Calculating Functions of Interval Type-2 Fuzzy Numbers for Fault Current Analysis

Julio Romero Agüero, *Member, IEEE*, and Alberto Vargas, *Senior Member, IEEE*

***Abstract*— In this work, functions of type-2 fuzzy numbers are analyzed. For the special case of interval type-2 fuzzy numbers, the type-2 membership function of the output variable is calculated using the lower and upper membership functions of the input variables and the vertex method. This procedure is used in an application where the type-2 fuzzy fault currents of an electric distribution system are calculated. The results are shown and the advantages of the approach are discussed.**

***Index Terms*— Interval type-2 fuzzy sets, extension principle.**

I. Introduction

In many engineering problems, the calculation of reliable solutions depends on the availability of exact values for the variables of model equations. However, in daily practice, these precise values can not be obtained because the existing information usually is incomplete, imprecise, noisy, vague, qualitative or linguistic. Therefore, in this case the results obtained by deterministic approaches, which uses one specific crisp value as the most likely value for a variable, cannot be considered to be representative for the whole spectrum of possible results [1]. Therefore, it is necessary to introduce uncertain variables for modeling the available information and to implement procedures for calculating functions of these variables. To solve this problem, a common practice is to model the uncertain variables as type-1 fuzzy numbers, with their shapes derived from experimental data and/or expert knowledge [2], and to implement extension principle-based methods for evaluating the corresponding functions. Thus, the type-1 fuzzy results reflect the influence of the input uncertainties on the output function variables. If the type-1 fuzzy results are defuzzified, the most likely crisp values for the output variables can be obtained. The defuzzified values generally differ from the crisp results obtained by means of deterministic methods, since in the type-1 fuzzy approach the uncertainties have been included and processed through the calculation procedure.

In the type-1 fuzzy modeling, the values used in developing the membership functions of type-1 fuzzy numbers are often overly precise, because they require that each element of the universe where the type-1 fuzzy number is defined be assigned a specific value of membership [3]. When the level of information is not adequate to specify membership functions with this precision, then it is necessary to use type-2 fuzzy numbers to represent the uncertainties of model variables. Examples of this situation have been identified in [4], [5], e.g., when a measurement is corrupted with non-stationary noise and the mathematical description of nonstationarity is unknown; when features in a pattern recognition application have statistical attributes that are non-stationary and the mathematical descriptions of the nonstationarities are unknown; when membership values are extracted from a group of experts using questionnaires, etc. Nevertheless, it is noted that for the problem under analysis, the type-2 fuzzy modeling of uncertain variables implies calculating functions of type-2 fuzzy numbers.

The computation of functions of type-1 fuzzy numbers is a well-known field of fuzzy set theory and has been documented in several publications [6]-[15]. However, less coverage has been given to the calculation of functions of type-2 fuzzy numbers [10], [14], [16]. In this work, type-2 fuzzy numbers are used to model the uncertainties associated to the most influential variables of conventional fault current calculation of electrical distribution systems [17], [18]. This modeling allows the introduction of the Fuzzy Fault Currents (FCC) concept, which is used in an application example.

The structure of this work is as follows. First in Section II, the generalities of type-2 fuzzy sets are explained. In Section III, the calculation of functions of type-2 fuzzy sets is analyzed. In Section IV, the FFC concept is described and in Section V it is applied to a real electric distribution system. Finally in Section VI, conclusions are presented.

II. Type-1 and Type-2 Fuzzy Sets

A. Type-1 fuzzy sets

A conventional type-1 fuzzy set A, defined on the universe of discourse X is characterized by a two-dimensional membership function (type-1 membership function), which is totally crisp. This is shown in (1), where $\mu_A(x)$ is the *grade of the membership function* (a crisp number) for a generic element $x \in X$ and $0 \le \mu_A(x) \le 1$. Alternatively, A can be expressed by means of (2), where the symbol $\int$ denotes union over all admissible x.

$$A = \left\{ \left(x, \mu_A(x) \right) \middle| \forall x \in X \right\} \tag{1}$$

$$A = \int_{x \in X} \mu_A(x) / x \tag{2}$$

This work was supported in part by the German Academic Exchange Service (Deutscher Akademischer Austauschdienst / DAAD).

J. Romero Agüero and A. Vargas are with Instituto de Energía Eléctrica, Universidad Nacional de San Juan, Av. Libertador San Martín 1109 Oeste, San Juan, Argentina (e-mail: jera@ieee.org; avargas@iee.unsj.edu.ar).

B. Type-2 fuzzy sets

A type-2 fuzzy set $\tilde{A}$ is characterized by a three-dimensional membership function (type-2 membership function) which itself is fuzzy [5], [10]. This is shown in (3), where $0 \le \mu_{\tilde{A}}(x,u) \le 1$. Alternatively, $\tilde{A}$ can be expressed by means of (4), where the symbol $\iint$ denotes union over all admissible *x* and *u*.

$$\tilde{A} = \left\{ \left((x,u), \mu_{\tilde{A}}(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\} \quad (3)$$

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u) / (x,u) \quad (4)$$

A very convenient notation of type-2 fuzzy sets is the *vertical slice* representation. This is shown in (5)-(7), where $\mu_{\tilde{A}}(x)$ is the *secondary membership function* (a type-1 fuzzy set) for a generic element $x \in X$. The domain (J_x) and amplitude ($f_x(u)$) of $\mu_{\tilde{A}}(x)$ are the *primary membership* of *x* and *secondary grade*, respectively.

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \middle| \forall x \in X \right\} \quad (5)$$

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x) / x \quad (6)$$

$$\mu_{\tilde{A}}(x) = \int_{u \in J_x} f_x(u) / u \ , \quad J_x \subseteq [0,1] \quad (7)$$

The union $(\bigcup)$ of all primary memberships of $\tilde{A}$ is a bounded region called *fingerprint of uncertainty* (FOU) (8). The upper and lower bounds of the FOU are two type-1 membership functions called the *upper and lower membership functions* of $\tilde{A}$, respectively (9), (10).

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \quad (8)$$

$$\bar{\mu}_{\tilde{A}}(x) = \overline{FOU(\tilde{A})} = \bigcup_{x \in X} \overline{J_x} \ , \quad \forall x \in X \quad (9)$$

$$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU(\tilde{A})} = \bigcup_{x \in X} \underline{J_x} \ , \quad \forall x \in X \quad (10)$$

If *X* and J_x are both discrete, either by problem formulation or by discretization of continuous universes of discourse, then $\tilde{A}$ can be expressed by means of (11), where the symbols Σ and + also denote union over all admissible *x* and *u*.

$$\tilde{A} = \sum_{x \in X} \left[\sum_{u \in J_x} f_x(u) / u \right] \Big/ x = \sum_{i=1}^{N} \left[\sum_{u \in J_{x_i}} f_{x_i}(u) / u \right] \Big/ x_i$$
$$= \left[\sum_{k=1}^{M_1} f_{x_1}(u_{1k}) / u_{1k} \right] \Big/ x_1 + \cdots + \left[\sum_{k=1}^{M_N} f_{x_N}(u_{Nk}) / u_{Nk} \right] \Big/ x_N \quad (11)$$

These last expressions allow the introduction of the useful concepts of embedded type-2 and embedded type-1 fuzzy sets. For discrete universes of discourse *X* and *U*, an *embedded type-2 fuzzy set* $\tilde{A}_e$ has *N* elements, where $\tilde{A}_e$ contains exactly one element from $J_{x_1}, J_{x_2}, \ldots, J_{x_N}$, each with its associated secondary grade (12). The union of all the primary memberships of $\tilde{A}_e$ is called an *embedded type-1 fuzzy set* A_e (13). Set $\tilde{A}_e$ is embedded in $\tilde{A}$, there are a total of $\prod_{i=1}^{N} M_i$ embedded type-2 and embedded type-1 fuzzy sets in $\tilde{A}$.

$$\tilde{A}_e = \sum_{i=1}^{N} \left[f_{x_i}(u_i) / u_i \right] \Big/ x_i \ , \quad u_i \in J_{x_i} \subseteq U = [0,1] \quad (12)$$

$$A_e = \sum_{i=1}^{N} u_i / x_i \ , \quad u_i \in J_{x_i} \subseteq U = [0,1] \quad (13)$$

