The complete failure of insulation is called a “short circuit” or a “fault”. A fault occurs on a distribution system when one or more energized conductors contact other conductors or ground [18]. Since they are mostly constructed of bare conductors, the overhead distribution lines are one of the most vulnerable points of distribution systems. A considerable amount of faults occurs in these lines, causing outages and economic losses to the utilities, as well as to the affected customers.

In order to reduce the effect of faults, protective devices are installed on distribution feeders. These devices are coordinated among them in such a way that only the device located closer to the fault must actuate when a fault current is detected. This way, the outage extension and total amount of affected customers is reduced. The starting point for coordinating protective devices is calculating fault currents. Unfortunately, the calculation of fault currents, particularly in distribution systems, strongly depends on uncertain variables, whose values are roughly known. If these uncertainties are not taken into account, coordination problems may occur in some situations. These problems can lead to incorrect actuations of protective devices, which cause extended outages and important economic losses.

The traditional approach for handling uncertainties in fault current calculations consists in computing maximum and minimum fault currents by using a bolted fault resistance ( $0\Omega$ ) and an assumed fault resistance (generally of  $40\Omega$ ),

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respectively. This approach has proven to be problematic, since very few faults are bolted faults. Moreover the fault current levels produced by 40Ω faults are lower than the trip levels of many protective devices. This complicates designing an economical and flexible protection system [21].

In this work, in order to take into account the uncertainties associated to conventional fault current calculations, the concept of Fuzzy Fault Currents (FFC) is introduced. In the FFC calculation, the input variables ( $x_i$ ) may be modeled as crisp numbers, type-1 fuzzy numbers or interval type-2 fuzzy numbers. Function  $f(x_1, \dots, x_n)$  is one of the classical fault equations and the output ( $y$ ) is a FFC, which is calculated by applying the extension principle and vertex method.

#### A. Fault resistance

The main advantage of the FFC concept is that it sets a mathematical framework for computing with uncertain variables. This is very useful for taking into account the uncertainty about fault resistance value ( $R_f$ ), which is the most uncertain and influential of all variables involved in the calculation of fault currents in distribution systems. In this work,  $R_f$  is modeled as a type-1 fuzzy number and as a type-2 fuzzy number. The parameters of these fuzzy numbers model the usual range of values for fault resistance, which can be obtained from the analysis of the historical data of faults or from procedures based on expert knowledge (e.g., knowledge mining through surveys). Procedures for defining type-2 membership function on the basis of statistical data and expert knowledge are proposed in [4], [10], [22] and [23]. For instance, through the analysis of historical data of faults, the values  $R_f \approx 15\Omega$ ,  $20\Omega$  and  $25\Omega$  and  $R_f \approx [5\Omega, 25\Omega]$  have been suggested in [19] and [20], respectively. These values depend on the particularities of each distribution system; however it is worth noting that the data used to calculate them are generally scarce and uncertain. Thus, a type-2 fuzzy modelling of  $R_f$  is an adequate proposal for considering these uncertainties.

#### B. Pre-fault voltage

Another important variable involved in the calculation of fault currents is the pre-fault voltage ( $U_{pfk}$ ). This variable generally is not monitored in real-time due to economic constraints, therefore only a rough estimation of its value is available. This estimation may be obtained by means of a power flow-based algorithm, like the one proposed in [24]. This algorithm iteratively tunes the estimated nodal demands of the feeder (the nodal demands also are not monitored in real-time) until the calculated power flow matches the measured power flow at the High Voltage/Medium Voltage Substation (HV/MV SE). The voltages of the tuned nodal demands are the pre-fault voltages of the feeder. The estimated nodal demands are obtained from the analysis of typical load curves, power factors and monthly (or bimonthly) energy consumptions of customers. The algorithm first calculates the losses and initial nodal voltages of the feeder by means of power flow software. The calculated nodal voltages are used to tune the estimated nodal demands. This is done in order to take into account the dependency of load with respect to voltage. In the next iteration, the tuned nodal demands are used to recalculate the losses and nodal voltages of the feeder,

and then the latter are used to retune the nodal demands. This procedure is repeated until a convergence condition is reached, this way the consistency among the calculated nodal demands, calculated voltages and the available measured values is guaranteed. However, it is noted that the calculated values are only approximations of the real nodal demands and voltages. In this work, in order to take into account this uncertainty, the pre-fault voltages are modelled as type-1 fuzzy numbers and as type-2 fuzzy numbers. The parameters of these fuzzy numbers are chosen on the basis of calculated pre-fault voltages ( $U_{pfk}$ ), expert knowledge and quality of service regulations, e.g., according to power quality standards, voltage levels must stay in a  $\pm 10\%$  range (or  $\pm 5\%$  in some countries) with respect to nominal voltage.

#### C. Fault current equations

The most common fault in distribution systems is the single line-to-ground fault [17], its fault current equation (1) is used in the next section to illustrate the application of the FFC concept. Nevertheless, for this analysis, other fault current equations (line-to-line fault, three-phase fault, etc) can be used as well. In (1),  $I_{fk}$  is the magnitude of the fault current for a single line-to-ground fault at point  $k$  and  $U_{pfk}$  is the corresponding pre-fault voltage. Moreover,  $(R_k^0, X_k^0)$  are the equivalent zero sequence impedances and  $(R_k^+, X_k^+)$  are the positive sequence impedances at point  $k$ . Here, an approximate equation –based on the principle of superposition– is used to take into account the pre-fault load current contribution (2). This equation is based on the assumption that load current remains constant during the fault, i.e., that pre-fault and post-fault load current are the same. Thus, the total fault current ( $I_{fk}$ ) is equal to the sum of two components:  $I_{fk}$ , which is calculated without considering load and only depends on network impedances and fault resistance, and the pre-fault load current ( $I_{pf}$ ), which depends on load characteristics. It is worth noting that  $I_{pf}$  also can be modeled as a fuzzy number. However, for simplicity and given that usually  $I_{fk} \gg I_{pf}$ , in this work  $I_{pf}$  is modeled as a crisp number.

$$I_{fk} = 3U_{pfk} / \sqrt{(2R_k^+ + R_k^0 + 3R_f)^2 + (2X_k^+ + X_k^0)^2} \quad (1)$$

$$I_k = I_{fk} + I_{pf} \quad (2)$$

For computing (1) and (2) is necessary to calculate functions of type-1 and type-2 fuzzy numbers. In the next sections, the basics of fuzzy sets and functions of fuzzy sets are presented. Then, a novel approach for calculating functions of uncertain variables is introduced. Finally, this procedure is applied to compute type-1 and type-2 FFC for a real distribution feeder. These results are compared, and the advantages of type-2 fuzzy modelling are discussed.

### III. TYPE-1 AND TYPE-2 FUZZY SETS

#### A. Type-1 fuzzy sets

A conventional type-1 fuzzy set  $A$ , defined on the universe of discourse  $X$  is characterized by a two-dimensional membership function (type-1 membership function), which is

totally crisp. This is shown in (3), where  $\mu_A(x)$  is the *grade of the membership function* (a crisp number) for a generic element  $x \in X$  and  $0 \leq \mu_A(x) \leq 1$ . Alternatively,  $A$  can be expressed by means of (4), where the symbol  $\int$  denotes union over all admissible  $x$ .

$$A = \{(x, \mu_A(x)) | \forall x \in X\} \quad (3)$$

$$A = \int_{x \in X} \mu_A(x) / x \quad (4)$$

### B. Type-2 fuzzy sets

A type-2 fuzzy set  $\tilde{A}$  is characterized by a three-dimensional membership function (type-2 membership function) which itself is fuzzy [5], [10]. This is shown in (5)-(7), where  $\mu_{\tilde{A}}(x)$  is the *secondary membership function* (a type-1 fuzzy set) for a generic element  $x \in X$ . The domain ( $J_x$ ) and amplitude ( $f_x(u)$ ) of  $\mu_{\tilde{A}}(x)$  are the *primary membership* of  $x$  and *secondary grade*, respectively.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | \forall x \in X\} \quad (5)$$

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x) / x \quad (6)$$

$$\mu_{\tilde{A}}(x) = \int_{u \in J_x} f_x(u) / u, \quad J_x \subseteq [0, 1] \quad (7)$$

The union ( $\cup$ ) of all primary memberships of  $\tilde{A}$  is a bounded region called *fingerprint of uncertainty* (FOU) (8). The upper and lower bounds of the FOU are two type-1 membership functions called the *upper and lower membership functions* of  $\tilde{A}$ , respectively (9), (10).

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \quad (8)$$

$$\bar{\mu}_{\tilde{A}}(x) = \overline{FOU(\tilde{A})} = \bigcup_{x \in X} \bar{J}_x, \quad \forall x \in X \quad (9)$$

$$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU(\tilde{A})} = \bigcup_{x \in X} \underline{J}_x, \quad \forall x \in X \quad (10)$$

### C. Interval type-2 fuzzy sets

When  $f_x(u) = 1, \forall u \in J_x \subseteq [0, 1], \forall x \in X$ , the secondary membership functions are interval sets and  $\tilde{A}$  is called an interval type-2 fuzzy set (11). Interval secondary membership functions reflect a uniform uncertainty at the primary memberships of  $x$  [10]. Due to this feature, interval type-2 fuzzy sets are the most widely used type-2 fuzzy sets to date [5], mostly in applications of Type-2 Fuzzy Logic Systems.

$$\tilde{A} = \int_{x \in X} \left[ \int_{u \in J_x} 1/u \right] / x = \int_{x \in X} \left[ \int_{u \in [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)]} 1/u \right] / x \quad (11)$$

### D. Centroid of an interval type-2 fuzzy set

The centroid of an interval type-2 fuzzy set  $\tilde{A}$ , whose domain,  $x \in X$ , is discretized into  $N$  points,  $x_1, \dots, x_N$  (12), is defined by

means of (13). The centroid of an interval type-2 fuzzy set is a type-1 interval (Fig. 1) and its defuzzified value is  $c_c = (c_l + c_r)/2$  [10]. Procedures for calculating (13) are presented in [10] and [25]. In this work, in order to avoid confusions, the term ‘‘type-reduced set’’ is used to refer to the ‘‘centroid of an interval type-2 fuzzy set’’ and the term ‘‘centroid’’ is used to refer to the ‘‘defuzzified value of a type-1 fuzzy set’’.

$$\tilde{A} = \sum_{i=1}^N \left[ \int_{u \in J_{x_i}} f_{x_i}(u) / u \right] / x_i \quad (12)$$

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x_1}} \dots \int_{\theta_N \in J_{x_N}} 1 / \frac{\sum_{i=1}^N x_i \theta_i}{\sum_{i=1}^N \theta_i} = [c_l, c_r] \quad (13)$$

The centroid provides a measure of the effect that uncertainties (associated to the grades of the membership function) have on the defuzzified crisp value.