Using these last definitions, the *Representation Theorem* is defined [5]. Let $\tilde{A}_e^j$ denote the *j*-th embedded type-2 fuzzy set for the type-2 fuzzy set $\tilde{A}$ (14), then $\tilde{A}$ can be represented as the union of its embedded type-2 fuzzy sets (15). This is called *wavy slice* representation of $\tilde{A}$.

$$\tilde{A}_e^j \equiv \left\{ \left(u_i^j, f_{x_i}(u_i^j) \right), i = 1, \ldots, N \right\}, \quad u_i^j \in \left\{ u_{ik}, k = 1, \ldots, M_i \right\} \quad (14)$$

$$\tilde{A} = \sum_{j=1}^{n} \tilde{A}_e^j \ , \quad n \equiv \prod_{i=1}^{N} M_i \quad (15)$$

C. Interval type-2 fuzzy sets

When $f_x(u) = 1, \forall u \in J_x \subseteq [0,1], \forall x \in X$, the secondary membership functions are interval sets and $\tilde{A}$ is called an interval type-2 fuzzy set (16). Interval secondary membership functions reflect a uniform uncertainty at the primary memberships of *x* [10]. Due to this feature, interval type-2 fuzzy sets are the most widely used type-2 fuzzy sets to date [5], mostly in applications of Type-2 Fuzzy Logic Systems.

$$\tilde{A} = \int_{x \in X} \left[\int_{u \in J_x} 1/u \right] \Big/ x = \int_{x \in X} \left[\int_{u \in \left[\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x) \right]} 1/u \right] \Big/ x \quad (16)$$

D. Centroid of an interval type-2 fuzzy set

The centroid of an interval type-2 fuzzy set $\tilde{A}$, whose domain, $x \in X$, is discretized into *N* points, $x_1, \ldots, x_N$ (17), is defined by means of (18). The centroid of an interval type-2 fuzzy set is a type-1 interval (Fig. 1) and its defuzzified value is $c_c = (c_l + c_r)/2$ [10]. Every combination of $\theta_1, \ldots, \theta_N$ and its associated secondary grade (which is equal to 1) forms an embedded type-2 fuzzy set $\tilde{A}_e^j$ and its corresponding embedded type-1 fuzzy set A_e^j. Therefore, the centroid of an interval type-2 fuzzy set $\tilde{A}$ is equivalent to the union of the centroids of all its embedded type-1 fuzzy sets A_e^j. Procedures for calculating (18) are presented in [10] and [19]. The centroid of an interval type-2 fuzzy set provides a measure of the effect that uncertainties (associated to the grades of the membership function) have on the defuzzified crisp value, this is shown in Fig. 1.

$$\tilde{A} = \sum_{i=1}^{N} \left[\int_{u \in J_{x_i}} f_{x_i}(u)/u \right] \Big/ x_i \tag{17}$$

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x1}} \cdots \int_{\theta_N \in J_{xN}} 1 \Bigg/ \frac{\sum_{i=1}^{N} x_i \theta_i}{\sum_{i=1}^{N} \theta_i} = [c_l, c_r] \tag{18}$$

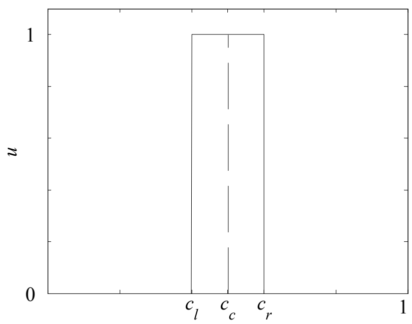

Fig. 1. Centroid of an interval type-2 fuzzy set.

In this work, in order to avoid confusions, the term “type-reduced set” is used to refer to the “centroid of an interval type-2 fuzzy set” and the term “centroid” is used to refer to the “defuzzified value of a type-1 fuzzy set”.

E. Type-1 and interval type-2 fuzzy numbers

A convex and normal type-1 fuzzy set that has a numerical domain is called a type-1 fuzzy number [13]. In [20], a type-2 fuzzy number is broadly defined as a type-2 fuzzy set that has a numerical domain. In [14], an interval type-2 fuzzy set is defined using the following four constraints, where $A_\alpha = \{[a^\alpha,b^\alpha],[c^\alpha,d^\alpha]\}$, $\forall \alpha \in [0,1]$, $\forall a^\alpha,b^\alpha,c^\alpha,d^\alpha \in \mathrm{R}$ (Fig. 2):

1. $a^\alpha \le b^\alpha \le c^\alpha \le d^\alpha$
2. $[a^\alpha,d^\alpha]$ and $[b^\alpha,c^\alpha]$ generate a function that is convex and $[a^\alpha,d^\alpha]$ generate a function that is normal.
3. $\forall \alpha_1, \alpha_2 \in [0,1] : (\alpha_2 > \alpha_1) \Rightarrow \left(\left[a^{\alpha_1}, c^{\alpha_1}\right] \supset \left[a^{\alpha_2}, c^{\alpha_2}\right], \right.$ $\left. \left[b^{\alpha_1}, d^{\alpha_1}\right] \supset \left[b^{\alpha_2}, d^{\alpha_2}\right] \right)$, for $c^{\alpha_2} \ge b^{\alpha_2}$.
4. If the maximum of the membership function generated by $[b^\alpha,c^\alpha]$ is the level α_m, that is, $\left[b^{\alpha_m}, c^{\alpha_m}\right]$, then $\left[b^{\alpha_m}, c^{\alpha_m}\right] \subset \left[a^{\alpha=1}, d^{\alpha=1}\right]$.

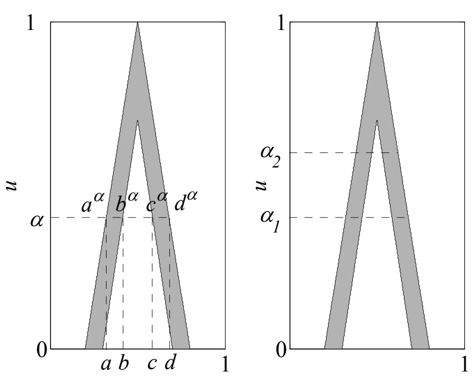

Fig. 2. Definition of an interval type-2 fuzzy number.

In this work, for the FFC calculations, only interval type-2 gaussian fuzzy numbers with uncertain standard deviation (σ) are used (19), (20). In addition to the aforementioned constraints, they also satisfy that both, $\bar{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$, are convex and normal. This is shown graphically in Fig. 3 and Fig. 4, where the type-2 membership function, as well as the FOU, lower and upper membership functions of an interval type-2 gaussian fuzzy number with uncertain σ are shown.

$$\bar{\mu}_{\tilde{A}}(x) = \exp\left[-\frac{1}{2} \left(\frac{x-m}{\sigma_2} \right)^2 \right] \tag{19}$$

$$\underline{\mu}_{\tilde{A}}(x) = \exp\left[-\frac{1}{2} \left(\frac{x-m}{\sigma_1} \right)^2 \right] \tag{20}$$

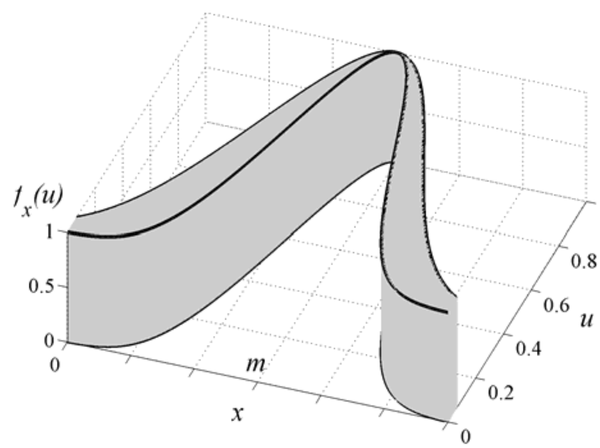

Fig. 3. Type-2 membership function of an interval type-2 gaussian fuzzy number with uncertain σ.