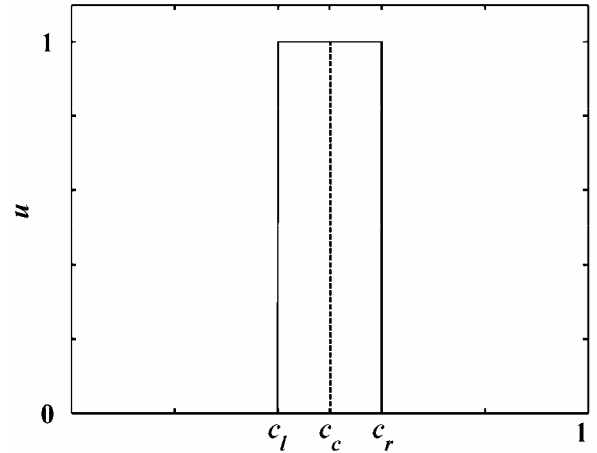


Fig. 1. Centroid of an interval type-2 fuzzy set.

### E. Type-1 and interval type-2 fuzzy numbers

A convex and normal type-1 fuzzy set that has a numerical domain is called a type-1 fuzzy number [13]. In [26], a type-2 fuzzy number is broadly defined as a type-2 fuzzy set that has a numerical domain. In [14], an interval type-2 fuzzy set is defined using the following four constraints, where  $A_\alpha = \{[a^\alpha, b^\alpha], [c^\alpha, d^\alpha]\}, \forall \alpha \in [0, 1], \forall a^\alpha, b^\alpha, c^\alpha, d^\alpha \in \mathbb{R}$  (Fig. 2):

1.  $a^\alpha \leq b^\alpha \leq c^\alpha \leq d^\alpha$
2.  $[a^\alpha, d^\alpha]$  and  $[b^\alpha, c^\alpha]$  generate a function that is convex and  $[a^\alpha, d^\alpha]$  generate a function that is normal.
3.  $\forall \alpha_1, \alpha_2 \in [0, 1]: (\alpha_2 > \alpha_1) \Rightarrow ([a^{\alpha_1}, c^{\alpha_1}] \supseteq [a^{\alpha_2}, c^{\alpha_2}], [b^{\alpha_1}, d^{\alpha_1}] \supseteq [b^{\alpha_2}, d^{\alpha_2}]), \text{ for } c^{\alpha_2} \geq b^{\alpha_2}$ .
4. If the maximum of the membership function generated by  $[b^\alpha, c^\alpha]$  is the level  $\alpha_m$ , that is,  $[b^{\alpha_m}, c^{\alpha_m}]$ , then  $[b^{\alpha_m}, c^{\alpha_m}] \subset [a^{\alpha=1}, d^{\alpha=1}]$ .

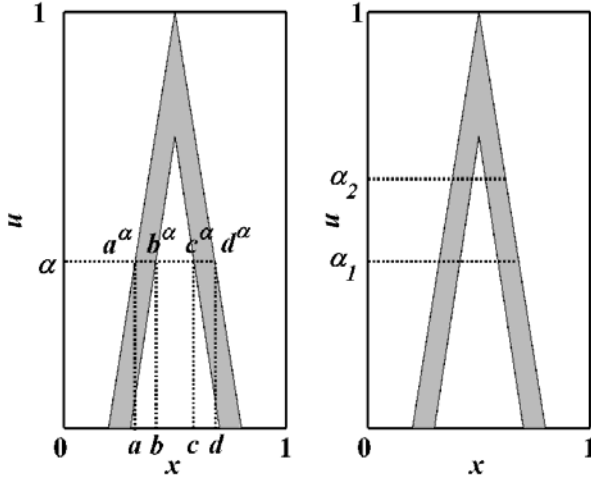


Fig. 2. Definition of an interval type-2 fuzzy number.

In this work, only interval type-2 gaussian fuzzy numbers with uncertain standard deviation ( $\sigma$ ) are used (14), (15). In addition to the aforementioned constraints, they also satisfy that both,  $\bar{\mu}_{\tilde{A}}(x)$  and  $\underline{\mu}_{\tilde{A}}(x)$ , are convex and normal. This is shown graphically in Fig. 3, where the FOU, lower and upper membership functions of an interval type-2 gaussian fuzzy number with uncertain  $\sigma$  are shown.

$$\bar{\mu}_{\tilde{A}}(x) = \exp \left[ -\frac{1}{2} \left( \frac{x-m}{\sigma_2} \right)^2 \right] \quad (14)$$

$$\underline{\mu}_{\tilde{A}}(x) = \exp \left[ -\frac{1}{2} \left( \frac{x-m}{\sigma_1} \right)^2 \right] \quad (15)$$

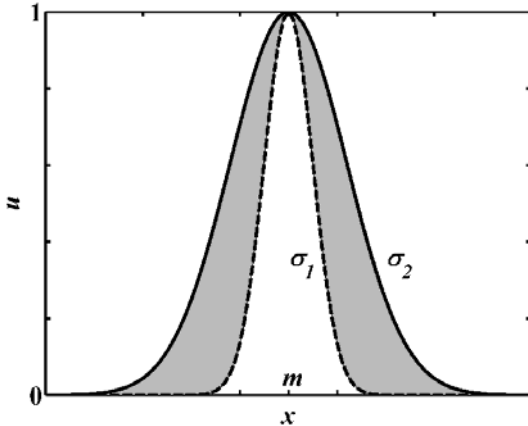


Fig. 3. FOU of an interval type-2 gaussian fuzzy number with uncertain  $\sigma$ : lower membership function (dashed line) and upper membership function (thick line).