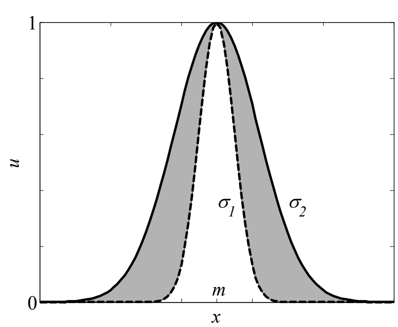

Fig. 4. FOU, lower membership function (dashed line) and upper membership function (thick line) of interval type-2 gaussian fuzzy number with uncertain σ

III. Functions of Type-1 and Type-2 Fuzzy Numbers

A fuzzy function can be understood in several ways according to where fuzziness occurs. Roughly there are three basic kinds of fuzzy functions: ordinary functions having fuzzy properties or satisfying fuzzy constraints, functions that just “carry” the fuzziness in their argument(s) without generating extra fuzziness themselves (the image of a non-fuzzy element is a non-fuzzy element) and ill-known functions of non-fuzzy arguments (the image of an element is blurred by the jiggling function) [12]. The second ones are the functions of interest for this work. In this section, the calculation procedure of the image of these functions when their argument(s) are type-1 and interval type-2 fuzzy numbers is explained.

A. Functions of type-1 fuzzy numbers

The mathematical tool for the calculation of functions of type-1 fuzzy numbers is the extension principle [21]. The extension principle is one of the most basic concepts of fuzzy set theory and provides a method to generalize crisp mathematical concepts to fuzzy sets [6]-[15]. Let $\mathbf{X}$ be the Cartesian product of universes $\mathbf{X} = X_1 \times \cdots \times X_n$ and let f be a mapping from $\mathbf{X}$ to a universe Y such that $y = f(x_1,\ldots, x_n)$. Next, let $A_1,\ldots, A_n$ be n type-1 fuzzy numbers in $X_1,\ldots, X_n$, respectively. Then, the extension principle allows to induce from the n type-1 fuzzy sets $A_1,\ldots, A_n$ a type-1 fuzzy set B on Y (21), through f, i.e., $B = f(A_1,\ldots, A_n)$. This is carried out by means of (22).

$$B = \left\{ \left(y, \mu_B(y) \right) \middle| y = f(x_1,\ldots,x_n), (x_1,\ldots,x_n) \in X \right\} \tag{21}$$

$$\mu_B(y) = \bigvee_{y=f(x_1,\ldots,x_n)} \left\{ \mu_{A_1}(x_1) \bigstar \cdots \bigstar \mu_{A_n}(x_n) \right\} \tag{22}$$

Here, maximum t-conorm and a general t-norm (minimum or product) are used [10]. To implement (22), first $f(x_1,\ldots,x_n)$ and $\left\{ \mu_{A_1}(x_1) \bigstar \cdots \bigstar \mu_{A_r}(x_n) \right\}$, $\forall\, x_1 \in X_1,\ldots, \forall\, x_n \in X_n$ are computed. If more than one set of values $x_1,\ldots, x_n$ satisfy $y = f(x_1,\ldots, x_n)$, then this procedure is repeated for all of them and the largest of minima or the largest of products is chosen as $\mu_B(y)$.

B. Functions of type-2 fuzzy numbers

When an operation of the form $f(x_1,\ldots, x_n)$ is extended to $f(\tilde{A}_1,\ldots,\tilde{A}_n)$, where $\tilde{A}_i$ are type-2 fuzzy numbers, the binary operations included in (22) must be extended in order to handle *type-1 fuzzy sets* instead of *crisp numbers* as *grades of membership*. A binary operation $(*)$ defined for *crisp numbers*, can be extended to *two type-1 fuzzy sets*, $F_1 = \int_v f_1(v)/v$ and $F_2 = \int_v f_2(w)/w$, by means of (23) [22].

$$F_1 * F_2 = \int_v \int_w \left[f_1(v) \bigstar f_2(w) \right] / (v * w) \tag{23}$$

Using (23), the extensions of the t-conorm and t-norm operations to type-1 fuzzy sets are defined. These operations are denoted as *join* (24) and *meet* (25), respectively [23]. Karnik and Mendel [10], [22] provided equations for evaluating (24) and (25) for gaussian fuzzy numbers when maximum t-conorm and minimum or product t-norms are used.

$$F_1 \sqcup F_2 = \int_v \int_w \left[f_1(v) \bigstar f_2(w) \right] / (v \vee w) \tag{24}$$

$$F_1 \sqcap F_2 = \int_v \int_w \left[f_1(v) \bigstar f_2(w) \right] / (v \wedge w) \tag{25}$$

Therefore, to calculate $\tilde{B} = f(\tilde{A}_1,\ldots,\tilde{A}_n)$, first $f(x_1,\ldots, x_n)$ and $\mu_{\tilde{A}_1}(x_1) \sqcap \cdots \sqcap \mu_{\tilde{A}_r}(x_n)$, $\forall x_1 \in X_1,\ldots,\forall x_n \in X_n$ are computed. If more than one set of $x_1,\ldots, x_n$ satisfy $y = f(x_1,\ldots, x_n)$, then this procedure is repeated for all of them and their *join* is chosen as $\mu_{\tilde{B}}(y)$. This calculation is in fact a special case of the composition of a type-2 fuzzy set with a relation [10] (26), when the type-2 fuzzy set is the Cartesian product of n type-2 fuzzy sets (27) and the relation $\mu_{\tilde{R}}(\mathbf{x}, y)$ is a *crisp many-to-one function*. The analogous case for type-1 fuzzy sets is explained in [15].

$$\mu_{\tilde{B}}(y) = \mu_{\tilde{A}_{\mathbf{x}} \circ \tilde{R}}(y) = \bigsqcup_{\mathbf{x} \in \mathbf{X}} \left[\mu_{\tilde{A}_{\mathbf{x}}}(\mathbf{x}) \sqcap \mu_{\tilde{R}}(\mathbf{x}, y) \right] \tag{26}$$

$$\mu_{\tilde{A}_{\mathbf{x}}}(\mathbf{x}) = \mu_{\tilde{A}_1 \times \cdots \times \tilde{A}_n}(x_1,\ldots,x_n) = \mu_{\tilde{A}_1}(x_1) \sqcap \cdots \sqcap \mu_{\tilde{A}_n}(x_n) \tag{27}$$

The results of applying this procedure for evaluating a function of type-2 fuzzy numbers are shown in (28)-(31). Here, x_1 and x_2 are discrete type-2 fuzzy numbers (29)-(31). For simplicity only four elements of y are described analytically in (31), the other elements are shown graphically in Fig. 5. These results have been obtained by using maximum t-conorm and product t-norm.

$$y = f(x_1, x_2) = x_1 x_2 + 1 \tag{28}$$

$$\tilde{x}_1 = \frac{\frac{0.3}{0.4} + \frac{0.8}{0.5}}{1} + \frac{\frac{0.8}{0.9} + \frac{0.9}{1.0}}{2} + \frac{\frac{0.7}{0.6} + \frac{0.8}{0.7}}{3} \tag{29}$$

$$\tilde{x}_2 = \frac{\frac{0.7}{0.3} + \frac{1.0}{0.4}}{2} + \frac{\frac{0.5}{0.8} + \frac{0.8}{1.0}}{3} + \frac{\frac{0.6}{0.4} + \frac{0.7}{0.5}}{4} \tag{30}$$

$$\tilde{y} = \frac{\frac{0.21}{0.12} + \frac{0.56}{0.15} + \frac{0.3}{0.16} + \frac{0.8}{0.2}}{3} + \cdots + \frac{\frac{0.31}{0.27} + \frac{0.35}{0.3} + \frac{0.45}{0.36} + \frac{0.50}{0.4}}{5} + \frac{\frac{0.32}{0.72} + \frac{0.36}{0.8} + \frac{0.51}{0.9} + \frac{0.58}{1.0}}{7} + \cdots + \frac{\frac{0.42}{0.24} + \frac{0.48}{0.28} + \frac{0.49}{0.3} + \frac{0.56}{0.35}}{13} \tag{31}$$