#### IV. FUNCTIONS OF TYPE-1 AND TYPE-2 FUZZY NUMBERS

A fuzzy function can be understood in several ways according to where fuzziness occurs. Roughly there are three basic kinds of fuzzy functions: ordinary functions having fuzzy properties or satisfying fuzzy constraints, functions that just “carry” the fuzziness in their argument(s) without generating extra fuzziness themselves (the image of a non-fuzzy element is a non-fuzzy element) and ill-known functions of non-fuzzy

arguments (the image of an element is blurred by the jiggling function) [12]. The second ones are the functions of interest for this work. In this section, the calculation procedure of the image of these functions when their argument(s) are type-1 and interval type-2 fuzzy numbers is explained.

##### A. Functions of type-1 fuzzy numbers

The mathematical tool for the calculation of functions of type-1 fuzzy numbers is the extension principle [27]. The extension principle is one of the most basic concepts of fuzzy set theory and provides a method to generalize crisp mathematical concepts to fuzzy sets [6]-[15]. Let  $\mathbf{X}$  be the Cartesian product of universes  $\mathbf{X} = X_1 \times \dots \times X_n$  and let  $f$  be a mapping from  $\mathbf{X}$  to a universe  $Y$  such that  $y = f(x_1, \dots, x_n)$ . Next, let  $A_1, \dots, A_n$  be  $n$  type-1 fuzzy numbers in  $X_1, \dots, X_n$ , respectively. Then, the extension principle allows to induce from the  $n$  type-1 fuzzy sets  $A_1, \dots, A_n$  a type-1 fuzzy set  $B$  on  $Y$  (16), through  $f$ , by means of (17), i.e.,  $B = f(A_1, \dots, A_n)$ .

$$B = \left\{ (y, \mu_B(y)) \mid y = f(x_1, \dots, x_n), (x_1, \dots, x_n) \in X \right\} \quad (16)$$

$$\mu_B(y) = \bigvee_{y=f(x_1, \dots, x_n)} \left\{ \mu_{A_1}(x_1) \star \dots \star \mu_{A_n}(x_n) \right\} \quad (17)$$

Here, the maximum t-conorm  $\bigvee$ , and a general t-norm  $\star$  (minimum or product) are used [10]. To implement (17), first  $f(x_1, \dots, x_n)$  and  $\left\{ \mu_{A_1}(x_1) \star \dots \star \mu_{A_n}(x_n) \right\}$ ,  $\forall x_1 \in X_1, \dots, \forall x_n \in X_n$  are computed. If more than one set of values  $x_1, \dots, x_n$  satisfy  $y = f(x_1, \dots, x_n)$ , then this procedure is repeated for all of them and the largest of minima or the largest of products is chosen as  $\mu_B(y)$ .

##### B. Functions of type-2 fuzzy numbers

When an operation of the form  $f(x_1, \dots, x_n)$  is extended to  $f(\tilde{A}_1, \dots, \tilde{A}_n)$ , where  $\tilde{A}_i$  are type-2 fuzzy numbers, the binary operations included in (17) must be extended in order to handle type-1 fuzzy sets instead of crisp numbers as grades of membership. The extensions of the t-conorm and t-norm operations to type-1 fuzzy sets are denoted as *join* (18) and *meet* (19), respectively [10], [28], [29].

$$F_1 \sqcup F_2 = \int_v \int_w [f_1(v) \star f_2(w)] / (v \vee w) \quad (18)$$

$$F_1 \sqcap F_2 = \int_v \int_w [f_1(v) \star f_2(w)] / (v \wedge w) \quad (19)$$

Therefore, to calculate  $\tilde{B} = f(\tilde{A}_1, \dots, \tilde{A}_n)$ , first  $f(x_1, \dots, x_n)$  and  $\mu_{\tilde{A}_1}(x_1) \sqcap \dots \sqcap \mu_{\tilde{A}_n}(x_n)$ ,  $\forall x_1 \in X_1, \dots, \forall x_n \in X_n$  are computed. If more than one set of  $x_1, \dots, x_n$  satisfy  $y = f(x_1, \dots, x_n)$ , then this procedure is repeated for all of them and their *join* is chosen as  $\mu_{\tilde{B}}(y)$ .

##### C. Functions of interval type-2 fuzzy numbers

To compute  $\tilde{B} = f(\tilde{A}_1, \dots, \tilde{A}_n)$ , when  $\tilde{A}_i$  are interval type-2 fuzzy numbers, is not necessary to take into account the

interval secondary grades. Only calculations with  $\bar{\mu}_{\tilde{A}_i}(x_i)$  and  $\underline{\mu}_{\tilde{A}_i}(x_i)$  are performed and then the interval secondary grades  $(f_y(u) = 1, \forall u \in J_y \subseteq [0, 1], \forall y \in Y)$  are appended to  $\mu_{\tilde{B}}(y)$ .