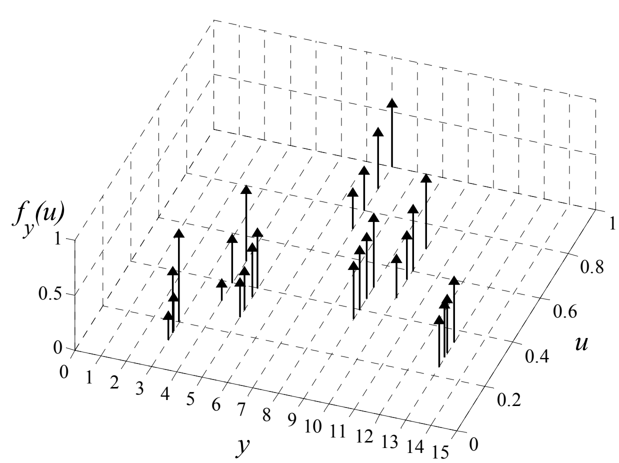

Fig. 5. Type-2 membership function of $y = x_1x_2+1$

C. Functions of interval type-2 fuzzy numbers

When an operation $f(x_1,\ldots,x_n)$ is extended to $f\left(\tilde{A}_1,\ldots,\tilde{A}_n\right)$, and the $\tilde{A}_i$ are interval type-2 fuzzy numbers, the calculation of the *join* and *meet* operations is simple. The *join* $\sqcup_{i=1}^{n} F_i$ of n interval type-1 sets $F_1,\ldots, F_n$, with domains $[l_1, r_1],\ldots,[l_n, r_n]$, respectively, is an interval set with domain $\left[\left(l_1 \vee l_2 \vee \cdots \vee l_n\right),\left(r_1 \vee r_2 \vee \cdots \vee r_n\right)\right]$, where $\vee$ denotes maximum. The *meet* $\sqcap_{i=1}^{n} F_i$ of n interval type-1 sets $F_1,\ldots, F_n$, having domains $[l_1, r_1],\ldots,[l_n, r_n]$, respectively, is an interval set with domain $\left[\left(l_1 \bigstar l_2 \bigstar \cdots \bigstar l_n\right),\left(r_1 \bigstar r_2 \bigstar \cdots \bigstar r_n\right)\right]$, where $\bigstar$ denotes either minimum or product t-norm [10]. From these definitions it is noted that the operations in (26) and (27) only imply calculations with the upper and lower membership functions of $\tilde{A}_i$, denoted as $\bar{\mu}_{\tilde{A}_i}(x_i)$ and $\underline{\mu}_{\tilde{A}_i}(x_i)$, respectively. Therefore, $\tilde{B}$ is also an interval type-2 fuzzy set (32), whose FOU can be calculated by means of (33) and (34).

$$\tilde{B}=\left\{\left(y,\mu_{\tilde{B}}(y)\right) \middle| y=f\left(x_1,\ldots,x_n\right),\left(x_1,\ldots,x_n\right)\in X\right\} \tag{32}$$

$$\bar{\mu}_{\tilde{B}}(y)=\bigvee_{y=f\left(x_1,\ldots,x_n\right)}\left\{\bar{\mu}_{\tilde{A}_1}\left(x_1\right)\bigstar\cdots\bigstar\bar{\mu}_{\tilde{A}_n}\left(x_n\right)\right\} \tag{33}$$

$$\underline{\mu}_{\tilde{B}}(y)=\bigvee_{y=f\left(x_1,\ldots,x_n\right)}\left\{\underline{\mu}_{\tilde{A}_1}\left(x_1\right)\bigstar\cdots\bigstar\underline{\mu}_{\tilde{A}_n}\left(x_n\right)\right\} \tag{34}$$

Then, to compute $\tilde{B}=f\left(\tilde{A}_1,\ldots,\tilde{A}_n\right)$, when $\tilde{A}_i$ are interval type-2 fuzzy numbers, it is not necessary to take into account the interval secondary grades. Only calculations with $\bar{\mu}_{\tilde{A}_i}(x_i)$ and $\underline{\mu}_{\tilde{A}_i}(x_i)$ are performed and after the interval secondary grades $\left(f_y(u)=1, \forall u\in J_y \subseteq [0,1], \forall y \in Y\right)$ are appended to $\mu_{\tilde{B}}(y)$. It is noted that (33) and (34) are equivalent to apply the extension principle, as stated in (22), to $\bar{\mu}_{\tilde{A}_i}(x_i)$ and $\underline{\mu}_{\tilde{A}_i}(x_i)$, respectively. Therefore, these calculations may be performed using the procedures proposed in the literature, e.g., the transformation method [1], the Fuzzy Weighted Averages (FWA) method [6] or the vertex method [7]. The latter one is used in this work due to its simplicity; its basic theory is explained next.

D. The vertex method

A disadvantage of the discretized form of the extension principle in propagating fuzziness for continuous-valued mappings is the irregular and erroneous membership functions determined for the output variable if the membership functions of the input variables are discretized for numerical convenience [3]. To overcome this difficulty, the vertex method [6] is used in this work to implement the extension principle. The vertex method is based on the combination of the α-cut concept and standard interval analysis. According to the resolution principle [13], [15] any continuous type-1 membership function can be represented by a continuous sweep of α-cut intervals from $\alpha = 0^+$ to $\alpha = 1$, where $I_\alpha = [a, b]$ is a generic α-cut as the one shown in Fig. 6.

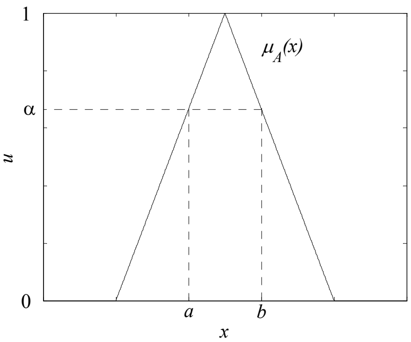

Fig. 6. A generic α-cut $I_\alpha = [a, b]$.

Suppose that a many-to-one mapping $y = f(x_1,\ldots, x_n)$ has to be extended for type-1 fuzzy sets $B = f(A_1,\ldots, A_n)$, then the input space can be represented by an n-dimensional Cartesian region. Each of the input variables can be described by an interval ($I_{i\alpha}$) at a specific α-cut (35).

$$I_{i\alpha}=\left[a_i,b_i\right] \quad i=1,\ldots,n \tag{35}$$

The endpoint pairs of each interval intersect in the n-dimensional space and form the vertices of the n-dimensional Cartesian space. The coordinates of these vertices are the values used in the vertex method to calculate the output interval for each α-cut. When the mapping $y = f(x_1,\ldots, x_n)$ is continuous in the n-dimensional Cartesian region and also there are no extreme points (maxima or minima of y) in this region or along the boundaries, the value of the interval function for a particular α-cut can be obtained by (36).

$$B_\alpha=f\left(I_{1\alpha},\ldots,I_{n\alpha}\right)=\left[\min_j\left(f\left(c_j\right)\right),\max_j\left(f\left(c_j\right)\right)\right] \tag{36}$$

Where $N = 2^n$ is the number of vertices (c_j) of the n-dimensional Cartesian region. Each vertex is defined by the values of a_i and b_i. For instance, when $n = 3$, there are eight vertices for each α-cut, with the following coordinates: $c_1 = (a_1, a_2, a_3)$, $c_2 = (a_1, a_2, b_3)$, $c_3 = (a_1, b_2, a_3)$, $c_4 = (a_1, b_2, b_3)$, $c_5 = (b_1, a_2, a_3)$, $c_6 = (b_1, a_2, b_3)$, $c_7 = (b_1, b_2, a_3)$, $c_8 = (b_1, b_2, b_3)$. When extreme points (maxima or minima of y) exist in the n-dimensional Cartesian region, they are identified and considered as additional vertices (E_k). Thus, when the mapping $y = f(x_1,\ldots, x_n)$ is continuous in the n-dimensional Cartesian region, the value of the interval function for a particular α-cut can be calculated by (37).

$$\begin{aligned}B_\alpha&=f\left(I_{1\alpha},\ldots,I_{n\alpha}\right)\\&=\left[\min_{j,k}\left(f\left(c_j\right),f\left(E_k\right)\right),\max_{j,k}\left(f\left(c_j\right),f\left(E_k\right)\right)\right]\end{aligned} \tag{37}$$

Where, c_j ($j = 1,\ldots,N$) is the coordinate of the j-th vertex representing the n-dimensional Cartesian region and E_k ($k = 1,\ldots,m$) is the coordinate of the k-th extreme point of $y = f(x_1,\ldots, x_n)$ identified in the n-dimensional Cartesian region (for

m extreme points). In the next section, the vertex method is used to implement the extension principle. This way (33) and (34) are calculated and the FFC concept is introduced.