$\tilde{B}$  is also an interval type-2 fuzzy set (20), whose FOU can be calculated by means of (21) and (22).

$$\tilde{B} = \left\{ (y, \mu_{\tilde{B}}(y)) \mid y = f(x_1, \dots, x_n), (x_1, \dots, x_n) \in X \right\} \quad (20)$$

$$\bar{\mu}_{\tilde{B}}(y) = \bigvee_{y=f(x_1, \dots, x_n)} \left\{ \bar{\mu}_{\tilde{A}_1}(x_1) \star \dots \star \bar{\mu}_{\tilde{A}_n}(x_n) \right\} \quad (21)$$

$$\underline{\mu}_{\tilde{B}}(y) = \bigvee_{y=f(x_1, \dots, x_n)} \left\{ \underline{\mu}_{\tilde{A}_1}(x_1) \star \dots \star \underline{\mu}_{\tilde{A}_n}(x_n) \right\} \quad (22)$$

It is noted that (21) and (22) are equivalent to apply the extension principle, as stated in (17), to  $\bar{\mu}_{\tilde{A}_i}(x_i)$  and  $\underline{\mu}_{\tilde{A}_i}(x_i)$ , respectively [30]. Therefore, these calculations may be performed using the procedures proposed in the literature, e.g., the transformation method [1], the Fuzzy Weighted Averages (FWA) method [6] or the vertex method [7]. The latter one is used in this work due to its simplicity, its basic theory is explained next.

#### D. The vertex method

The vertex method is based on the combination of the  $\alpha$ -cut concept and standard interval analysis. According to the resolution principle [13], [15] any continuous type-1 membership function can be represented by a continuous sweep of  $\alpha$ -cut intervals from  $\alpha = 0^+$  to  $\alpha = 1$ , where  $I_\alpha = [a, b]$  is a generic  $\alpha$ -cut as the one shown in Fig. 4.

Suppose that a many-to-one mapping  $y = f(x_1, \dots, x_n)$  has to be extended for type-1 fuzzy sets  $B = f(A_1, \dots, A_n)$ , then the input space can be represented by an  $n$ -dimensional Cartesian region. Each of the input variables can be described by an interval ( $I_{i\alpha}$ ) at a specific  $\alpha$ -cut (23).

$$I_{i\alpha} = [a_i, b_i] \quad i = 1, \dots, n \quad (23)$$

The endpoint pairs of each interval intersect in the  $n$ -dimensional space and form the vertices of the  $n$ -dimensional Cartesian space. The coordinates of these vertices are the values used in the vertex method to calculate the output interval for each  $\alpha$ -cut. When the mapping  $y = f(x_1, \dots, x_n)$  is continuous in the  $n$ -dimensional Cartesian region and also there are no extreme points (maxima or minima of  $y$ ) in this region or along the boundaries, the value of the interval function for a particular  $\alpha$ -cut can be obtained by (24).

$$B_\alpha = f(I_{1\alpha}, \dots, I_{n\alpha}) = \left[ \min_j (f(c_j)), \max_j (f(c_j)) \right] \quad (24)$$

Where  $N = 2^n$  is the number of vertices ( $c_j$ ) of the  $n$ -dimensional Cartesian region. Each vertex is defined by the values of  $a_i$  and  $b_i$ . When extreme points (maxima or minima of  $y$ ) exist in the  $n$ -dimensional Cartesian region, they are identified and considered as additional vertices ( $E_k$ ). Thus,

when the mapping  $y = f(x_1, \dots, x_n)$  is continuous in the  $n$ -dimensional Cartesian region, the value of the interval function for a particular  $\alpha$ -cut can be calculated by (25).

$$B_\alpha = f(I_{1\alpha}, \dots, I_{n\alpha}) = \left[ \min_{j,k} (f(c_j), f(E_k)), \max_{j,k} (f(c_j), f(E_k)) \right] \quad (25)$$

Where,  $c_j$  ( $j = 1, \dots, N$ ) is the coordinate of the  $j$ -th vertex representing the  $n$ -dimensional Cartesian region and  $E_k$  ( $k = 1, \dots, m$ ) is the coordinate of the  $k$ -th extreme point of  $y = f(x_1, \dots, x_n)$  identified in the  $n$ -dimensional Cartesian region (for  $m$  extreme points). In the next section, the vertex method is used to implement the extension principle. This way (21) and (22) are calculated and the FFC concept is introduced.

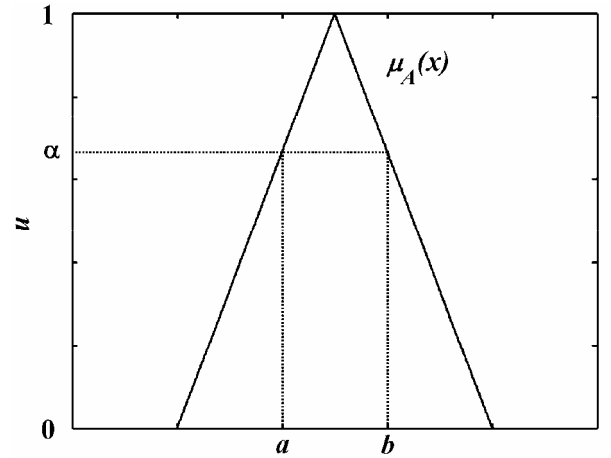


Fig. 4. A generic  $\alpha$ -cut  $I_\alpha = [a, b]$ .

## V. RESULTS

The FFC approach has been tested on a real distribution feeder of the city of San Juan, Argentina (Fig. 5). This feeder has a nominal voltage level of 13.2 kV, 172 nodes and 55 protective devices. The type-1 and type-2 FFC of  $n_1$ - $n_4$  are calculated with the parameters of Tables I-II, the pre-fault voltages ( $U_{pjk}$ ) and pre-fault load current ( $I_{pf} = 53.8$  A) are computed using the algorithm proposed in [24].

TABLE I  
TEST FEEDER PARAMETERS

$n_k$	$U_{pjk}$ (V)	$R_k^0$ ( $\Omega$ )	$X_k^0$ ( $\Omega$ )	$R_k^+$ ( $\Omega$ )	$X_k^+$ ( $\Omega$ )
$n_1$	7505.9	1.872	4.229	1.557	0.953
$n_2$	7486.1	5.226	10.895	4.392	2.470
$n_3$	7449.6	6.424	12.707	5.938	2.765
$n_4$	7437.4	13.153	23.533	12.836	5.020

TABLE II  
FAULT IMPEDANCE AND PRE-FAULT VOLTAGE PARAMETERS

Variable	Type-1		Type-2		
	$m$	$\sigma$	$m$	$\sigma_1$	$\sigma_2$
$R_f$	15 $\Omega$	4 $\Omega$	15 $\Omega$	3 $\Omega$	5 $\Omega$
$U_{pjk}$	$U_{pjk}$	$0.04 \cdot U_{pjk}$	$U_{pjk}$	$0.02 \cdot U_{pjk}$	$0.06 \cdot U_{pjk}$

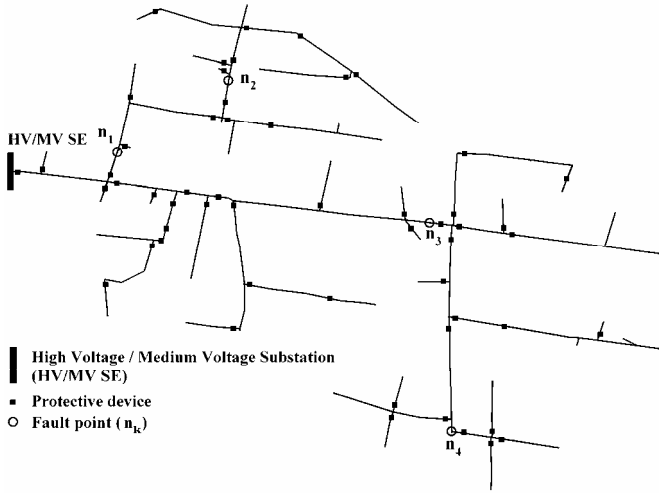


Fig. 5. Test feeder.

### A. Type-1 Fuzzy Fault Currents

In order to show the advantages of the proposed approach, first type-1 FFC are calculated using the extension principle (16), (17) and the vertex method (25), then these results are compared with the ones obtained using type-2 FFC and conclusions are drawn.

In Fig. 6, the type-1 membership functions with solid lines (Case 1) have been calculated by modeling  $R_f$  as a type-1 gaussian fuzzy number and  $U_{pfk}$  as a crisp number. The type-1 membership functions with dashed lines (Case 2) have been calculated by modeling both variables as type-1 gaussian fuzzy numbers. It is noted that in Case 2 the FFC are slightly more uncertain than in Case 1, this is expected so, because the calculation have been made using two uncertain variables. Moreover, the effect of the uncertainties associated to  $R_f$  and  $U_{pfk}$  becomes evident for points located close to the HV/MV SE, where the membership functions are broader than the ones corresponding to points located far. This is because the influence of  $R_f$  is greater in these sites than in points located far, where the feeder impedance is also significant.

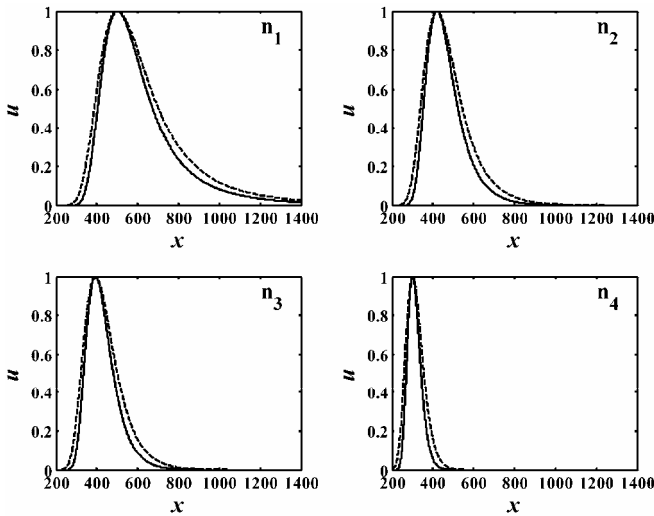


Fig. 6. Type-1 FFC for Case1 (solid lines) and Case 2 (dashed lines).

The effect of modeling  $R_f$  and  $U_{pfk}$  as type-1 fuzzy numbers can be noted in Table III, where the defuzzified values of the type-1 FFC of Case 1 ( $I_{k \text{ Case } 1}$ ) and Case 2 ( $I_{k \text{ Case } 2}$ ) are compared with the results obtained through conventional deterministic (crisp) calculations ( $I_{k \text{ crisp}}$ ). These defuzzified values have been obtained through type-1 centroid defuzzification. In Table III it is noted that  $I_{k \text{ Case } 1}$  and  $I_{k \text{ Case } 2}$  differ from  $I_{k \text{ crisp}}$ . This effect is caused by the uncertainties associated to  $R_f$  and  $U_{pfk}$ . Also it is noted that the more uncertainties are taken into account, the more different the defuzzified value of  $I_k$  with respect to its crisp alternative.