IV. Fuzzy Fault Currents

The normal operation of an electric distribution system may be disturbed or disrupted due to a system fault when abnormally high currents flow through an abnormal path as a result of the partial or complete failure of insulation at one or more points of the system. The complete failure of insulation is called a "short circuit" or a "fault". A fault occurs on a distribution system when one or more energized conductors contact other conductors or ground [18]. Since they are mostly constructed of bare conductors, the overhead distribution lines are one of the most vulnerable points of distribution systems. A considerable amount of faults occurs in these lines, causing outages and economic losses to the utilities, as well as to the affected customers.

In order to reduce the effect of faults, protective devices are installed on distribution feeders. These devices are coordinated among them in such a way that only the device located closer to the fault must actuate when a fault current is detected. This way, the outage extension and total amount of affected customers is reduced. The starting point for coordinating protective devices is calculating fault currents. Unfortunately, the calculation of fault currents, particularly in distribution systems, strongly depends on uncertain variables, whose values are roughly known. If these uncertainties are not taken into account, coordination problems may occur in some situations. These problems can lead to incorrect actuations of protective devices, which cause extended outages and important economic losses.

The traditional approach for handling uncertainties in fault current calculations consists in computing maximum and minimum fault currents by using a bolted fault resistance (0Ω) and an assumed fault resistance (generally of 40Ω), respectively. This approach has proven to be problematic, since very few faults are bolted faults. Moreover the fault current levels produced by 40Ω faults are lower than the trip levels of many protective devices. This complicates designing an economical and flexible protection system [26].

In this work, in order to take into account the uncertainties associated to conventional fault current calculations, the concept of Fuzzy Fault Currents (FFC) is introduced. In the FFC calculation, the input variables (x_i) may be modeled as crisp numbers, type-1 and/or interval type-2 fuzzy numbers. Function $f(x_1,\ldots, x_n)$ is one of the classical fault equations and the output (y) is a FFC, which is calculated by applying the extension principle (32)-(34) and vertex method (37) to the upper and lower membership functions of input variables.

A. Fault resistance

The main advantage of the FFC concept is that it sets a mathematical framework for computing with uncertain variables. This is very useful for taking into account the uncertainty about fault resistance value (R_f), which is the most uncertain and influential of all variables involved in the calculation of fault currents in distribution systems. In this work, R_f is modelled as an interval type-2 gaussian fuzzy number with uncertain σ (Fig. 3). The parameters of $\tilde{R}_f$ model the usual range of values for fault resistance, which can be obtained from the analysis of the historical data of faults or from procedures based on expert knowledge (e.g., knowledge mining through surveys). Procedures for defining type-2 membership function on the basis of statistical data and expert knowledge are proposed in [4], [10], [27] and [28]. For instance, through the analysis of historical data of faults, the values $R_f \approx 15\Omega$, 20Ω and 25Ω and $R_f \approx [5\Omega, 25\Omega]$ have been suggested in [24] and [25], respectively. These values depend on the particularities of each distribution system; however it is worth noting that the data used to calculate them are generally scarce and uncertain. Thus, a type-2 fuzzy modelling of R_f is an adequate proposal for considering these uncertainties.

B. Pre-fault voltage

Another important variable involved in the calculation of fault currents is the pre-fault voltage (U_{pfk}). This variable generally is not monitored in real-time due to economic constraints, therefore only a rough estimation of its value is available. This estimation may be obtained by means of a power flow-based algorithm, like the one proposed in [29]. This algorithm iteratively tunes the estimated nodal demands of the feeder (the nodal demands also are not monitored in real-time) until the calculated power flow matches the measured power flow at the High Voltage/Medium Voltage Substation (HV/MV SE). The voltages of the tuned nodal demands are the pre-fault voltages of the feeder. The estimated nodal demands are obtained from the analysis of typical load curves, power factors and monthly (or bimonthly) energy consumptions of customers. The algorithm first calculates the losses and initial nodal voltages of the feeder by means of power flow software. The calculated nodal voltages are used to tune the estimated nodal demands. This is done in order to take into account the dependency of load with respect to voltage. In the next iteration, the tuned nodal demands are used to recalculate the losses and nodal voltages of the feeder, and then the latter are used to retune the nodal demands. This procedure is repeated until a convergence condition is reached, this way the consistency among the calculated nodal demands, calculated voltages and the available measured values is guaranteed. However, it is noted that the calculated values are only approximations of the real nodal demands and voltages. In this work, in order to take into account this uncertainty, the pre-fault voltages are modelled as interval type-2 gaussian fuzzy numbers with uncertain σ ($\tilde{U}_{pfk}$). The mean (m) of $\tilde{U}_{pfk}$ are the calculated pre-fault voltages (U_{pfk}) and its standard deviations (σ_1, σ_2) are chosen on the basis of expert knowledge and quality of service regulations, e.g., according to power quality standards, voltage levels must stay in a ± 10% range (or ± 5% in some countries) with respect to nominal voltage.

C. Fault current equations

The most common fault in distribution systems is the single line-to-ground fault [17], its fault current equation (38) is used in the next section to illustrate the application of the FFC concept. Nevertheless, for this analysis, other fault current equations (line-to-line fault, three-phase fault, etc) can be used as well. In (38), I_{fk} is the magnitude of the fault current for a single line-to-ground fault at point k and U_{pfk} is the corresponding pre-fault voltage. Moreover, (R_k^0, X_k^0) are the equivalent zero sequence impedances and (R_k^+, X_k^+) are the equivalent positive sequence impedances at point k. Here, an approximate equation –based on the principle of superposition– is used to take into account the pre-fault load current contribution (39). This equation is based on the assumption that load current remains constant during the fault, i.e., that pre-fault and post-fault load current are the same. Thus, the total fault current (I_{fk}) is equal to the sum of two components: I_{fk}, which is calculated without considering load and only depends on network impedances and fault resistance, and the pre-fault load current (I_{pf}), which depends on load characteristics. It is worth noting that I_{pf} also can be modeled as a fuzzy number. However, for simplicity and given than usually $I_{fk} \gg I_{pf}$, in this work I_{pf} is modeled as a crisp number.

$$I_{fk} = 3U_{pfk} \Big/ \sqrt{\left(2R_k^+ + R_k^0 + 3R_f\right)^2 + \left(2X_k^+ + X_k^0\right)^2} \tag{38}$$

$$I_k = I_{fk} + I_{pf} \tag{39}$$

V. Results

The FFC approach has been tested on a real distribution feeder of the city of San Juan, Argentina (Fig. 7). This feeder has a nominal voltage level of 13.2 kV, 172 nodes and 55 protective devices. The type-1 and type-2 FFC of n_1-n_4 are calculated with the parameters of Tables I-II, the pre-fault voltages (U_{pfk}) and pre-fault load current (I_{pf} = 53.8 A) are computed using the algorithm proposed in [29].