TABLE III  
CRISP FAULT CURRENT – DEFUZZIFIED VALUES OF TYPE-1 FFC

$n_k$	$I_k$		
	$I_{k \text{ crisp}}$	$I_{k \text{ Case } 1}$	$I_{k \text{ Case } 2}$
$n_1$	506.34	536.24	541.21
$n_2$	435.19	452.02	455.38
$n_3$	406.49	419.74	422.60
$n_4$	316.70	321.90	323.42

### B. Type-2 Fuzzy Fault Currents

In Fig. 7, the FOU with dark shading (Case 3) have been calculated by modeling  $R_f$  as an interval type-2 gaussian fuzzy number and  $U_{pfk}$  as a crisp number. The FOU with light shading (Case 4) have been calculated by modeling both variables as interval type-2 gaussian fuzzy numbers. These results show that fault location also affects the FOU of type-2 FFC, which are broader for points located closer to the HV/MV SE than for points located far. Moreover, the FOU of Case 4 are more uncertain than the FOU of Case 3, this effect is a consequence of modeling the uncertainties of both variables (Case 4) instead of considering only one (Case 3). Similar conclusions can be drawn by analyzing Fig. 8, where the type-reduced sets of the type-2 FFC of Fig. 7 are shown. The type-reduced sets are a measure of the effect that input variables uncertainties have on the output variable ( $I_k$ ). These type-1 intervals may be used for protection coordination, instead of using the values derived from conventional deterministic and type-1 fuzzy calculations. This way, the uncertainties associated to  $R_f$  and  $U_{pfk}$  are taken into account.

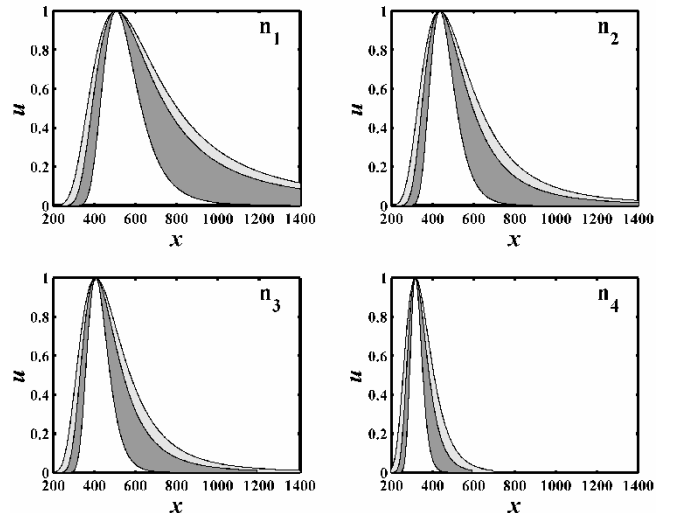


Fig. 7. Type-2 FFC for Case 3 (dark FOU) and Case 4 (light FOU).

In Fig. 8, it is noted that the type-reduced sets ( $[I_{kl}, I_{kr}]$ ) of Case 4 are more uncertain than the type-reduced sets of Case 3. Furthermore, when the domains of the type-reduced sets are analyzed, the effect of the uncertainties associated to  $R_f$  and  $U_{p/fk}$  becomes evident. The domains are wide for points located close to the HV/MV SE and are narrow for points located far. Both conclusions have been obtained by analyzing Fig. 7, but, they turn out to be clear by inspecting Fig. 8.

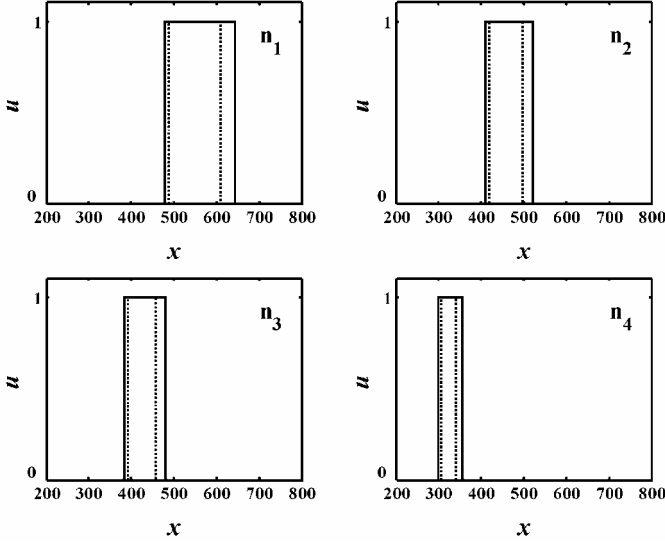


Fig. 8. Type-reduced sets of the type-2 FFC of Fig. 7: Case 3 (dashed line), Case 4 (solid line).

### C. Comparison of crisp, type-1 and type-2 Fuzzy Fault Currents

Finally, in Table IV and Fig. 9 a comparison among the values of  $I_k$  obtained through deterministic (crisp), type-1 and type-2 fuzzy approaches is presented.