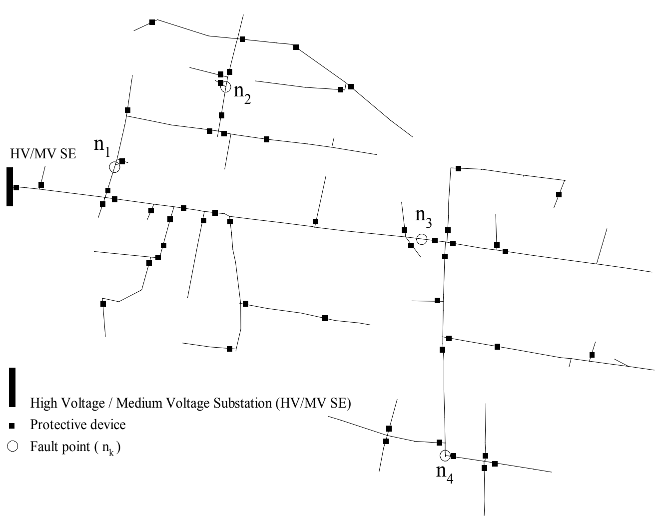

Fig. 7. Test feeder.

Table I

n_k	U_{pfk} (V)	R_k^0 (Ω)	X_k^0 (Ω)	R_k^+ (Ω)	X_k^+ (Ω)
n_1	7505.9	1.872	4.229	1.557	0.953
n_2	7486.1	5.226	10.895	4.392	2.470
n_3	7449.6	6.424	12.707	5.938	2.765
n_4	7437.4	13.153	23.533	12.836	5.020

Table II

Variable	Type-1		Type-2		
	m	σ	m	σ_1	σ_2
R_f	15Ω	4Ω	15Ω	3Ω	5Ω
U_{pfk}	U_{pfk}	$0.04{\cdot}U_{pfk}$	U_{pfk}	$0.02{\cdot}U_{pfk}$	$0.06{\cdot}U_{pfk}$

A. Type-1 Fuzzy Fault Currents

In order to show the advantages of the proposed approach, first type-1 FFC are calculated using the extension principle (21), (22) and the vertex method (37), then these results are compared with the ones obtained using type-2 FFC, and finally conclusions are drawn.

In Fig. 8, the type-1 membership functions with solid lines (Case 1) have been calculated by modeling R_f as a type-1 gaussian fuzzy number and U_{pfk} as a crisp number. The type-1 membership functions with dashed lines (Case 2) have been calculated by modeling both variables as type-1 gaussian fuzzy numbers. It is noted that in Case 2 the FFC are slightly more uncertain than in Case 1, this is expected so, because the calculation have been made using two uncertain variables. Moreover, the effect of the uncertainties associated to R_f and U_{pfk} becomes evident for points located close to the HV/MV SE, where the membership functions are broader than the ones corresponding to points located far. This is because the influence of R_f is greater in these sites than in points located far, where the feeder impedance is also significant.

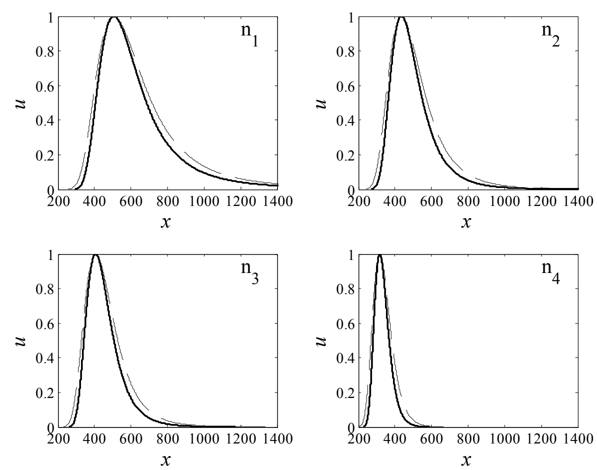

Fig. 8. Type-1 FFC for Case1 (solid lines) and Case 2 (dashed lines).

The effect of modeling R_f and U_{pfk} as type-1 fuzzy numbers can be noted in Table III, where the defuzzified values of the type-1 FFC of Case 1 ($I_{k\ Case\ 1}$) and Case 2 ($I_{k\ Case\ 2}$) are compared with the results obtained through conventional deterministic (crisp) calculations ($I_{k\ crisp}$). These defuzzified values have been obtained through type-1 centroid defuzzification. In Table III it is noted that $I_{k\ Case\ 1}$ and $I_{k\ Case\ 2}$, differ from $I_{k\ crisp}$. This effect is caused by the uncertainties associated to R_f and U_{pfk}. Also it is noted that if more uncertainties are taken into account, more different is the defuzzified value of I_k with respect to its crisp alternative. These effects are observed graphically in Fig. 9 where the three cases have been plotted for node n_2.

Table III

n_k	I_k		
	$I_{k\ crisp}$	$I_{k\ Case\ 1}$	$I_{k\ Case\ 2}$
n_1	506.34	536.24	541.21
n_2	435.19	452.02	455.38
n_3	406.49	419.74	422.60
n_4	316.70	321.90	323.42

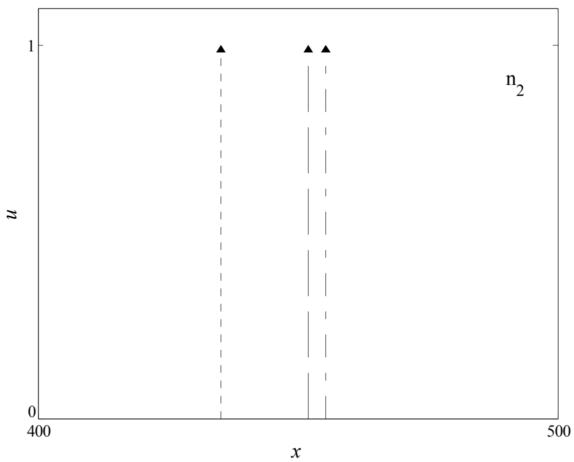

Fig. 9. Comparison of the defuzzified values of I_k for Case 1 (dashed line) and Case 2 (dashed-dotted line) with respect to the crisp value of I_k obtained through conventional deterministic calculations (dotted line).

B. Type-2 Fuzzy Fault Currents

In Fig. 10, the FOU with dark shading (Case 3) have been calculated by modeling R_f as an interval type-2 gaussian fuzzy number and U_{pfk} as a crisp number. The FOU with light shading (Case 4) have been calculated by modeling both variables as interval type-2 gaussian fuzzy numbers. These results show that fault location also affects the FOU of type-2 FFC, which are broader for points located closer to the HV/MV SE than for points located far. Moreover, the FOU of Case 4 are more uncertain than the FOU of Case 3, this effect is a consequence of modeling the uncertainties of both variables (Case 4) instead of considering only one (Case 3).

Similar conclusions can be drawn by analyzing Fig. 11, where the type-reduced sets of the type-2 FFC of Fig. 10 are shown. The type-reduced sets are a measure of the effect that input variables uncertainties have on the output variable (I_k). These type-1 intervals may be used for protection coordination, instead of using the values derived from conventional deterministic and type-1 fuzzy calculations (Fig. 9). This way, the uncertainties associated to R_f and U_{pfk} are taken into account. In Fig. 11, it is noted that the type-reduced sets ($[I_{kl}, I_{kr}]$) of Case 4 are more uncertain than the type-reduced sets of Case 3. Furthermore, when the domains of the type-reduced sets are analyzed, the effect of the uncertainties associated to R_f and U_{pfk} becomes evident. The domains are wide for points located close to the HV/MV SE and are narrow for points located far. Both conclusions have been obtained by analyzing Fig. 10, but, they turn out to be clear by inspecting Fig. 11.

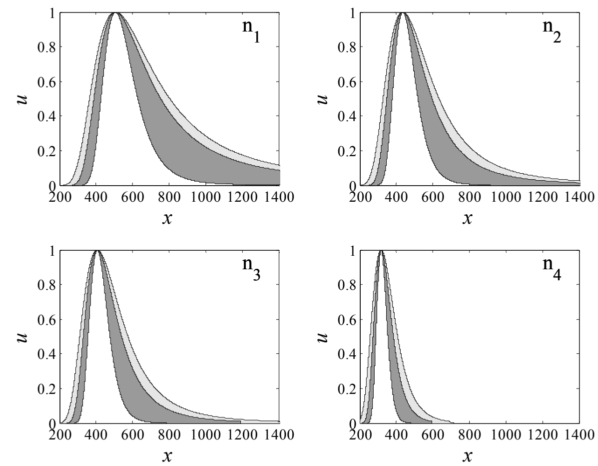

Fig. 10. Type-2 FFC for Case 3 (dark FOU) and Case 4 (light FOU).

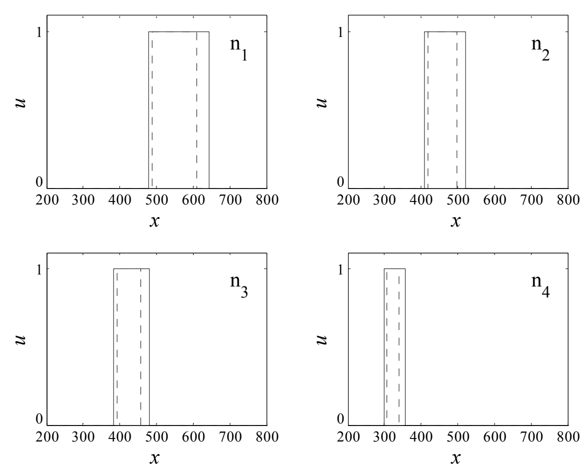

Fig. 11. Type-reduced sets of the type-2 FFC of Fig. 10: Case 3 (dashed line), Case 4 (solid line).

C. Comparison of crisp, type-1 and type-2 Fuzzy Fault Currents

The type-reduced sets of Fig. 11 have been calculated by using centroid type reduction. Here is important to highlight that the type-reduced set is the union of the centroids of all embedded type-1 fuzzy sets of the original type-2 fuzzy set. For instance, by analyzing Table II and Fig. 12 it can be seen that the type-1 FFC of Fig. 8 are only one of the uncountable number of embedded type-1 fuzzy sets of the type-2 FFC of Fig. 10. Thus, the defuzzified values of Fig. 9 are only one of the centroids that compose the type-reduced sets of Fig. 11. Furthermore, in Fig. 12 it is noted that the lower and upper membership functions also are only two of the uncountable number of embedded type-1 fuzzy sets of the type-2 FFC. Therefore, the defuzzified values of these membership functions not necessarily coincide with the leftmost (I_{kl}) and rightmost (I_{kr}) limits of the type-reduced set of the type-2 FFC. This can be noted in Table IV, where the type-reduced sets of Cases 3 and 4 are compared with the defuzzified values of their lower and upper membership functions. Also, this is shown graphically in Fig. 13, for the type-2 FFC of n_3. From these results it can be concluded that performing one type-2

FFC calculation is equivalent to executing an uncountable number of type-1 FFC computations. These results show that the type-2 FFC approach outperforms conventional crisp and type-1 fuzzy calculations.

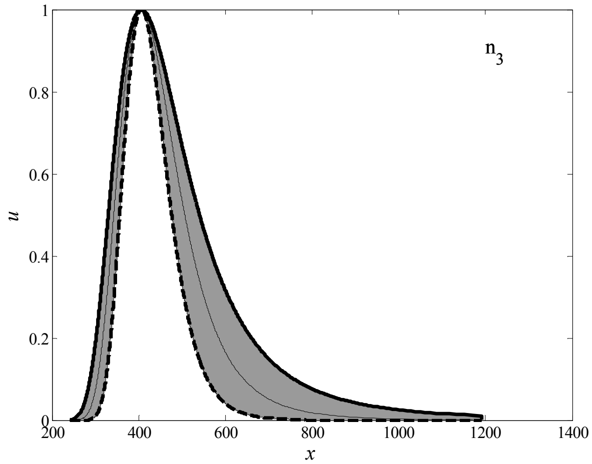

Fig. 12. Type-1 FFC of n_3 (thin line) and lower (dashed line) and upper (thick line) membership functions of the type-2 FFC of n_3.

Table IV

n_k	Lower / Upper MF		Type-2 (Case 3)		Type-2 (Case 4)	
	$I_{k\ lower}$	$I_{k\ upper}$	I_{kl}	I_{kr}	I_{kl}	I_{kr}
n_1	522.14	557.89	487.95	609.57	477.61	643.10
n_2	444.29	462.96	418.70	497.79	408.97	522.54
n_3	413.71	428.16	391.28	457.75	381.95	479.72
n_4	319.58	325.00	306.32	340.96	298.49	355.38

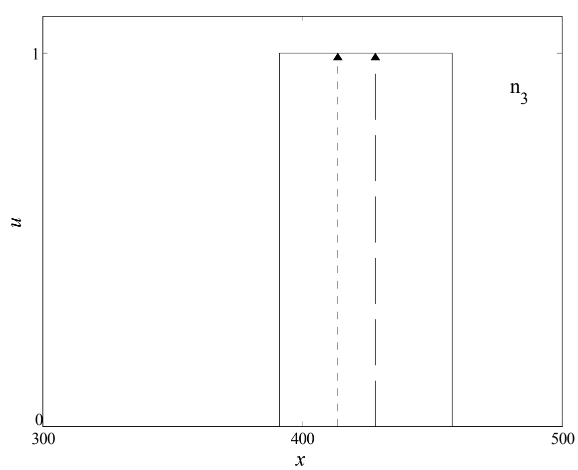

Fig. 13. Type-reduced set of the type-2 FFC of n_3 (solid line) and defuzzified values of its lower (dotted line) and upper (dashed line) membership functions.

Finally, in Table V and Fig. 14 a comparison among the values of I_k obtained through deterministic (crisp), type-1 and type-2 fuzzy approaches is presented. The values of the fourth column of Table V are the mean of the type-reduced set of the type-2 FFC, i.e., the defuzzified values of I_k for Case 3 (40).

$$I_k = \frac{I_{kl} + I_{kr}}{2} \tag{40}$$

Table V

n_k	I_k		
	$I_{k\ crisp}$	$I_{k\ type\ 1\ (Case\ 1)}$	$I_{k\ type\ 2\ (Case\ 3)}$
n_1	506.34	536.24	548.76
n_2	435.19	452.02	458.24
n_3	406.49	419.74	424.52
n_4	316.70	321.90	323.64

In Fig. 14, the dominion of the type-reduced set covers both, the crisp value of I_k obtained through conventional deterministic calculations and also the defuzzified value of I_k obtained trough type-1 FFC calculations. From these results it can be concluded that in type-2 FFC calculations, the uncertainties of input variables flow along the computations and their effect on the defuzzified output is measured by the type-reduced set. This can not be achieved through conventional deterministic or type-1 fuzzy approaches. The type-reduced set provides a measure of the uncertainties of the defuzzified output; in the same way that standard deviation provides a measure of the uncertainties of the mean in probability-based modeling.

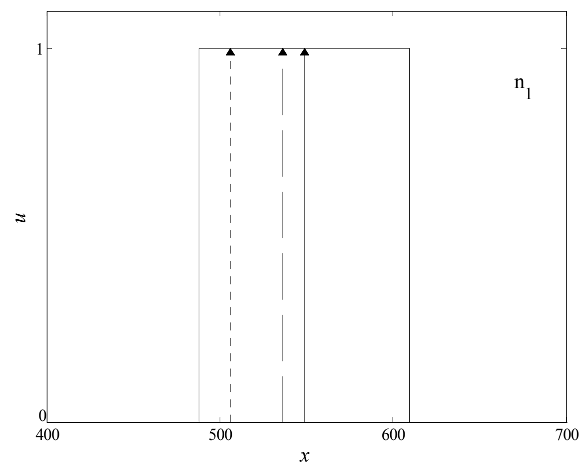

Fig. 14. Comparison of the defuzzified values of I_k for Case 1 (dashed line) and Case 3 (solid line) with respect to the crisp value of I_k obtained through conventional deterministic calculations (dotted line).