TABLE IV  
CRISP FAULT CURRENT – DEFUZZIFIED VALUES OF TYPE-1 AND TYPE-2 FFC

$n_k$	$I_k$		
	$I_{k \text{ crisp}}$	$I_{k \text{ type 1 (Case 1)}}$	$I_{k \text{ type 2 (Case 3)}}$
$n_1$	<b>506.34</b>	<b>536.24</b>	<b>548.76</b>
$n_2$	<b>435.19</b>	<b>452.02</b>	<b>458.24</b>
$n_3$	<b>406.49</b>	<b>419.74</b>	<b>424.52</b>
$n_4$	<b>316.70</b>	<b>321.90</b>	<b>323.64</b>

The values of the fourth column of Table IV are the mean of the type-reduced set of the type-2 FFC, i.e., the defuzzified values of  $I_k$  for Case 3 (26).

$$I_k = \frac{I_{kl} + I_{kr}}{2} \quad (26)$$

In Fig. 9, the dominion of the type-reduced set covers both, the crisp value of  $I_k$  obtained through conventional deterministic calculations and also the defuzzified value of  $I_k$  obtained through type-1 FFC calculations. From these results it can be concluded that in type-2 FFC calculations, the uncertainties of input variables flow along the computations and their effect on the defuzzified output is measured by the

type-reduced set. This can not be achieved through conventional deterministic or type-1 fuzzy approaches. The type-reduced set provides a measure of the uncertainties of the defuzzified output; in the same way that standard deviation provides a measure of the uncertainties of the mean in probability-based modeling.

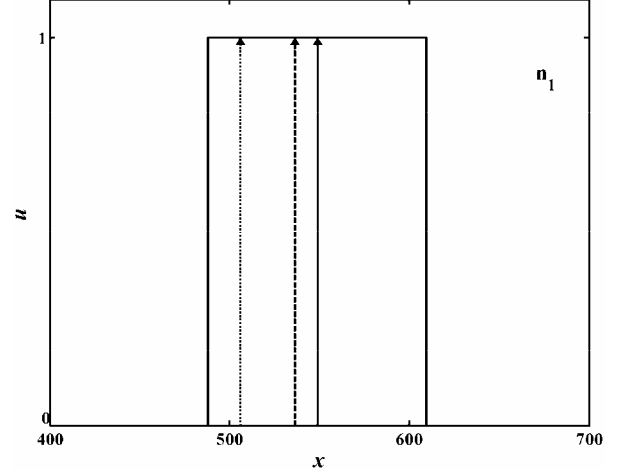


Fig. 9. Comparison of the defuzzified values of  $I_k$  for Case 1 (dashed line) and Case 3 (solid line) with respect to the crisp value of  $I_k$  obtained through conventional deterministic calculations (dotted line).

### D. Decision making with Fuzzy Fault Currents

Regarding decision making, the results obtained by means of the FFC methodology supply more information than those attained through conventional deterministic calculations and provide a more realistic modeling of the system under analysis. For decision making, two approaches can be used. If only the most possible value for a fault current is required, then the defuzzified value of a type-1 FFC, or the defuzzified value and type-reduced set of a type-2 FFC must be used. The selection of the FFC modeling (type-1 or type-2) will depend on the certainty level of the available information.

If an interval of possible fault currents is needed, then the possibility level can be selected by using the  $\alpha$ -cut concept of Figure 4. By applying this concept to Figures 6 and 7 it is noted that the interval of possible fault currents widens as the value of  $\alpha$  decreases, and vice versa. For instance, the interval for  $\alpha = 0.6$  is wider than the one corresponding to  $\alpha = 0.8$ . The  $\alpha$ -cut concept has several applications and the value of  $\alpha$  will depend on the “level of certainty” the user wants to achieve. For instance, by using an interval of possible fault currents, a designer can coordinate protective devices for clearing several fault conditions, instead of verifying coordination only for minimum or maximum fault impedance values.

An important advantage of the FFC modeling is that a single type-1 or type-2 FFC calculation provides more realistic values for decision making than a single deterministic computation, because all uncertainties are taken into account during the fuzzy calculation procedure. Similar results can be obtained through conventional modeling; however, several deterministic computations must be performed in order to take into account all possible combinations of values for all variables.

## VI. CONCLUSIONS

In this work, in order to take into account the uncertainties associated to conventional fault current calculation, an approach aiming at computing functions of interval type-2 fuzzy numbers has been presented. This approach sets a methodological framework to calculate with uncertain variables, allowing the introduction of the FFC concept, which is used to model the uncertain data and expert knowledge typically available in distribution systems. Interval type-2 fuzzy sets can be interpreted as a collection of an uncountable number of embedded type-1 fuzzy sets. Therefore, the type-reduced set of an interval type-2 fuzzy set can be thought as the union of the defuzzified outputs of all their embedded type-1 fuzzy sets. Through this important feature, the FFC calculation takes into account the uncertainties associated to  $R_f$  and  $U_{pf}$ , obtaining a range (instead of a crisp number) of possible values of fault currents ( $I_k$ ). These results provide a more significant assessment for further applications (e.g., coordination of protective devices), than the results obtained by means of conventional deterministic methodologies or type-1 fuzzy approaches. Moreover, interval type-2 fuzzy numbers have the advantage to be general enough to facilitate the modeling of the available approximate knowledge. The results obtained by applying the proposed approach to a real distribution system are satisfactory and are encouraging for future engineering applications of type-2 fuzzy sets.

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