VI. Conclusions

In this work, in order to take into account the uncertainties associated to conventional fault current calculation, an approach aiming at computing functions of interval type-2 fuzzy numbers has been presented. This approach sets a methodological framework to calculate with uncertain variables, allowing the introduction of the FFC concept, which is used to model the uncertain data and expert knowledge typically available in distribution systems. The flexibility of the proposed approach allows modeling the variables of interest as crisp, type-1 and/or interval type-2 fuzzy numbers, depending on the nature of the uncertainties of each variable. It is important to highlight that interval type-2 fuzzy sets can be interpreted as a collection of an uncountable number of embedded type-1 fuzzy sets. Therefore, the type-reduced set of an interval type-2 fuzzy set can be thought as the union of the defuzzified outputs of all their embedded type-1 fuzzy sets. Through this important feature, the proposed type-2 FFC calculation takes into account the uncertainties associated to R_f and U_{pf}, obtaining a range (instead of a crisp number) of possible values for fault currents (I_k). These results provide a more significant assessment for further applications (e.g., coordination of protective devices), than the results obtained by means of conventional deterministic methodologies or type-1 fuzzy approaches. Moreover, interval type-2 fuzzy numbers have the advantage to be general enough to facilitate the modeling of the available approximate knowledge. The results obtained by applying the proposed approach to a real distribution system are satisfactory and are encouraging for future engineering applications of type-2 fuzzy sets theory.

REFERENCES

[1] M. Hanss, "The transformation method for the simulation and analysis of systems with uncertain parameters", *Fuzzy Sets and Systems*, Vol. 130, No. 3, pp. 227-289, Sept. 2002.

[2] I.B. Türksen, "Measurement of membership functions and their acquisition", *Fuzzy Sets and Systems*, Vol. 40, No. 1, pp. 5-38, Mar. 1991.

[3] T.J. Ross, *Fuzzy Logic with Engineering Applications*, 2nd Ed., John Wiley and Sons, USA, 2004.

[4] I.B. Türksen, "Type 2 representation and reasoning for CWW", *Fuzzy Sets and Systems*, Vol. 127, No. 1, pp. 17-36, Apr. 2000.

[5] J.M. Mendel, R.I. John, "Type-2 fuzzy sets made simple", *IEEE Trans. Fuzzy Systems*, Vol. 10, No. 2, pp. 117-127, Apr. 2002.

[6] W.M. Dong, F.S. Wong, "Fuzzy weighted averages and implementation of the extension principle", *Fuzzy Sets and Systems*, Vol. 21, No. 2, pp. 183-199, Feb. 1987.

[7] W. Dong, H. Shah, "Vertex method for computing functions of fuzzy variables", *Fuzzy Sets and Systems*, Vol. 24, No. 1, pp. 65-78, Oct. 1987.

[8] H.Q. Yang, H. Yao, J.D. Jones, "Calculating functions of fuzzy numbers", *Fuzzy Sets and Systems*, Vol. 55, No. 3, pp. 273-283, May 1993.

[9] H.Q. Yang, Z. Yang, "Calculating inverse functions of fuzzy numbers", *Applied Mathematical Letters*, Vol. 8, No. 3, pp. 43-48, May 1995.

[10] J.M. Mendel, *Uncertain Rule-Based Fuzzy Logic Systems. Introduction and New Directions*, Prentice Hall PTR, USA, 2001.

[11] H.-J. Zimmermann, *Fuzzy Set Theory and its Applications*, Kluwer Academic Publishers, USA, 1996.

[12] D. Dubois, H. Prade, *Fuzzy Sets and Systems*, Academic Press, Inc., USA, 1980.

[13] L.H. Tsoukalas, R.E. Urigh, *Fuzzy and Neural Approaches in Engineering*, John Wiley and Sons, Inc., USA, 1997.

[14] A. Kaufmann, M.M. Gupta, *Introduction to Fuzzy Arithmetic. Theory and Application*, Van Nostrand Reinhold Company Inc., USA, 1985.

[15] J.-S.R. Jang, C.-T. Sun, E. Mizutani, *Neuro-Fuzzy and Soft Computing*, Prentice Hall, USA, 1997.

[16] M. Wagenknecht, K. Hartmann, "Application of fuzzy sets of type 2 to the solution of fuzzy equation systems", *Fuzzy Sets and Systems*, Vol. 25, No. 2, pp. 183-190, Feb. 1988.

[17] T. Gönen, *Electric Power Distribution System Engineering*, McGraw-Hill Inc., USA, 1986.

[18] T. Gönen, *Modern Power System Analysis*, John Wiley and Sons, USA, 1988.

[19] N.N. Karnik, J.M. Mendel, "Centroid of a type-2 fuzzy set", *Information Sciences*, Vol. 132, No. 1-4, pp. 195-220, Feb. 2001.

[20] S. Coupland, R. John, "An approach to type-2 fuzzy arithmetic", in *Proc. UK Workshop on Computational Intelligence*, pp. 107-114, Sep. 2003.

[21] L.A. Zadeh, "The concept of a linguistic variable and its applications to approximate reasoning-I", *Information Sciences*, Vol. 8, No. 3, pp. 199-249, 1975.

[22] N.N. Karnik, J.M. Mendel, "Operations on type-2 fuzzy sets", *Fuzzy Sets and Systems*, Vol. 122, No. 2, pp. 327-348, Sept. 2001.

[23] M. Mizumoto, K. Tanaka, "Some properties of fuzzy sets of type-2", *Information and Control*, Vol. 31, No. 4, pp. 312-340, Aug. 1976.

[24] R. Rosés, D. Castro, "Estimación de la localización de protecciones accionadas en sistemas de distribución con sistemas de telemedida limitado", in *Proc. Congreso Internacional de Distribución Eléctrica, CIDEL Argentina 2002*, Paper No. 2.2.13, Dic. 2002.

[25] *Electrical Distribution-System Protection*, 3rd Ed., Cooper Power Systems, 1990.

[26] J. Dagenhart, "The 40-Ω ground fault phenomenon", *IEEE Trans. Industry Applications*, Vol. 36, No. 1, pp. 30-32, Jan./Feb. 2000.

[27] J.M. Mendel, "Fuzzy sets for words: a new beginning", in *Proc. of IEEE International Conference on Fuzzy Systems*, Vol. 1, pp.37-42, May 2003.

[28] Q. Liang, N.N. Karnik, J.M. Mendel, "Connection admission control in ATM networks using survey-based type-2 fuzzy logic systems", IEE Trans. on Systems, Man and Cybernetics, Part C, Vol. 30, No. 3, pp. 329-339, Aug. 2000.

[29] J. Romero Agüero, D. Hirshmann, A. Vargas, "Inference of the operative configuration of medium voltage distribution networks", in *Proc. of IEEE 2003 Bologna Power Tech Conference*, Vol. 3, pp. 562-569, Jun. 2003.

Julio Romero Agüero (S'00–M'05) received the Electrical Engineer degree in 1996, from Universidad Nacional Autónoma de Honduras (UNAH). He was part of Empresa Nacional de Energía Eléctrica (ENEE), Honduras from 1995 to 2000. In 2000 he was awarded with a scholarship from the German Academic Exchange Service (DAAD) for pursuing postgraduate studies at Instituto de Energía Eléctrica, Universidad Nacional de San Juan (IEE-UNSJ), Argentina, where he received the Ph.D. degree in Electrical Engineering in 2005 (with honors). Dr. Romero Agüero is a member of the IEEE Power Engineering Society, IEEE Industry Applications Society and IEEE Computational Intelligence Society.

Alberto Vargas (M'97–SM'02) received the Electromechanical Engineer degree in 1975 from Universidad Nacional de Cuyo, Mendoza, Argentina and the Ph.D. degree in Electrical Engineering in 2001 from the Universidad Nacional de San Juan (IEE-UNSJ), San Juan, Argentina. Currently, he is a Postgraduate Professor at the Instituto de Energía Eléctrica, IEE-UNSJ. Since 1985, he has been the Chief Researcher of the Electrical Market Team at IEE-UNSJ. He is a Consulting Program Manager of ASINELSA S.A, a specialized software company for electric distribution development dealing with Electrical AM/FM GIS and DMS applications. Dr. Vargas is a member of the IEEE Power Engineering Society.